$$E_s(T) \triangleq \int_{-T/2}^{T/2} |s(t)|^2 dt$$

seg. ad. energia finita

se $\exists$, finito e $\neq 0$

$$E_s = \lim_{T \to \infty} \int_{-T/2}^{T/2} |s(t)|^2 dt$$

$$P_s(T) \triangleq \frac{1}{T} \int_{-T/2}^{T/2} |s(t)|^2 dt$$

seg. potenza media finita

se $\exists$, finito e $\neq 0$

$$P_s = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |s(t)|^2 dt$$

segnali periodici

$$E_s \to \infty$$

$$P_s = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} |s(t)|^2 dt = \frac{1}{T_0} \int_{[T_0]} |s(t)|^2 dt$$

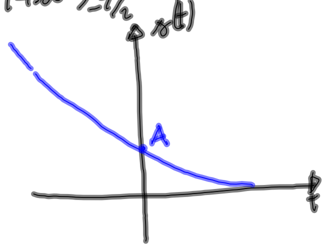
$s(t) = A e^{-\alpha t} \qquad \alpha > 0$

$$E_s = \lim_{T\to\infty} \int_{-T/2}^{T/2} |s(t)|^2 dt = \lim_{T\to\infty} \int_{-T/2}^{T/2} |A e^{-\alpha t}|^2 dt =$$

$$= \lim_{T\to\infty} \int_{-T/2}^{T/2} A^2 e^{-2\alpha t} dt =$$

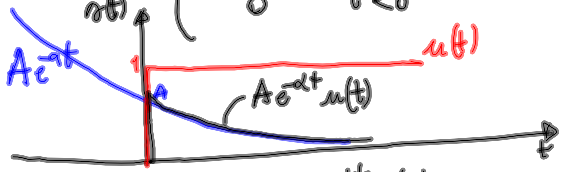
$$= \lim_{T\to\infty} A^2 \frac{1}{-2\alpha} e^{-2\alpha t}\Big|_{-T/2}^{T/2} =$$

$$= \lim_{T\to\infty} \frac{A^2}{-2\alpha} \left(\cancel{e^{-\alpha T}} - e^{\alpha T}\right) = +\infty$$



$$P_s = \lim_{T\to\infty} \frac{1}{T} E_s(T) = \lim_{T\to\infty} \frac{A^2}{-2\alpha}\left(\frac{-e^{\alpha T}}{T}\right) = +\infty$$

$$s(t) = \begin{cases} A e^{-\alpha t} & t \geq 0 \\ 0 & t < 0 \end{cases} \qquad , \alpha > 0$$



$$s(t) = A e^{-\alpha t} u(t)$$

$$E_s = \lim_{T\to\infty} \int_{-T/2}^{T/2} |s(t)|^2 dt = \lim_{T\to\infty} \int_{-T/2}^{T/2} |A e^{-\alpha t} u(t)|^2 dt$$

$$= \lim_{T\to\infty} \int_0^{T/2} A^2 e^{-2\alpha t} dt = \lim_{T\to\infty} \frac{A^2}{(-2\alpha)} e^{-2\alpha t}\Big|_0^{T/2} =$$

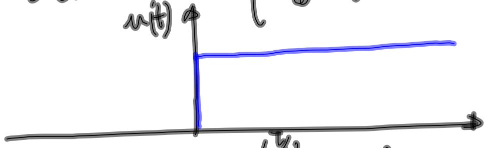
$$= \lim_{T\to\infty} -\frac{A^2}{2\alpha}\left(e^{-\alpha T} - 1\right) = \frac{A^2}{2\alpha}$$

$$P_s = \lim_{T\to\infty} \frac{1}{T} \frac{A^2}{2\alpha} = 0$$

se seg. E_s finita $\Longrightarrow P_s = 0$

seg. P_s finita $\Longrightarrow E_s \to \infty$

$s(t) = u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$

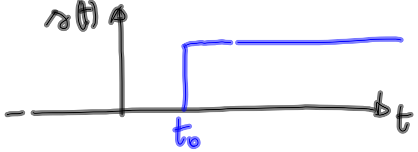


$E_s = \lim_{T \to \infty} \int_{-T/2}^{T/2} |u(t)|^2 dt = \lim_{T \to \infty} \int_0^{T/2} 1 \, dt =$

$= \lim_{T \to \infty} \frac{T}{2} \to \infty$

$P_s = \lim_{T \to \infty} \frac{1}{T} \int_0^{T/2} 1 \, dt = \frac{1}{2}$

$s(t) = A \, u(t - t_0)$



$P_s = \frac{A^2}{2}$

■ $s(t) = A \cos(\omega t)$ $\omega = 2\pi f$

$s(t)$ periodico $\Rightarrow E_s \to \infty$

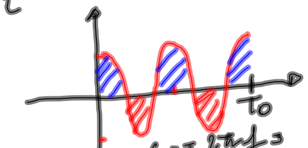
$P_s = \frac{1}{T_0} \int_0^{T_0} A^2 \cos^2(\omega t) \, dt =$

