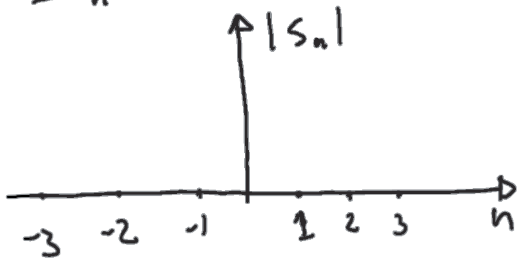
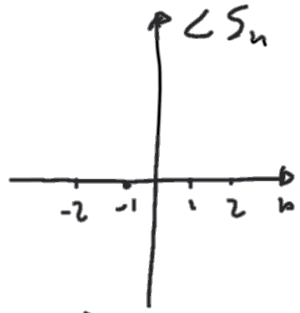
$e^{j2\pi n \frac{t}{T_0}}$ $\qquad f_0 = \frac{1}{T_0}$

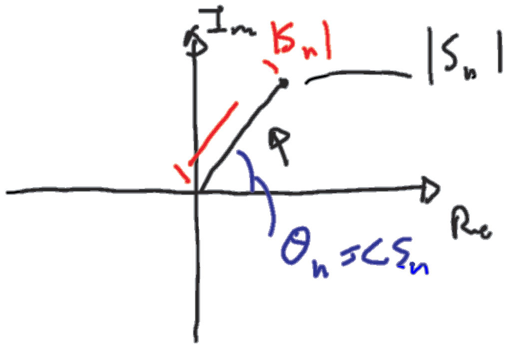
$$s(t) = \sum_{n=-\infty}^{+\infty} S_n \, e^{j2\pi n \frac{t}{T_0}}$$

$$S_n = |S_n| \, e^{j\angle S_n} = R_n + jI_n$$





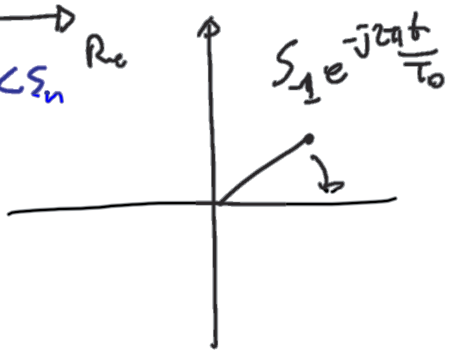
$n = 2 \qquad f_2 = \frac{2}{T_0} = 2f_0$

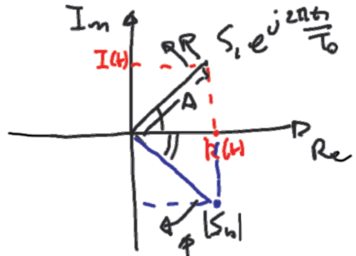


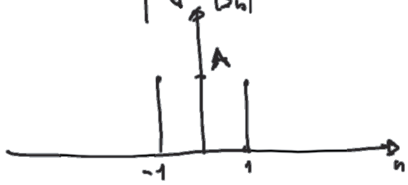
$|S_n| \, e^{j\theta_n}$

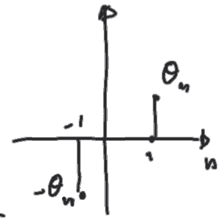
N.B. $\left(S_n e^{j2\pi n \frac{t}{T_0}}\right)$

$t = 0$







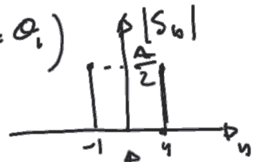


$$s(t) = S_1 e^{j\frac{2\pi t}{T_0}} + S_{-1} e^{-j\frac{2\pi t}{T_0}} =$$

$$= |S_1| e^{j\theta_1} e^{j\frac{2\pi t}{T_0}} + |S_{-1}| e^{j\theta_{-1}} e^{-j\frac{2\pi t}{T_0}} =$$

