$$s(t) \in \mathbb{C}$$

$$s(t) = a(t) + j\,b(t) = A(t)\,e^{j\theta(t)}$$

$$a(t),\ b(t) \in \mathbb{R}$$

$$A(t) = |s(t)|$$

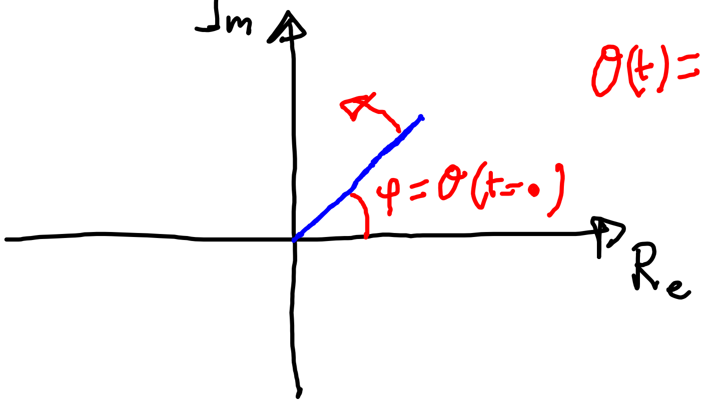
$$\theta(t) = \angle s(t)$$

— Fasori

$$s(t) = A\,e^{j(\omega t + \varphi)}$$

ω veloc. angolare (pulsazione)

φ fase iniziale $(t=0)$



$$\theta(t) = \omega t + \varphi$$

$$s(t) = A e^{j(\omega t+\varphi)} = A\cos(\omega t+\varphi) + jA\sin(\omega t+\varphi)$$

$$A\cos(\omega t+\varphi) = \mathrm{Re}\{s(t)\}$$

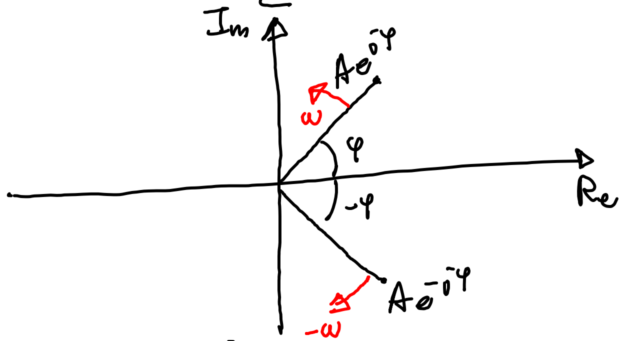
$$A\sin(\omega t+\varphi) = \mathrm{Im}\{s(t)\}$$

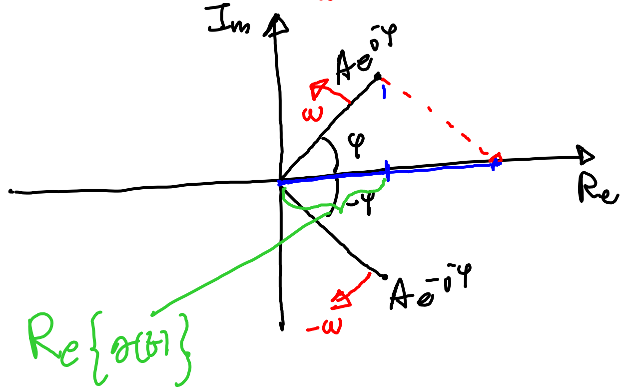
$$\mathrm{Re}\{s(t)\} = \frac{s(t)+s^*(t)}{2} =$$

