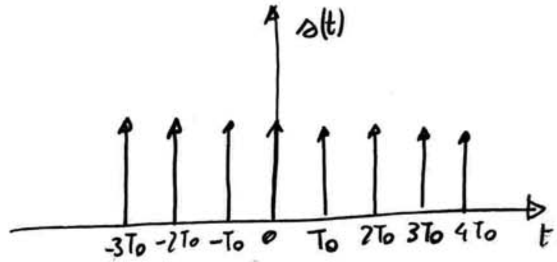
Lezione 11/11/2005

Riprendiamo il treno di Dirac

$$s(t) = rep_{T_0}(t) = \sum_{n=-\infty}^{+\infty} \delta(t - nT_0)$$



$$S_n = \frac{1}{T_0}\int_{t_0}^{t_0+T_0/2} \sum_{n=-\infty}^{+\infty} \delta(t-nT_0)\, e^{-j\frac{2\pi n t}{T_0}} dt = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} \delta(t)\, e^{-j\frac{2\pi n t}{T_0}} dt = \frac{1}{T_0}$$

$$s(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_0) = \frac{1}{T_0}\sum_{n=-\infty}^{\infty} e^{j\frac{2\pi n t}{T_0}}$$

→ $s(t) = A\cos(2\pi f_0 t)$ $\qquad T_0 = 1/f_0$

$$S_n = \frac{1}{T_0}\int_0^{T_0} A\cos(2\pi f_0 t)\, e^{-j2\pi n f_0 t} dt = \frac{1}{T_0}\int_0^{T_0} A\left(\frac{e^{j2\pi f_0 t} + e^{-j2\pi f_0 t}}{2}\right) e^{-j2\pi f_0 n t} dt =$$

$$= \frac{A}{2T_0}\int_0^{T_0}\left[e^{-j2\pi(n-1)f_0 t} + e^{-j2\pi(n+1)f_0 t}\right] dt = \frac{a}{2T_0}\left[\frac{e^{-j2\pi(n-1)} - 1}{-j2\pi(n-1)f_0} + \frac{e^{-j2\pi(n+1)} - 1}{-j2\pi(n+1)f_0}\right]$$

per $n \neq 1, n \neq -1$ $\quad S_n = 0$

per $n = 1$ o $n = -1$ si ha $0/0$, quindi si riparte da

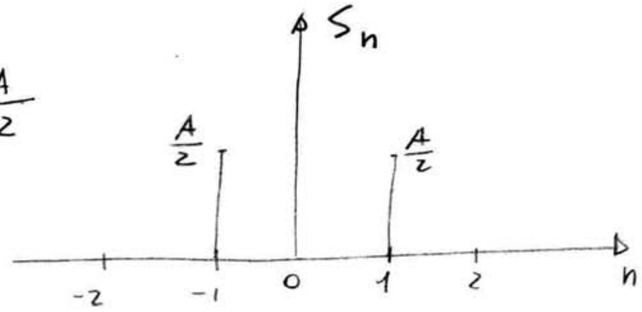
$$S_n = \frac{A}{2T_0}\int_0^{T_0}\left[e^{-j2\pi(n-1)f_0 t} + e^{-j2\pi(n+1)f_0 t}\right] dt$$

per $n = 1$

$$S_1 = \frac{A}{2T_0}\int_0^{T_0}\left[1 + e^{-j4\pi f_0 t}\right] dt = \frac{A}{2}$$