$\cos^2 \alpha = \frac{1 + \cos 2\alpha}{2}$

$= \frac{1}{T_0} \int_0^{T_0} \left(\frac{A^2}{2} + \frac{A^2 \cos 2\omega t}{2} \right) dt =$

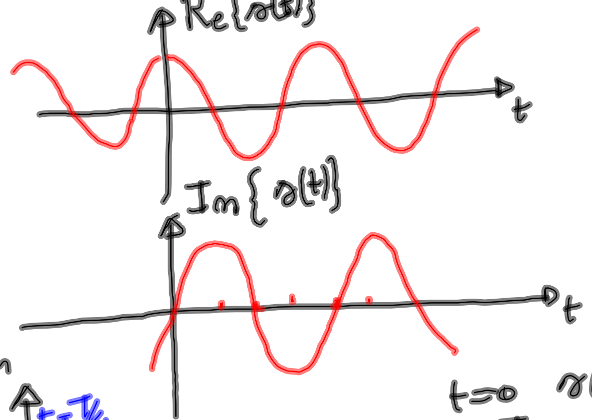
$= \frac{1}{T_0} \int_0^{T_0} \frac{A^2}{2} dt + \cancel{\frac{1}{T_0} \int_0^{T_0} \frac{A^2}{2} \cos 2\omega t \, dt} =$

$= \frac{A^2}{2}$

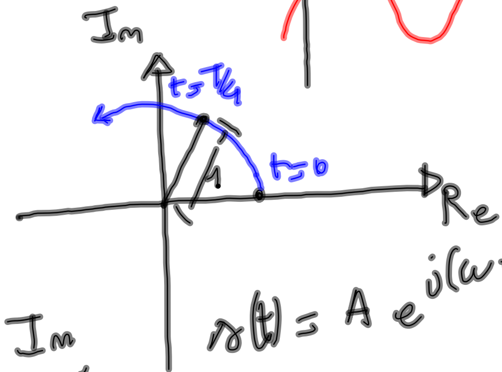


$\omega = 2\pi f_0 = 2\pi \frac{1}{T_0}$

$$s(t) = e^{j\omega t} = \cos\omega t + j\sin\omega t$$



$$\omega = \frac{2\pi}{T}$$

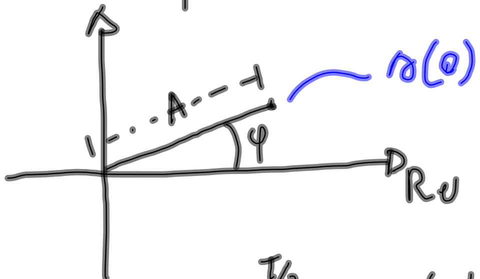


$t = 0 \quad s(0) = 1$

$t = T/4$

$$s(T/4) = e^{j\omega\frac{T}{4}} = e^{j\frac{\pi}{2}}$$

$$s(t) = A e^{j(\omega t + \varphi)}$$



$\omega t + \varphi$ fase istantanea

φ fase iniziale

$$P_s = \frac{1}{T}\int_{-T/2}^{T/2} \left|A e^{j(\omega t+\varphi)}\right|^2 dt = \frac{1}{T}\int_{-T/2}^{T/2} A^2\, dt$$

$$= A^2$$

Funzione delta di Dirac

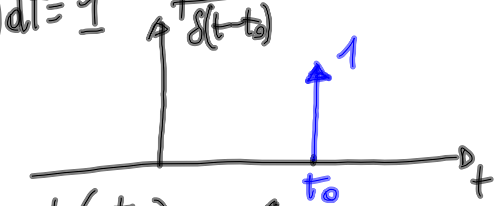
- $\displaystyle\int_{-\infty}^{\infty} f(t)\,\delta(t-t_0)\,dt = \int_{-\infty}^{\infty} f(t)\,\delta(t_0-t)\,dt =$

$$= f(t_0)$$

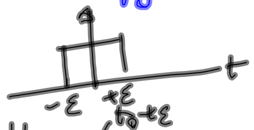
- $\delta(t) = \delta(-t)$ → segnale pari

$$\boxed{\begin{array}{ll} s(t) = s(-t) & \text{pari} \\ s(t) = -s(-t) & \text{dispari} \end{array}}$$

- $\displaystyle\int_{-\infty}^{\infty} \delta(t-t_0)\,dt = \underline{1}$



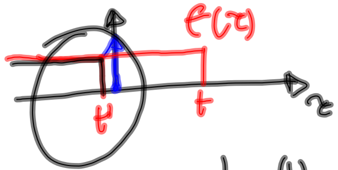
- $g_{2\varepsilon}(t) = \mathrm{rect}\left(\frac{t}{2\varepsilon}\right)$



$$\int_{-\infty}^{\infty} g_{2\varepsilon}(t-t_0)\,\delta(t-t_0)\,dt = \int_{t_0-\varepsilon}^{t_0+\varepsilon} \delta(t-t_0)\,dt =$$

