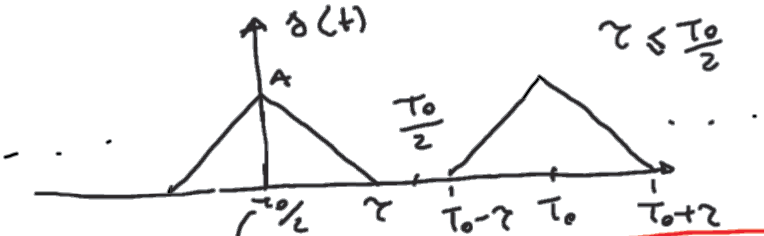


$\tau \le \frac{T_0}{2}$

$$S_n = R_n = \frac{2}{T_0}\int_0 s(t)\cos 2\pi n\frac{t}{T_0}\,dt$$

$$\boxed{\int f'(x)\,g(x)\,dx = f(x)\,g(x) - \int f(x)\,g'(x)\,d}$$

$$s(t) = A\left(1-\frac{t}{\tau}\right), \quad t \in [0, \tau]$$

$$s(t) = 0, \quad t \in \left(\tau, \frac{T_0}{2}\right]$$

$$S_n = \frac{2}{T_0}\int_0^\tau A\left(1-\frac{t}{\tau}\right)\cos 2\pi n\frac{t}{T_0}\,dt = \frac{2}{T_0}\int_0^\tau A\cos 2\pi n\frac{t}{T_0}\,dt +$$

$$-\frac{2}{T_0}\int_0^\tau A\frac{t}{\tau}\cos 2\pi n\frac{t}{T_0}\,dt = \frac{2}{T_0}A\frac{T_0}{2\pi n}\int_0^\tau \frac{d}{dt}\sin 2\pi n\frac{t}{T_0}\,dt +$$

$$-\frac{2A}{T_0\tau}\int_0^\tau t\frac{T_0}{2\pi n}\frac{d}{dt}\sin 2\pi n\frac{t}{T_0}\,dt = \frac{2A}{T_0}\frac{T_0}{2\pi n}\left[\sin 2\pi n\frac{t}{T_0}\right]_0^\tau +$$

$$\frac{2A}{T_0\tau}\left[t\frac{T_0}{2\pi n}\sin 2\pi n\frac{t}{T_0}\right]_0^\tau + \frac{2A}{T_0\tau}\int_0^\tau \frac{T_0}{2\pi n}\sin 2\pi n\frac{t}{T_0}\,dt =$$

$$= \frac{2A}{T_0}\cancel{\frac{T_0}{2\pi n}\sin 2\pi n\frac{\tau}{T_0}} - \cancel{\frac{2A}{T_0\tau}\frac{\tau T_0}{2\pi n}\sin 2\pi n\frac{\tau}{T_0}} +$$

$$+\frac{2A}{T_0\tau}\frac{T_0}{2\pi n}\frac{T_0}{2\pi n}\left[-\cos 2\pi n\frac{t}{T_0}\right]_0^\tau =$$

$$\boxed{\cos 2\alpha = 1 - 2\sin^2\alpha}$$

$$= \frac{2A\,T_0^2}{T_0\tau(2\pi n)^2}\left[1-\cos 2\pi n\frac{\tau}{T_0}\right] =$$

$$= \frac{2A\,T_0^2}{T_0\tau(2\pi n)^2}\left[1-1+2\sin^2\left(\frac{\pi n\tau}{T_0}\right)\right] =$$

$$= \frac{2AT_0^2}{T_0\tau(2\pi n)^2}\left[1-1+2\sin^2\left(\frac{\pi n\tau}{T_0}\right)\right]=$$

$$= \frac{2AT_0^3}{T_0\tau(2\pi n)^2}\; 2\sin^2\left(\frac{\pi n\tau}{T_0}\right)=$$

$$= \frac{\cancel{2}A}{T_0\cancel{4}}\frac{1}{\left(\frac{\pi n}{T_0}\right)^2}\cancel{2}\sin^2\left(\frac{\pi n\tau}{T_0}\right)=$$

