Es 3

$p = 0,7 \qquad q = 1-p = 0,3$

a) $P_5(2) = \binom{5}{2} p^2 q^3 = \frac{5!}{3!\,2!}(0,7)^2(0,3)^3 = 10 \cdot 0,7^2 0,3^3 = 0,13$

b) $P\{\text{almeno 3 valide}\} = P_5(3) + P_5(4) + P_5(5) = \binom{5}{3}(0,7)^3\, 0,3^2 + \binom{5}{4}\, 0,7^4\, 0,3 + \binom{5}{5}\, 0,7^5 =$

$= \frac{5!}{3!\,2!}\, 0,7^3\, 0,3^2 + \frac{5!}{4!\,1!}\, 0,7^4\, 0,3 + 0,7^5 = 10 \cdot 0,7^3 \cdot 0,3^2 + 5 \cdot 0,7^4 \cdot 0,3 + 0,7^5 = 0,837$

$\boxed{N.B.}$ $P\{X \geq x_i\} = \sum_{X \geq x_i} p_i = P_5(3) + P_5(4) + P_5(5)$

$P\{X \geq x_i\} = 1 - P\{X < x_i\} = 1 - \sum_{X < x_i} p_i =$

$= 1 - P_5(1) - P_5(2)$

Es. 4

$\text{Prev.} = 1/20 = 0.05$

$\text{spec} = 0,99$

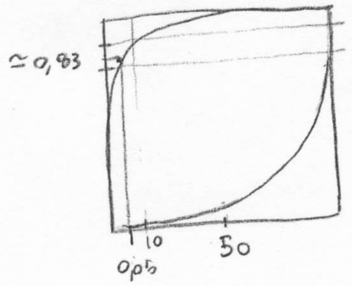
$$P_{m,tp} = \frac{\text{sens} \cdot \text{prev}}{\text{sens} \cdot \text{prev} + (1-\text{spec}) \cdot (1-\text{prev})}$$

$$0,83 = \frac{\text{sens} \cdot 0,05}{\text{sens} \cdot 0,05 + 0,01 \cdot 0,95}$$

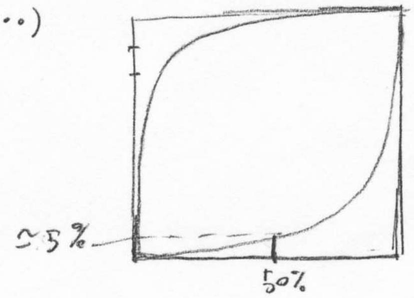
$0,83 \cdot 0,05 \cdot \text{sens} + 0,83 \cdot 0,01 \cdot 0,95 = \text{sens} \cdot 0,05$

$$\text{sens} = \frac{0,83 \cdot 0,01 \cdot 0,95}{0,17 \cdot 0,05} = 0,93$$

.) il grafico corretto è il b, infatti



..)



Es. 5.

$$S_n = \frac{1+(-1)^n}{n^2} e^{-j\frac{\pi n}{2}}$$

a) $s(t) \in \mathbb{R} \Leftrightarrow S_n = S_{-n}^*$

$$S_{-n} = \frac{1+(-1)^{-n}}{(-n)^2} e^{j\frac{\pi n}{2}} = \frac{1+(-1)^n}{n^2} e^{j\frac{\pi n}{2}} = S_n^* \Rightarrow s(t) \text{ reale}$$

b) $s(t)$ pari $\Leftrightarrow S_n = S_{-n}$

$s(t)$ dispari $\Leftrightarrow S_n = -S_{-n}$

$S_{-n} = \frac{1+(-1)^n}{n^2} e^{j\frac{\pi n}{2}}$ si nota che per n dispari $S_n = 0$ e $S_{-n} = 0$ $\Rightarrow S_n = S_{-n}$, $S_n = -S_{-n}$ per n dispari

per n pari

$$S_n = \frac{2}{n^2} e^{-j\frac{\pi n}{2}}$$

$$S_{-n} = \frac{2}{n^2} e^{j\frac{\pi n}{2}}$$

n pari si può scrivere come

$n = 2m$ con $m \in \mathbb{Z}$

$$S_n = \frac{2}{n^2} e^{-j\frac{\pi 2m}{2}} = \frac{2}{n^2} e^{-j\pi m} \qquad S_{-n} = \frac{2}{n^2} e^{j\pi m}$$

se $m \in \mathbb{Z}$ $e^{-j\pi m} = e^{j\pi m}$ quindi $S_{-n} = S_n$ se n pari

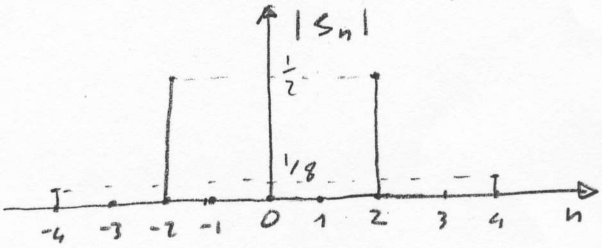
in complesso considerando n pari o dispari si ha $S_n = S_{-n} \Rightarrow s(t)$ è pari