per $n = -1$

$$S_{-1} = \frac{A}{2}$$



→ $s(t) = A \sin(2\pi f_0 t)$

$$s(t) = A \sin(2\pi f_0 t) = A \cos(2\pi f_0 t - \pi/2)$$

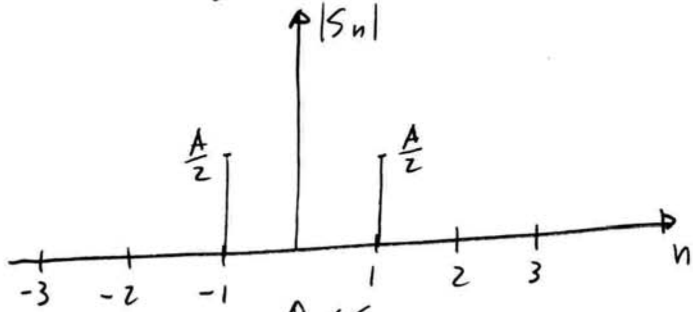
Si utilizza la Formula di Eulero

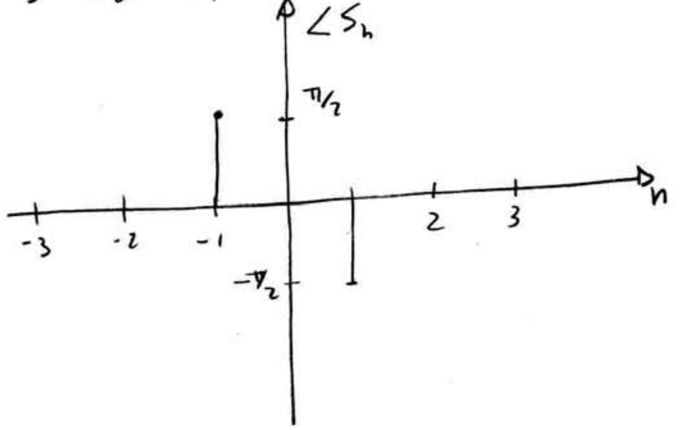
$$s(t) = A \, \frac{e^{j(2\pi f_0 t - \pi/2)} + e^{-j(2\pi f_0 t - \frac{\pi}{2})}}{2} = \frac{A}{2} e^{-j\pi/2} e^{j2\pi f_0 t} + \frac{A}{2} e^{j\pi/2} e^{-j2\pi f_0 t}$$

se confrontiamo con $s(t) = \sum_{n=-\infty}^{+\infty} S_n \, e^{j2\pi n f_0 t}$

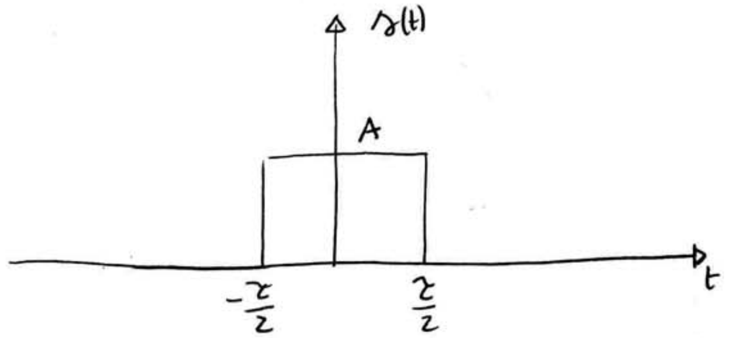
Si possono dedurre i coeff. dello sviluppo in serie di Fourier

$$S_{-1} = \frac{A}{2} e^{j\pi/2} \qquad S_1 = \frac{A}{2} e^{-j\pi/2}$$





Trasformata Continua di Fourier



$$s(t) = A\,\mathrm{rect}\left(\frac{t}{\tau}\right)$$

$$S(f) = \int_{-\infty}^{\infty} s(t)\, e^{-j2\pi f t}\, dt = \int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} A\, e^{-j2\pi f t}\, dt = A\left[\frac{e^{-j2\pi f t}}{-j2\pi f}\right]_{-\frac{\tau}{2}}^{+\frac{\tau}{2}} =$$