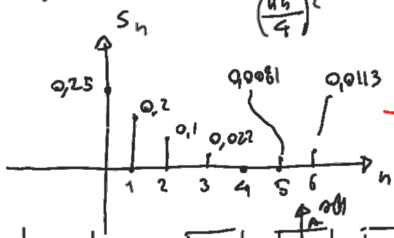
$$\boxed{\mathrm{sinc}(\alpha)=\frac{\sin\pi\alpha}{\pi\alpha}}$$

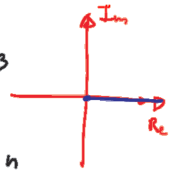
$$= \frac{A\tau}{T_0}\frac{\sin^2\left(\pi n\frac{\tau}{T_0}\right)}{\left(\frac{\pi n\tau}{T_0}\right)^2} = \frac{A\tau}{T_0}\,\mathrm{sinc}^2\left(\frac{n\tau}{T_0}\right)$$

$$S_0 = \frac{2}{T_0}\int_0^{T_0/2} s(t)\,dt = \frac{A\tau}{T_0}$$

$A=1 \quad \tau=\frac{T_0}{4}$

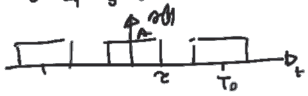
$$S_n=\frac{1}{4}\,\mathrm{sinc}^2\left(\frac{n}{4}\right)=\frac{1}{4}\frac{\sin^2\left(\frac{\pi n}{4}\right)}{\left(\frac{\pi n}{4}\right)^2} \qquad S_0=\frac{1}{4}$$



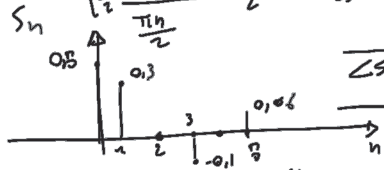


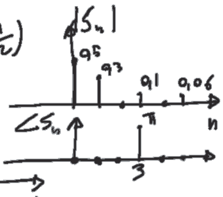
onda quadra

$A=1 \quad \tau=\frac{T_0}{4}$

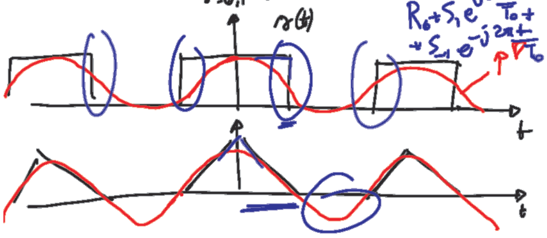


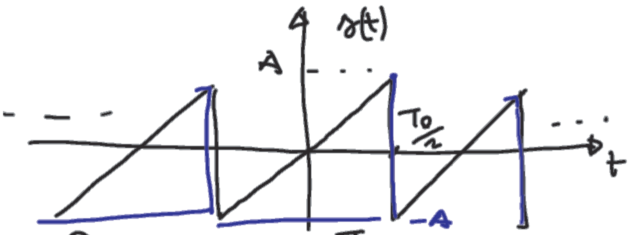
$$S_n=\begin{cases}1/2 & n=0\\ \frac{1}{2}\frac{\sin\left(\frac{n\pi}{2}\right)}{\frac{\pi n}{2}}=\frac{1}{2}\sin\left(\frac{n}{2}\right)\end{cases}$$





$R_0+S_1e^{j\frac{2\pi t}{T_0}}+S_{-1}e^{-j\frac{2\pi t}{T_0}}$





$R_n = 0 \qquad S_n = j\, I_n$

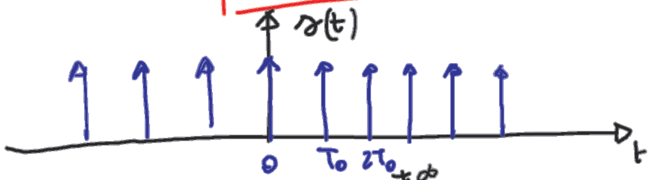
$$I_n = -\frac{2}{T_0}\int_0^{T_0/2} s(t)\, \sin 2\pi \frac{nt}{T_0}\, dt$$