$$= \frac{A e^{j(\omega t+\varphi)} + A e^{-j(\omega t+\varphi)}}{2} =$$

$$= \frac{A e^{j\varphi} e^{j\omega t} + A e^{-j\varphi} e^{-j\omega t}}{2}$$

$t=0$
$\Downarrow$
$*=1$





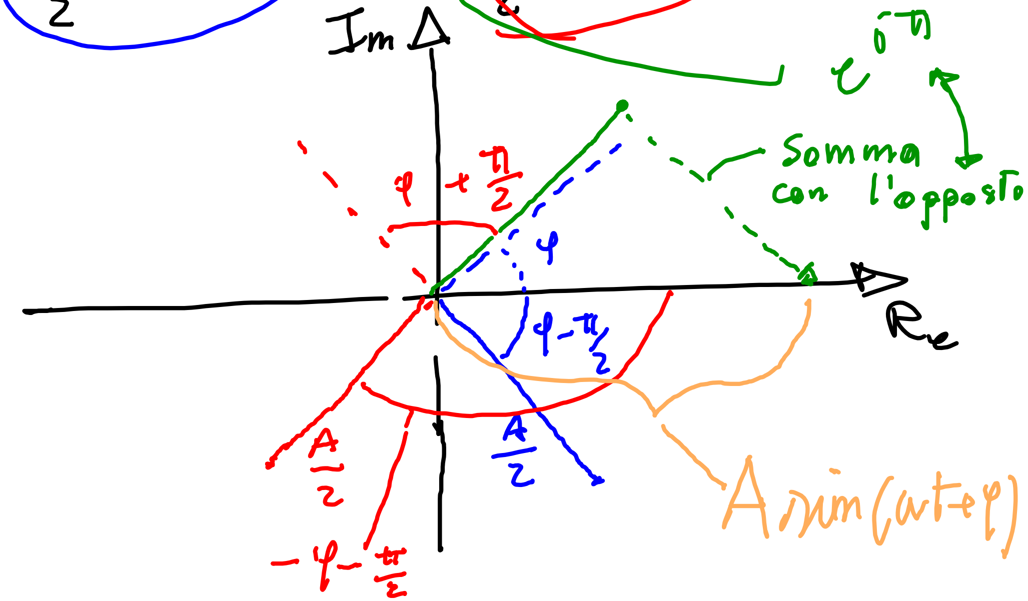
$$\mathrm{Im}\{s(t)\} = \frac{s(t) - s^*(t)}{2j} =$$

$$= \frac{A e^{j(\omega t + \varphi)} - A e^{-j(\omega t + \varphi)}}{2j} =$$

$$= \frac{A}{2j} e^{j\varphi} e^{j\omega t} - \frac{A}{2j} e^{-j\varphi} e^{-j\omega t} =$$

$= ($ per facilit. il grafico $\frac{1}{j} = \frac{1}{e^{j\frac{\pi}{2}}} = e^{-j\frac{\pi}{2}})$

$$= \frac{A}{2} e^{j(\varphi - \frac{\pi}{2})} e^{j\omega t} - \frac{A}{2} e^{-j(\varphi + \frac{\pi}{2})} e^{-j\omega t}$$



$s(t)$ periodico di periodo T_0

$$s(t) = s(t+T_0), \forall t$$

$$s(t) = \sum_{n=-\infty}^{+\infty} S_n e^{j\frac{2\pi n t}{T_0}}$$

$\left\{ e^{j2\pi n \frac{t}{T_0}} \right\}$ base di Fourier

$\omega = \frac{2\pi n}{T_0}$ $\quad f_0 = \frac{1}{T_0}$ $\quad f_n = \frac{n}{T_0}$

armoniche

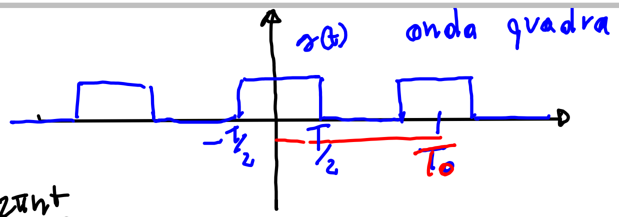
$$S_n = |S_n| e^{j\angle S_n} = R_n + j I_n$$

grafico di:

