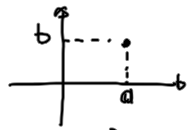
$A, B \qquad A \times B \qquad (a,b) \quad a \in A$
$\qquad\qquad\qquad\qquad\qquad\qquad b \in B$

Es

- $\mathbb{R}^2 = \mathbb{R} \times \mathbb{R}$



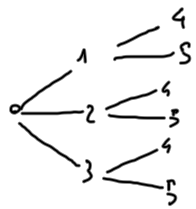
- $A = \{1,2,3\} \quad B = \{4,5\}$

$$A \times B = \{(1,4),(1,5),(2,4),(2,5),(3,4),(3,5)\}$$

si estende a n insiemi $A_1, A_2, \dots, A_n$

$A_1 \times A_2 \times \dots \times A_n \qquad (a_1, a_2, \dots, a_n)$

Diagramma ad albero

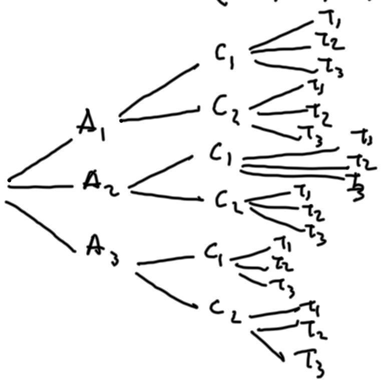


1, 4
1, 5
2, 4
2, 5
3, 4
3, 5

A e B
$p \quad q$ elem.

$A \times B = p\,q$

- 3 Abiti (A_1, A_2, A_3), 2 Camicie (C_1, C_2)
3 Cravatte (T_1, T_2, T_3)



$A_1\ C_1\ T_1$
$A_1\ C_1\ T_2$
$\vdots$

$3 \times 2 \times 3 = 18$ modi diversi

$E = \{ n \in \mathbb{N} : 10 \leq n < 80 \}$

1) el. di E con la prima cifra pari e la sec. dispari

2) el. di E con 1 # dispari, 2 cifra pari

1) 2- 4- 6-   -1 -3 -5 -7 -9

$3 \times 5 = 15$

2) 1- 3- 5- 7-   -2 -4 -6 -8 -0

$4 \cdot 5 = 20$

Disp. con rip.

$$D^{(r)}_{n,K} = n^K$$

$\{1,2,3\}$ $D^{(r)}_{3,2}$

1,1   1,2   1,3
2,1   2,2   2,3
3,1   3,2   3,3

Disp. semplici

$$D_{n,K} = \frac{n!}{(n-k)!}$$

1,2   1,3
2,1   2,3
3,1   3,2

$n = K$ Permutazioni

$$D_{n,n} = P_n = n!$$

Combinazioni

- con ripetizione

$$C^{(r)}_{n,k} = \binom{n+k-1}{k}$$

1,1
2,1   2,2
3,1   3,2   3,3

$$C^{(r)}_{3,2} = \binom{3+2-1}{2} = \binom{4}{2} = \frac{4!}{2!\,2!} = 6$$

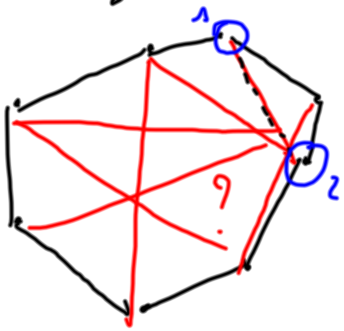
- senza rip.

$$C_{n,k} = \binom{n}{k} = \frac{n!}{(n-k)!\,k!}$$

2,1
3,1   3,2

ES 1.

Num Diagonali Poligono Convesso di n lati?



I) $n(n-3) \Rightarrow \frac{n(n-3)}{2}$

II) $V_1, V_2 \ldots V_n$

$V_i V_j$

$$C_{n,2} = \frac{n!}{(n-2)!\,2!}$$

$$\text{Num} = C_{n,2} - n = \frac{n \cdot (n-1)}{2} - n = \frac{n^2 - n - 2n}{2} = \frac{n(n-3)}{2}$$

- Valigetta serratura a 6 cifre
Num. Combinazioni?