$\delta(t)$

$$\boxed{\int_{-\infty}^{+\infty} f(t)\, \delta(t-t_0)\, dt = f(t_0)}$$

$$f(t) = A\, \mathrm{rect}\left(\frac{t-t_0}{2\varepsilon}\right)$$

$$\int_{t_0-\varepsilon}^{t_0+\varepsilon} A\, \delta(t-t_0)\, dt = A$$



$$s(t) = \mathrm{rep}_{T_0}(\delta(t)) = \sum_{n=-\infty}^{+\infty} \delta(t-nT_0)$$

$$S_n = \frac{1}{T_0}\int_{[T_0]} s(t)\, e^{-j2\pi\frac{nt}{T_0}}\, dt = \frac{1}{T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} \delta(t)\, e^{-j2\pi\frac{nt}{T_0}}\, dt =$$

$$= \frac{1}{T_0}\, e^{-j2\pi n\frac{(t=0)}{T_0}} = \frac{1}{T_0}$$

$$s(t) = \underbrace{\sum_{n=-\infty}^{\infty} \frac{1}{T_0}\, e^{j2\pi\frac{nt}{T_0}}} = \sum_{n=-\infty}^{\infty} \delta(t-nT_0)$$

$$S(f) = F_c[s(t)] = \int_{-\infty}^{\infty} s(t)\, e^{-j2\pi f t}\, dt$$

$$s(t) = F_c^{-1}[s(t)] = \int_{-\infty}^{\infty} S(f)\, e^{j2\pi f t}\, df$$

$|S(f)|\, df \qquad \theta(f) = \angle S(f)$

$S(f) = |S(f)|\, e^{j\theta(f)}$

$|S(f)|$ spettro di ampiezza

$\theta(f)$ spettro di fase

$$\boxed{s(t) \in \mathbb{R} \qquad S(f) = S^*(-f)}$$

<u>Linearità</u>

$$s(t) = \sum_{i=1}^{N} \alpha_i\, s_i(t) \overset{\mathcal{F}}{\Longleftrightarrow} S(f) = \sum_{i=1}^{N} \alpha_i\, S_i(f)$$

$$s_i(t) \overset{\mathcal{F}}{\Longleftrightarrow} S_i(f),\ \forall i$$

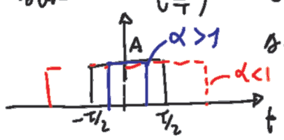
<u>Cambiamento di scala</u>

$\alpha \in \mathbb{R}$ costante $\qquad s(t) \Longleftrightarrow S(f)$

$s(\alpha t)$ $\qquad$ se $\alpha > 1 \Rightarrow$ compressione

$s(t) = A\,\mathrm{rect}\left(\frac{t}{T}\right)$ $\qquad s(\alpha t) = A\,\mathrm{rect}\left(\frac{\alpha t}{T}\right)$

se $\alpha < 1 \Rightarrow$ espansione



$$s(\alpha t) \Longleftrightarrow \frac{1}{|\alpha|}\, S(f/\alpha)$$

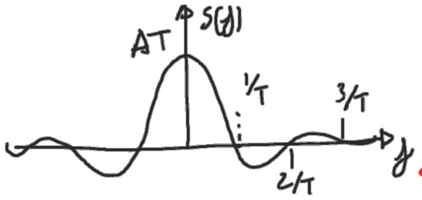
$s(t) = A\,\mathrm{rect}\left(\frac{t}{T}\right)$

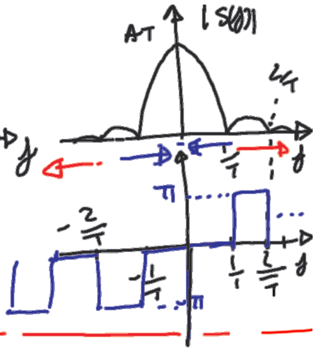
$$S(f) = \int_{-\infty}^{\infty} A\,\mathrm{rect}\left(\frac{t}{T}\right) e^{-j2\pi f t}\, dt =$$

$$= \int_{-T/2}^{T/2} A\, e^{-j2\pi f t}\, dt = \frac{A}{-j2\pi f}\, e^{-j2\pi f t}\Big|_{-T/2}^{T/2} =$$

$$= \frac{A}{-j2\pi f}\left[e^{-j2\pi f T/2} - e^{j2\pi f T/2}\right] = \frac{A}{\cancel{-j\pi f}}\, \cancel{-2j}\sin(\pi f T) =$$