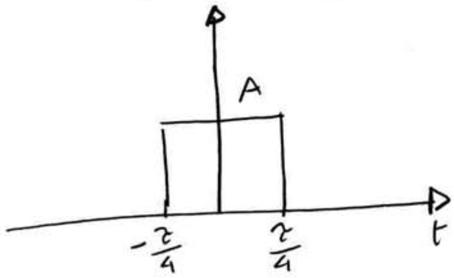
$$= A\left[\frac{e^{-j2\pi f\frac{\tau}{2}} - e^{j2\pi f\frac{\tau}{2}}}{-j2\pi f}\right] = A\,\frac{-2j\sin(\pi f\tau)}{-j2\pi f} = A\,\frac{\sin(\pi f\tau)}{\pi f} =$$

$$= A\tau\,\frac{\sin(\pi f\tau)}{\pi f\tau} = A\tau\,\mathrm{sinc}(f\tau)$$

- Proprietà della TCF

Cambiamento di scala

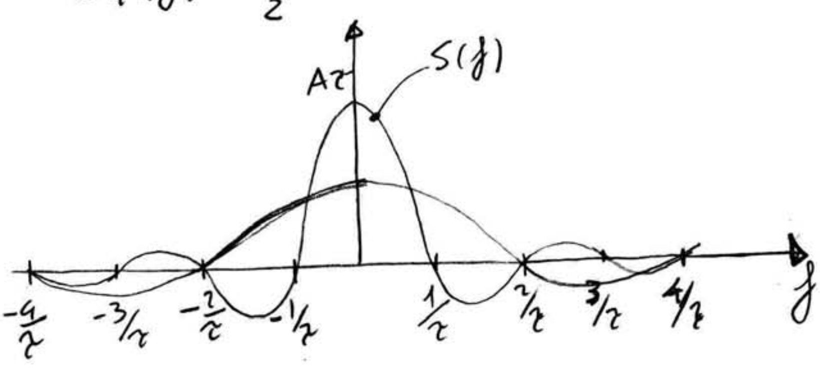
$$s(t) \overset{\mathcal{F}}{\Longleftrightarrow} S(f) \qquad s(\alpha t) \Longleftrightarrow \frac{1}{|\alpha|} S(f/\alpha)$$



$$s_1(t) = A\,\mathrm{rect}\left(\frac{2t}{\tau}\right) \qquad S_1(f)\,?$$

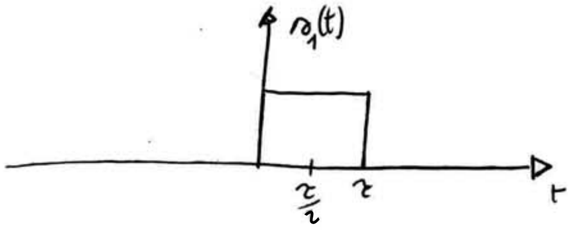
$$s_1(t) = A\,\mathrm{rect}\left(\frac{2t}{\tau}\right) = s(\alpha t) \qquad \text{con } \alpha = 2 \text{ e } s(t) = A\,\mathrm{rect}\left(\frac{t}{\tau}\right)$$

$$S_1(f) = \frac{1}{2} A\tau\,\mathrm{sinc}\left(f\frac{\tau}{2}\right)$$



→ Teorema del ritardo: $s(t) \Leftrightarrow S(f) \Rightarrow s(t-t_0) \Leftrightarrow S(f)\, e^{-j2\pi f t_0}$

$s_1(t) = A\ \mathrm{rect}\left(\frac{t-\frac{\tau}{2}}{\tau}\right)$



$s(t) = A\,\mathrm{rect}\left(\frac{t}{\tau}\right) \Leftrightarrow A\tau\,\mathrm{sinc}(f\tau)$

$s_1(t) = s(t-\frac{\tau}{2})$

$S_1(f) = A\tau\,\mathrm{sinc}(f\tau)\, e^{-j2\pi f \frac{\tau}{2}} = A\tau\,\mathrm{sinc}(f\tau)\, e^{-j\pi f\tau}$