$\begin{cases} |S_n| \Rightarrow \text{Spettro di Ampiezza} \\ \angle S_n \Rightarrow \text{Spettro di Fase} \end{cases}$

in alternativa

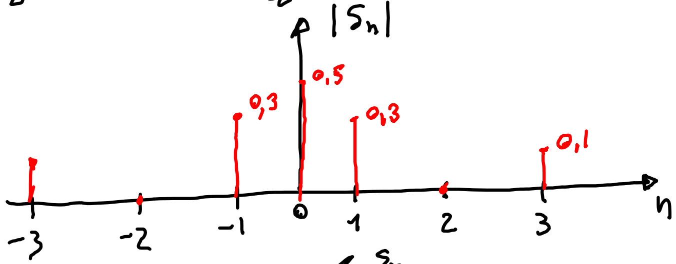
$\begin{cases} R_n \\ I_n \end{cases}$

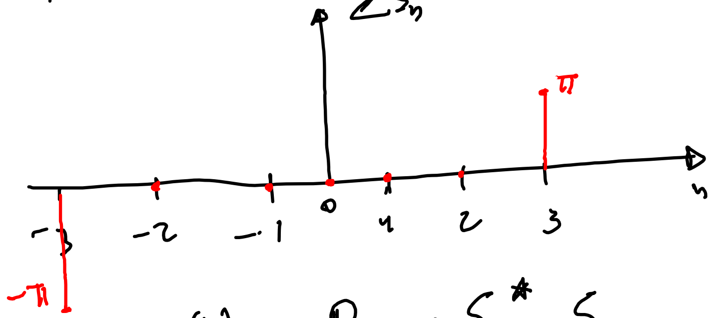


$e^{j\frac{2\pi n t}{T_0}}$ $\quad n = 0, \pm 1, \pm 2, \pm \cdots (\to \infty)$

$$S_n = \begin{cases} 1/2 & n = 0 \\ \frac{1}{2}\,\frac{\sin(n\frac{\pi}{2})}{n\frac{\pi}{2}} & n \neq 0 \end{cases}$$

$T = \frac{T_0}{2}$



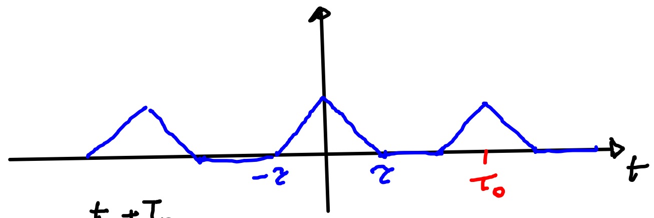


$$s(t) \in \mathbb{R} \Rightarrow S_n^* = S_{-n}$$

Nota

$n = 3$

$$S_3 = \frac{1}{2}\,\frac{\sin(3\frac{\pi}{2})}{3\frac{\pi}{2}} = \frac{1}{\cancel{2}}\,\frac{-1}{3\pi/\cancel{2}} = \frac{1}{3\pi}e^{j\pi}$$



$$S_n = \frac{1}{T_0}\int_{t_0}^{t_0+T_0} s(t)\, e^{-j\frac{2\pi n t}{T_0}}\, dt =$$

$$= \underbrace{\frac{1}{T_0}\int_{t_0}^{t_0+T_0} s(t)\cos\frac{2\pi n t}{T_0}\, dt}_{1} - j\underbrace{\frac{1}{T_0}\int_{t_0}^{t_0+T_0} s(t)\sin\frac{2\pi n t}{T_0}\, dt}_{2}$$

se $s(t) \in \mathbb{R} \Rightarrow$ 1 R_n

2 $\left(-\frac{1}{T_0}\int \dots\right)$ I_n

- $s(t) \in \mathbb{R}$, pari $\Rightarrow$ $S_n = S_{-n}$

$S_n^* = S_{-n}$

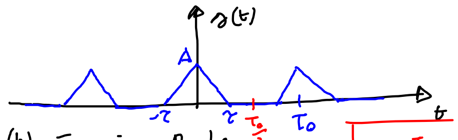
$I_n = 0 \quad \forall n$

$R_n = R_{-n}$

$$S_n = R_n = \frac{1}{T_0}\int_{t_0}^{t_0+T_0} s(t)\cos\frac{2\pi n t}{T_0}\, dt =$$