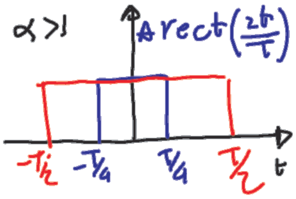
$$= \frac{A}{\pi f}\sin(\pi f T) = \frac{AT}{\pi f T}\sin(\pi f T) = AT\,\mathrm{sinc}(fT)$$

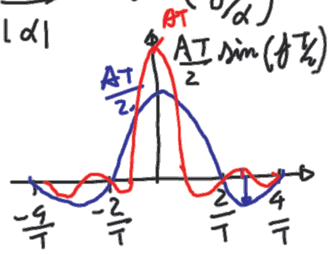




$$s(\alpha t) \Longleftrightarrow \frac{1}{|\alpha|} S\left(f/\alpha\right)$$

$$A\,\mathrm{rect}\left(\frac{t\alpha}{T}\right) \Longleftrightarrow \frac{1}{|\alpha|} AT\,\mathrm{sinc}\left(f\,T/\alpha\right)$$





Proprietà del ritardo temporale

$$s(t) \Longleftrightarrow S(f)$$

$$s(t-t_0) \Longleftrightarrow S(f)\, e^{-j2\pi f t_0} = S_1(f)$$

$$|S_1(f)| = |S(f)|$$

$$\angle S_1(f) = \angle S(f) - 2\pi f t_0$$

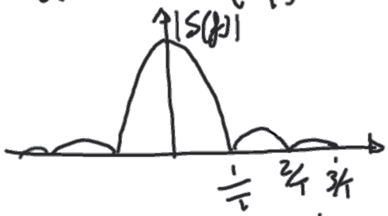
$\boxed{\text{DIM}}$

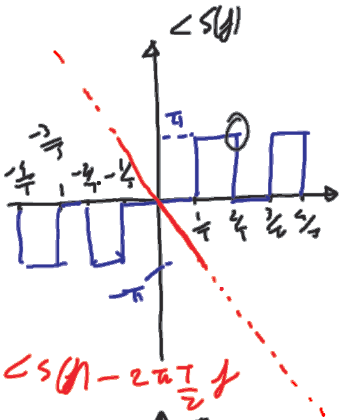
$$F_c[s(t-t_0)] = \int_{t=-\infty}^{+\infty} s(t-t_0)\, e^{-j2\pi f t}\, dt = \quad x = t - t_0 \quad t = x + t_0$$

$$= \int_{x=-\infty}^{+\infty} s(x)\, e^{-j2\pi f x}\, e^{-j2\pi f t_0}\, dt = e^{-j2\pi f t_0} \int_{-\infty}^{+\infty} s(x)\, e^{-j2\pi f x}\, dx$$

$$= S(f)\, e^{-j2\pi f t_0}$$

$s(t) = A\,\mathrm{rect}\left(\frac{t}{T}\right)$

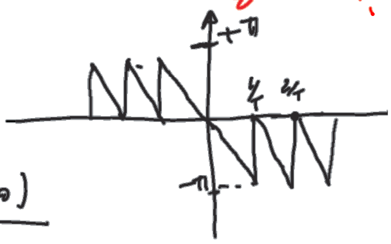




$s(t-t_0) \Longleftrightarrow S(f)e^{-j2\pi f t_0}$

$\angle S_1(f) = \angle S(f) - 2\pi f t_0 = \angle S(f) - 2\pi \frac{T}{2} f$

$\boxed{es}$ $t_0 = \frac{T}{2}$



Derivazione (nel tempo)

$$s(t) \Longleftrightarrow S(f)$$

$$s_1(t) = \frac{d}{dt}s(t) \Longleftrightarrow j2\pi f\, S(f)$$

Integrazione (nel tempo)

$$s(t) \Longleftrightarrow S(f) \quad e \quad S(0) = 0$$

$$s_1(t) = \int_{-\infty}^{t} s(\alpha)\,d\alpha \Longleftrightarrow \frac{S(f)}{j2\pi f}$$