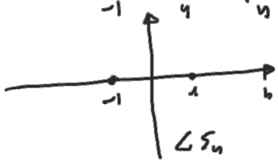
$$= |S_1| e^{j\theta_1} e^{j\frac{2\pi t}{T_0}} + |S_1| e^{-j\theta_1} e^{-j\frac{2\pi t}{T_0}} =$$

$$= |S_1| \, 2\cos\left(2\pi \frac{t}{T_0} + \theta_1\right)$$



$\boxed{\text{es}}$ $s(t) = A \cos 2\pi \frac{t}{T_0}$

$S_1 = \frac{A}{2}$ $\quad S_{-1} = \frac{A}{2}$

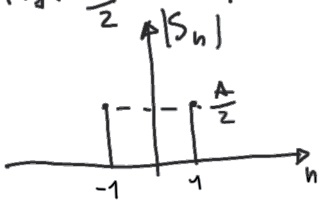


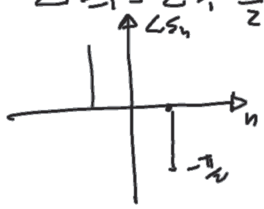
$$s(t) = A \sin 2\pi \frac{t}{T_0} =$$

$$= A \cos\left(\frac{2\pi t}{T_0} - \frac{\pi}{2}\right)$$

$|S_1| = \frac{A}{2}$ $\quad \angle S_1 = -\frac{\pi}{2}$ $\quad |S_{-1}| = \frac{A}{2}$

$\angle S_{-1} = -\angle S_1 = \frac{\pi}{2}$





$s(t) \in \mathbb{R} \qquad S_n = S_{-n}^*$

$$S_n = \frac{1}{T_0}\int_{[T_0]} s(t)\, e^{-j\frac{2\pi n t}{T_0}}\, dt =$$

$$= \frac{1}{T_0}\int_{[T_0]} s(t)\left(\cos\frac{2\pi n t}{T_0} - j\sin\frac{2\pi n t}{T_0}\right) dt =$$

$$= \frac{1}{T_0}\Big[\underbrace{\int_{[T_0]} s(t)\cos\frac{2\pi n t}{T_0}\,dt}_{R(t)} - j\underbrace{\int_{[T_0]} s(t)\sin\frac{2\pi n t}{T_0}\,dt}_{I(t)}\Big] =$$

$$S_{-n} = \frac{1}{T_0}\int_{[T_0]} s(t)\cos\left(-\frac{2\pi n t}{T_0}\right) dt - \frac{j}{T_0}\int_{[T_0]} s(t)\sin\left(-\frac{2\pi n t}{T_0}\right) dt$$

$$= \frac{1}{T_0}\int_{[T_0]} s(t)\cos\left(\frac{2\pi n t}{T_0}\right) dt - \frac{j}{T_0}\int_{[T_0]} s(t)\left(-\sin\frac{2\pi n t}{T_0}\right) dt$$

$$S_{-n} = R_n - j I_n$$

$$|S_n| = |S_{-n}| \qquad \angle S_n = -\angle S_{-n}$$

Es. esame 11/2/2008

$$S_n = \frac{1}{n^2} + \frac{e^{j\left(\pi n + \frac{\pi}{2}\right)}}{n}\ , \quad S_0 = 0$$

$s(t)$ è reale?

$S_n = S_{-n}^*$?

$$S_n = \frac{1}{n^2} + \frac{e^{j\pi n}\, e^{j\frac{\pi}{2}}}{n} = \frac{1}{n^2} + \frac{j\, e^{j\pi n}}{n} =$$

$$= \frac{1}{n^2} + j\,\frac{(-1)^n}{n}$$

N.B. $(-1)^{-n} = (-1)^n$

$$S_{-n} = \frac{1}{(-n)^2} + j\,\frac{(-1)^{-n}}{(-n)} = \frac{1}{n^2} - j\,\frac{(-1)^n}{n}$$

$$S_{-n} = S_n^* \Rightarrow s(t) \in \mathbb{R}$$

$\boxed{\text{ES}}$

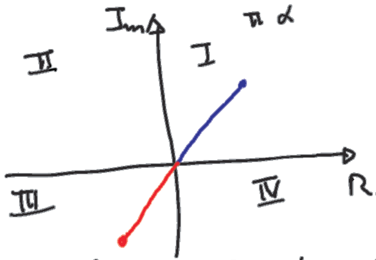
$$S_n = \frac{\cos \pi n}{n^2} + j\,\frac{\sin\frac{\pi n}{2}}{n}$$

$$S_{-n} = \frac{\cos \pi n}{n^2} + j\,\frac{\sin\frac{-\pi n}{2}}{-n} = S_n$$

$s(t) \in \mathbb{C}$

$$S_n = \begin{cases} 1/2 & n=0 \\ \frac{1}{2}\frac{\sin\left(n\frac{\pi}{2}\right)}{n\frac{\pi}{2}} = \frac{1}{2}\,\mathrm{sinc}\left(\frac{n}{2}\right) \end{cases}$$

$$\mathrm{sinc}(\alpha) \triangleq \frac{\sin(\pi\alpha)}{\pi\alpha}$$



$$|S_n| = \sqrt{R_n^2 + I_n^2}$$

$$\angle S_n = \mathrm{atg}\,\frac{I_n}{R_n}$$ se $S_n \in$ I, IV quad.