1 2 3 - . . 2 1 3 . . .

$$D^{(r)}_{10,6} = 10^6$$

- "albergo" Num. parole di 4 lettere?

$$D_{7,4} = \frac{7!}{3!} = 7 \cdot 6 \cdot 5 \cdot 4$$

- Num. parole 5 vocali?

$$D_{5,5} = P_5 = 5!$$

- Num. parole che si formano con:
1) erba 2) mietere 3) tratteggiare

1) $P_4 = 4!$

2) $m\,i\,e_1\,t\,e_2\,r\,e_3$ $m\,i\,e_2\,t\,e_1\,r\,e_3$

$$\frac{P_7}{P_3} = \frac{7!}{3!}$$

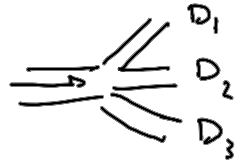
n oggetti dei quali n_1 sono uguali
n_2 " "
n_r " "

$$\frac{n!}{n_1!\, n_2! \cdots n_r!}$$

3) 3t 2g 2r 2e 2a

$$\frac{12!}{3!\,2!\,2!\,2!\,2!}$$

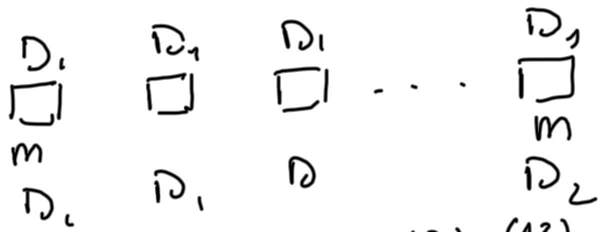
10 macchine



1) macchine indist.

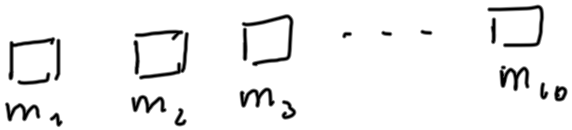
2) tutte diverse

3) 5 classe A, 3 classe B, 2 classe C



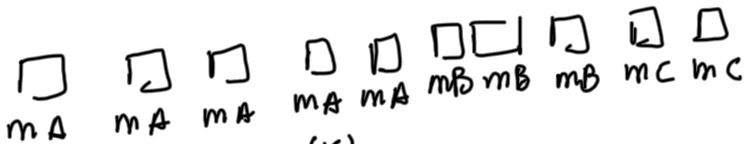
$$C^{(r)}_{3,10} = \binom{10+3-1}{10} = \binom{12}{10} = \binom{12}{2} = \frac{12\cdot 11}{2} = 66$$

2)



$$D^{(r)}_{3,10} = 3^{10}$$

3)



$$C^{(r)}_{3,5} \cdot C^{(r)}_{3,3} \cdot C^{(r)}_{3,2}$$

20 pneumatici, 3 difettosi

scelta 4 pneumatici

$P(1 \text{ sia difettoso}) = ?$

$$\frac{\text{n. favorevoli}}{\text{n. casi possibili}}$$

casi possibili $C_{20,4} = \binom{20}{4} = 4845$

casi favorevoli $C_{17,3} \cdot C_{3,1}$

$$p = \frac{C_{17,3} \cdot C_{3,1}}{C_{20,4}}$$

- estraiamo 2 pneumatici

$A = \{\text{entrambi diff}\}$ $\quad P(A) = ?$

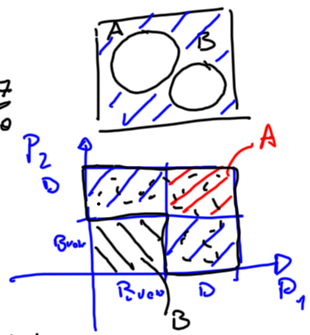
$B = \{\text{entrambi non diff}\}$ $\quad P(B) = ?$