$s(t)$ pari, $\cos(\cdot)$ pari $\Rightarrow$ $s(t)\cos(\cdot)$ è pari

$$R_n = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} s(t)\cos\frac{2\pi n t}{T_0}\, dt = \frac{2}{T_0}\int_{0}^{T_0/2} s(t)\cos\frac{2\pi n t}{T_0}\, dt$$



x(t) è pari e Reale

$\tau < \frac{T_0}{2}$ Ipot.

$I_n = 0 \quad \forall n$

$$R_n = \frac{2}{T_0}\int_0^{T_0/2} x(t)\cos\frac{2\pi n t}{T_0}\,dt = \frac{2}{T_0}\int_0^{\tau} x(t)\cos\frac{2\pi n t}{T_0}\,dt$$

$x(t) \quad t \in [0,\tau] \quad x(t) = a + bt$

$$\begin{cases} x(0) = A = a \\ x(\tau) = 0 = a + b\tau \end{cases} \qquad \begin{cases} a = A \\ b = -\frac{A}{\tau} \end{cases} \qquad x(t) \ t\in[0,\tau] = A\left(1 - t/\tau\right)$$

$$R_n = \frac{2}{T_0}\int_0^{\tau} A\left(1-\frac{t}{\tau}\right)\cos\frac{2\pi n t}{T_0}\,dt =$$

$$= \frac{2}{T_0}\int_0^{\tau} A\cos\frac{2\pi n t}{T_0}\,dt \mp \frac{2}{T_0}\int_0^{\tau}\frac{At}{\tau}\cos\frac{2\pi n t}{T_0}\,dt =$$

$$= \frac{2}{T_0}\int_0^{\tau} A\frac{T_0}{2\pi n} D\left(\sin\left(\frac{2\pi n t}{T_0}\right)\right)dt \mp \dots$$

Int. per. parti $\int_a^b g(x) f'(x)\,dx = g(x)f(x)\Big|_a^b - \int_a^b f(x) g'(x)\,dx$

$$\mp \frac{2}{T_0}\int_0^{\tau}\frac{At}{\tau}\frac{1}{\frac{2\pi n}{T_0}} D\left(\sin\frac{2\pi n t}{T_0}\right)dt =$$

$$= \frac{2}{T_0}\left[\frac{AT_0}{2\pi n}\sin\left(\frac{2\pi n t}{T_0}\right)\Big|_0^{\tau}\right] - \frac{2}{T_0}\left[\frac{AT_0}{2\pi n\tau}\, t\sin\frac{2\pi n t}{T_0}\Big|_0^{\tau} - \frac{AT_0}{2\pi n\tau}\int_0^{\tau}\sin\frac{2\pi n t}{T_0}\,dt\right] = \cancel{\frac{2}{T_0}\left[\frac{AT_0}{2\pi n}\sin\left(\frac{2\pi n \tau}{T_0}\right)\right]} -$$

$g(x)$

$$\cancel{-\frac{2}{T_0}\left[\frac{AT_0\,\tau\sin\frac{2\pi n\tau}{T_0}}{2\pi n\tau}\right]} + \frac{2}{T_0}\frac{AT_0}{2\pi n\tau}\frac{1}{\frac{2\pi n}{T_0}}\left[-\cos\frac{2\pi n t}{T_0}\right]_0^{\tau} =$$

$$= \frac{2AT_0}{(2\pi n)^2\tau}\left[1 - \cos\frac{2\pi n\tau}{T_0}\right]$$