se $S_n \in$ II quad. $\angle S_n = \pi + \mathrm{atg}\,\frac{I_n}{R_n}$

se $S_n \in$ III quad. $\angle S_n = -\pi + \mathrm{atg}\,\frac{I_n}{R_n}$

$$S_n \in [-\pi; \pi]$$

$$S_n = \frac{1}{2}\frac{\sin\left(\frac{\pi n}{2}\right)}{\frac{\pi n}{2}} \qquad S_0 = \frac{1}{2}$$

$n=1$ $\quad S_1 = \frac{1}{2}\frac{1}{\pi/2} = \frac{1}{\pi}$ $\quad n=-1$ $S_{-1} = \frac{1}{\pi}$

$$|S_1| = \frac{1}{\pi} \quad \angle S_1 = 0$$

$n=2$ $\quad S_2 = \frac{1}{2}\frac{0}{\pi} = 0 \implies n=-2$ $S_2 = \emptyset$

$n=3$ $\quad S_3 = \frac{1}{2}\frac{\sin\frac{3\pi}{2}}{\frac{3\pi}{2}} = -\frac{1}{3\pi}$

$$|S_3| = \frac{1}{3\pi} \quad \angle S_3 = \pi$$

$n=-3$ $\quad S_{-3} = \frac{1}{2}\frac{\sin -\frac{3\pi}{2}}{-\frac{3\pi}{2}} = -\frac{1}{3\pi}$

$$|S_{-3}| = \frac{1}{3\pi} \quad \angle S_{-3} = -\pi$$

$s(t) \in \mathbb{R}$ $\qquad S_n = R_n + j I_n$

$$s(t) = \sum_{n=-\infty}^{+\infty} S_n e^{j\frac{2\pi n t}{T_0}} =$$

$$= \sum_{n=-\infty}^{\infty} (R_n + j I_n)\left(\cos\frac{2\pi n t}{T_0} + j \sin\frac{2\pi n t}{T_0}\right) =$$

$$= \sum_{n=-\infty}^{n=+\infty} R_n \cos\frac{2\pi n t}{T_0} - I_n \sin\frac{2\pi n t}{T_0} +$$

$$+ j \sum_{n=-\infty} R_n \sin\frac{2\pi n t}{T_0} + I_n \cos\frac{2\pi n t}{T_0} =$$

$S_n = S_{-n}^*$ $\qquad R_n = R_{-n} \Rightarrow$ pari

$\qquad I_n = -I_{-n} \Rightarrow$ disp.

$\boxed{\text{N.B.}}$ se $s(t) \in \mathbb{R}$

$I_0 = 0$

sono pari $\quad \sum_{n=-\infty}^{\infty} a_n = \sum_{n=1}^{\infty} a_n + a_0 + \sum_{n=-\infty}^{-1} a_n =$

$$= a_0 + 2\sum_{n=1}^{+\infty} a_n$$

$$= R_0 + 2\sum_{n=+1}^{\infty}\left[R_n \cos\frac{2\pi n t}{T_0} - I_n \sin\frac{2\pi n t}{T_0}\right]$$

Forma trigonometrica o rettangolare

$$R_n = \frac{1}{T_0}\int_{[T_0]} s(t)\cos\frac{2\pi n t}{T_0}\,dt$$

$$I_n = -\frac{1}{T_0}\int_{[T_0]} s(t)\sin\frac{2\pi n t}{T_0}\,dt$$

Forma Polare

$$s(t) = \sum_{n=-\infty}^{+\infty} S_n e^{j\frac{2\pi n t}{T_0}}$$

$S_n = |S_n| e^{j\theta_n} \qquad S_{-n} = |S_{-n}| e^{j\theta_{-n}}$

$S_n = S_{-n}^*$

$$s(t) = S_0 + 2\sum_{n=1}^{\infty} |S_n| \left(\cos\frac{2\pi n t}{T_0} + \theta_n\right)$$