$C = \{\text{almeno 1 è diff}\}$ $\quad P(C) = ?$

casi possibili $C_{20,2} = \binom{20}{2} = \frac{20 \cdot 19}{2} = 190$

$$P(A) = \frac{C_{3,2}}{C_{20,2}}$$

$$P(B) = \frac{C_{17,2}}{C_{20,2}} = \frac{9 \cdot 17}{190}$$



$P(C)$

$C = \overline{B}$

$$P(C) = 1 - P(B) = \frac{44}{190}$$

1) lancio due dadi $P(R_1+R_2=7)$

casi possibili $D^{(r)}_{6,2}$

$$P=\frac{6}{D^{(r)}_{6,2}}=\frac{6}{6^2}=\frac{1}{6}$$

2) lancio 3 dadi

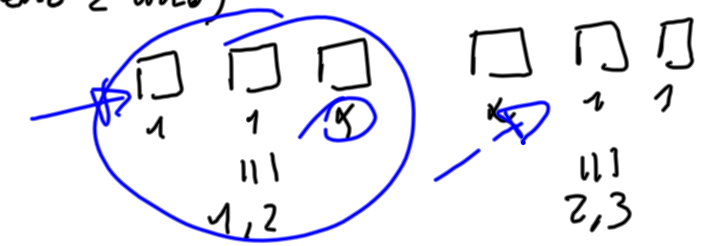
P(3 dispari)

casi possibili $D^{(r)}_{6,3}=6^3$

casi favorevoli $D^{(r)}_{3,3}=3^3$

$$P=\frac{3^3}{6^3}=\frac{1}{8}$$

P(almeno 2 uno)



$5\cdot C_{3,2}+1$ — 1 1 1

$$P=\frac{5\cdot C_{3,2}+1}{D^{(r)}_{6,3}}$$

Prob. condizionata

A, B

$P(A|B)$

$P(B|A)$

es Mazzo carte da poker 52

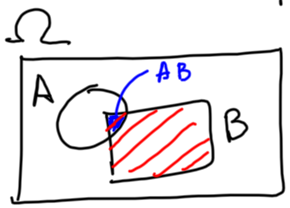
$A = \{$ estrazione donna cuori $\}$

$B = \{$ estrazione di una donna $\}$

$$P(A) = \frac{1}{52}$$

$$P(A|B) = \frac{1}{4}$$

$$P(A|B) = \frac{P(AB)}{P(B)} \qquad P(B) \neq 0$$



$$P(A|B) = \frac{n(AB)}{n} \cdot \frac{n}{n(B)}$$

$n(\cdot)$ num. elementi

$AB = A \cap B$

$\boxed{\text{Es}}$ $A = \{1\}$ $\quad B = \{1, 2, 3\}$

$$P(A|B) = \frac{1}{3} = \frac{P(AB)}{P(B)} = \frac{1}{6}\Big/\frac{1}{2} = \frac{1}{3}$$

$AB = \{1\}$

Regola moltiplicazione

$$P(AB) = P(A|B)\,P(B)$$

$A_1, A_2, A_3 \ \dots A_n$

$$P(A_1 A_2 \cdots A_n) = P(A_1)\,P(A_2|A_1)\,P(A_3|A_2 A_1) \cdots$$
$$\cdots P(A_n|A_1 A_2 \cdots A_{n-1})$$

$\boxed{es}$ 12 elem. 4 difettosi

estrazione 3 el. senza reintr.

$N_i = \{$ i-esimo non difettoso $\}$

$$P(N_1 N_2 N_3) = ?$$

$$P(N_1 N_2 N_3) = P(N_1)\,P(N_2|N_1)\,P(N_3|N_2 N_1) =$$

$$= \frac{8}{12} \cdot \frac{7}{11} \cdot \frac{6}{10}$$

eventi indipendenti

$$P(A|B) = P(A)$$
$$P(B|A) = P(B)$$

$$P(AB) = P(A|B)\,P(B) = P(A)\,P(B)$$

es lancio 2 volte dado

$$P(R_1 = 5, R_2 = 5) = ?$$

A: $R_1 = 5$

B: $R_2 = 5$

$$P(AB) = P(A)\,P(B) = \frac{1}{36}$$

$$P(\text{almeno 1 cinque}) = ?$$

$$P(A \cup B) = P(A) + P(B) - P(AB) =$$
$$= \frac{1}{6} + \frac{1}{6} - \frac{1}{36} = \frac{11}{36}$$