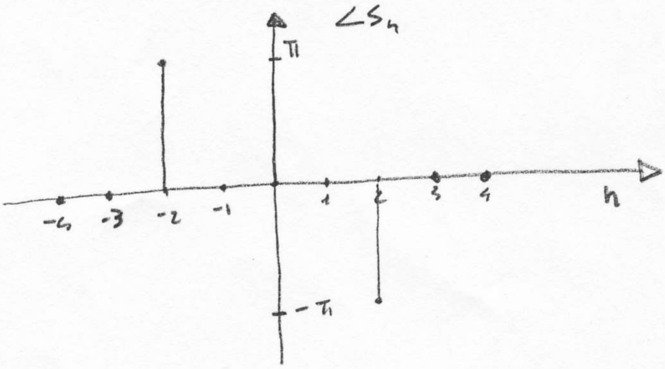
c) $S_1 = 0 \qquad S_{-1} = 0$

$S_2 = \frac{2}{4} e^{-j\pi} = -\frac{1}{2} \qquad S_{-2} = -\frac{1}{2}$

$S_3 = 0 \quad S_{-3} = 0$

$S_4 = \frac{2}{16} e^{-j2\pi} = \frac{1}{8} \quad S_{-4} = \frac{2}{16} e^{j2\pi} = \frac{1}{8}$





cont.
↓

↓ segue

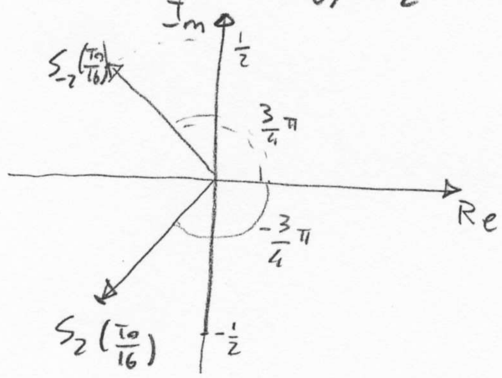
Es. 5 d)

$$S_2(t) = S_2\, e^{j4\pi \frac{t}{T_0}} = -\frac{1}{2}\, e^{j4\pi \frac{t}{T_0}}$$

$$S_{-2}(t) = S_{-2}\, e^{-j4\pi \frac{t}{T_0}} = -\frac{1}{2}\, e^{-j4\pi \frac{t}{T_0}}$$

$$S_2\left(\frac{T_0}{16}\right) = -\frac{1}{2}\, e^{j\frac{\pi}{4}} = \frac{1}{2}\, e^{-j\pi}\, e^{j\frac{\pi}{4}} = \frac{1}{2}\, e^{-j\frac{3}{4}\pi}$$

$$S_{-2}\left(\frac{T_0}{16}\right) = -\frac{1}{2}\, e^{-j\frac{\pi}{4}} = -\frac{1}{2}\, e^{j\pi}\, e^{-j\frac{\pi}{4}} = \frac{1}{2}\, e^{j\frac{3}{4}\pi}$$



Es 6.

a) $y(t) = t\, x(t-t_0)$

lineare?

$x_1(t)$ $\quad y_1(t) = t\, x_1(t-t_0)$

$x_2(t)$ $\quad y_2(t) = t\, x_2(t-t_0)$

$x_3(t) = a\, x_1(t) + b\, x_2(t)$

$$y_3(t) = t\, x_3(t) = t\,(a x_1(t) + b x_2(t)) =$$
$$= a t x_1(t) + b t x_2(t) = a\, y_1(t) + b\, y_2(t)$$
$$\Downarrow$$

sist. lineare.

tempo invariante?

$x_1(t)$ $\quad y_1(t) = t\, x_1(t-t_0)$

$x_2(t) = x_1(t - T_A)$ $\quad y_2(t) = t\, x_2(t) = t\, x_1(t-T_A) \neq y_1(t-T_A) = (t-T_A)\, x_1(t-T_A-$

non tempo invariante

b) $y(t) = t\sqrt{x(t)}$

lineare?

$x_1(t)$ $\quad y_1(t) = t\sqrt{x_1(t)}$ $\quad x_2(t) = t\sqrt{x_2(t)}$

$x_3(t) = a\, x_1(t) + b\, x_2(t)$ $\quad y_3(t) = t\sqrt{x_3(t)} = t\sqrt{a x_1(t) + b x_2(t)} \neq a\, y_1(t) + b\, y_2(t)$

non lineare

tempo invariante?

$x_1(t)$ $\quad y_1(t) = t\sqrt{x_1(t)}$

$x_2(t) = x_1(t-T_A)$ $\quad y_2(t) = t\sqrt{x_2(t)} = t\sqrt{x_1(t-T_A)} \neq y_1(t-T_A) = (t-T_A)\sqrt{x_1(t-}$

non tempo invariant.

ES.7

$f_{min} = 13\,KHz \quad f_{max} = 20\,KHz$ T secondi lunghezza

I) PASSA BASSO $B = f_{max} - 0 = f_{max}$

$$f_c \geq 2 f_{max} = 40\,KHz$$

II) PASSA BANDA

$$B = f_{max} - f_{min} = 7\,KHz$$

$$\frac{f_{max}}{B} = \frac{20\,KHz}{7\,KHz} = 2.86 \Rightarrow m = 2 \qquad f_c = \frac{2 f_{max}}{m} = f_{max} = 20\,KHz$$

$$f_{c\,min} = 20\,KHz$$

In uscita al filtro abbiamo $N_{out} = N + 100 - 1$ campioni, con $\Delta T = \frac{1}{f_c} = \frac{1}{20\,KHz}$

N è il numero di campioni in ingresso al filtro, pari a $N = T \cdot f_c = T \cdot 20\,KHz$

$$\Delta f = \frac{1}{N_{out} \cdot \Delta T} = \frac{1}{(N+99)\Delta T} = \frac{20\,KHz}{N+99} = 1\,Hz$$

$$N = 20000 - 99 = 19901$$

$$T = N \Delta T = \frac{19901}{20000\,Hz} = 0{,}995\ s$$