$s(t)$ generico $(\in \mathbb{C})$

$$S_n = \frac{1}{T_0}\int_{[T_0]} s(t)\, e^{-j\frac{2\pi n t}{T_0}}\, dt =$$

$$= \frac{1}{T_0}\int_{[T_0]} s(t)\cos\frac{2\pi n t}{T_0}\, dt - \frac{j}{T_0}\int_{[T_0]} s(t)\sin\left(\frac{2\pi n t}{T_0}\right) dt$$

1) $s(t) = s(-t)$

$$= \frac{2}{T_0}\int_0^{T_0/2} s(t)\cos\frac{2\pi n t}{T_0}\, dt = S_n = S_{-n}$$

2) $s(t) = -s(-t)$

$$S_n = -\frac{2j}{T_0}\int_0^{T_0/2} s(t)\sin\frac{2\pi n t}{T_0}\, dt$$

$s(t)$ è pari $\Longleftrightarrow$ $S_n = S_{-n}$

$s(t)$ è dispari $\Longleftrightarrow$ $S_n = -S_{-n}$

$\boxed{es}$ $S_n = \dfrac{\cos \pi n}{n^2} + j\dfrac{\sin\frac{\pi n}{2}}{n}$

$S_{-n} \neq S_n^*$ $\quad s(t) \in \mathbb{C}$

$S_{-n} = S_n \Longrightarrow s(t)$ è pari

ES

$$S_n = \frac{(-1)^n - \cos \pi n}{n^2} + j\,\frac{\sin \pi n/2}{n^2}$$

$s(t) \in \mathbb{R}$?

$$S_n = \frac{(-1)^n - (-1)^n}{n^2} + j\,\frac{\sin \frac{\pi n}{2}}{n^2} = j\,\frac{\sin \frac{\pi n}{2}}{n^2}$$

$$S_{-n} = -j\,\frac{\sin \frac{\pi n}{2}}{n^2} = S_n^* \implies s(t) \in \mathbb{R}$$

$$S_{-n} = -S_n \implies s(t) \text{ è dispari}$$

---

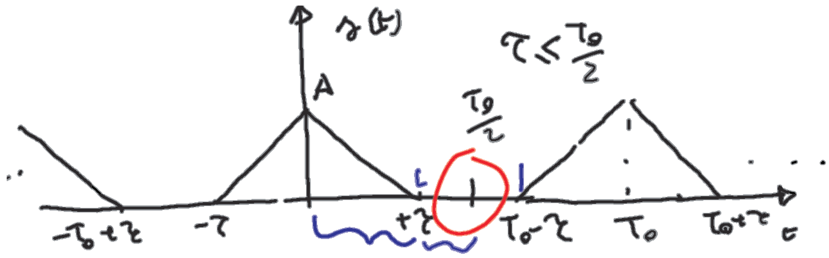
se $s(t) \in \mathbb{R}$ $\quad S_n = S_{-n}^*$

1) $\left.\begin{array}{l} s(t) = s(-t) \\ s(t) \in \mathbb{R} \end{array}\right\} \implies S_n = R_n$

2) $\left.\begin{array}{l} s(t) = -s(-t) \\ s(t) \in \mathbb{R} \end{array}\right\} \implies S_n = j I_n$

$$R_n = \frac{2}{T_0}\int_0^{T_0/2} s(t)\cos\frac{2\pi n t}{T_0}\,dt$$

$$I_n = -\frac{2}{T_0}\int_0^{T_0/2} s(t)\sin\frac{2\pi n t}{T_0}\,dt$$



$T_0 \Rightarrow f_0 = \frac{1}{T_0}$ $\qquad f_n = n f_0$

$$S_n = \frac{1}{T_0} \int_{[T_0]} s(t)\, e^{-j 2\pi n \frac{t}{T_0}}\, dt$$

$s(t) \in \mathbb{R}$, $s(t) = s(-t)$

$S_n = S_{-n}^*$ $\qquad \Longrightarrow S_n = R_n$

$R_n = R_{-n}$

$$R_n = \frac{2}{T_0} \int_0^{T_0/2} s(t) \cos 2\pi n \frac{t}{T_0}\, dt =$$

$$= \frac{2}{T_0} \int_0^{\tau} f(t) \cos 2\pi n \frac{t}{T_0}\, dt$$

$f(t) = s(t)$

$t \in [0, \tau]$

$f(t) = at + b$

$$\begin{cases} f(0) = A = b \\ f(\tau) = a\tau + b = 0 \end{cases} \qquad \begin{cases} b = A \\ a = -\frac{b}{\tau} = -\frac{A}{\tau} \end{cases}$$

$$f(t) = A\left(1 - \frac{t}{\tau}\right)$$