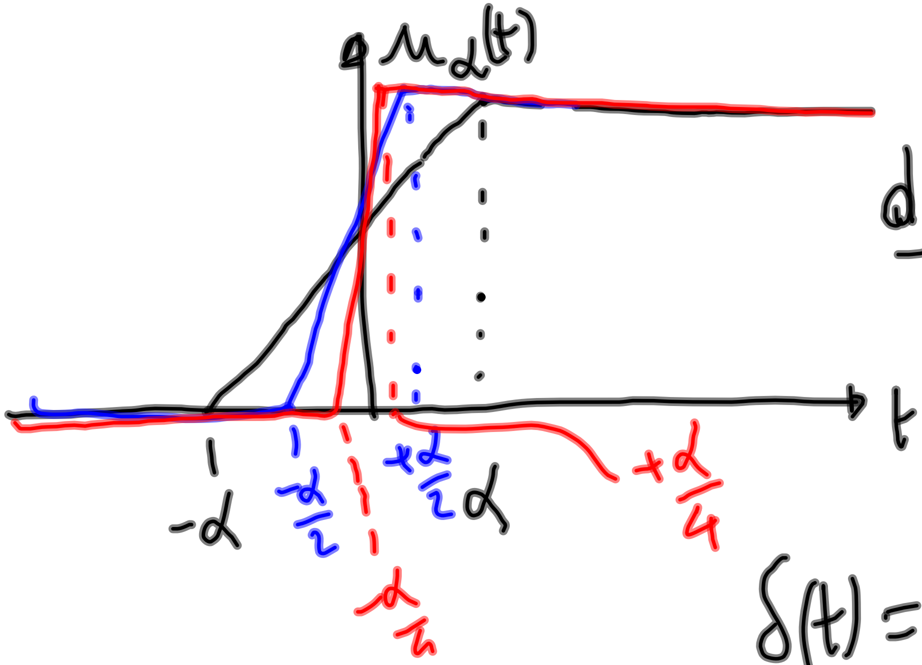
$$= 1$$

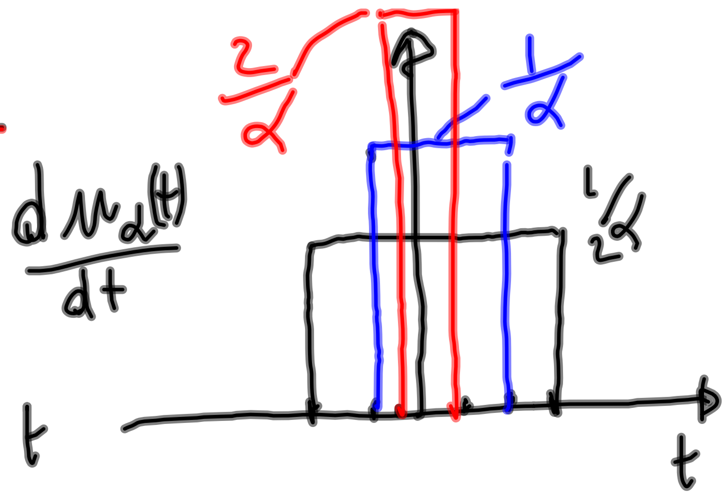
- $\displaystyle\int_{-\infty}^{t} \delta(\tau)\,d\tau = \begin{cases} 1 & \text{se } t \geq 0 \\ 0 & t < 0 \end{cases} = u(t)$



$$\int_{-\infty}^{\infty} f(\tau)\,\delta(\tau)\,d\tau$$

$$\frac{du(t)}{dt} = \delta(t)$$





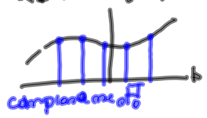
$$\delta(t) = \lim_{\alpha \to 0} \frac{d\, u_\alpha(t)}{dt}$$

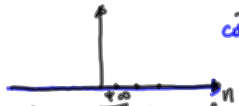
$\delta(t)$ P_s finita

$x[n]$

Sequenza

$x[n] = x(nT)$





$$E_x \triangleq \sum_{n=-\infty}^{+\infty} |x[n]|^2$$

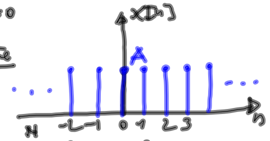
$$P_x = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2$$

caso periodico $\quad x[n] = x[n+N_0] \ \forall n$

$$P = \frac{1}{N_0} \sum_{n=0}^{N_0-1} |x[n]|^2$$

Segnale costante

$x[n] = A$



$E_x \to \infty$

$$P_x = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} A^2 = A^2$$

Segn. esponenziale

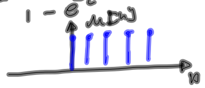
$$x[n] = e^{-n} u[n] = \begin{cases} e^{-n} & n \geq 0 \\ 0 & n < 0 \end{cases}$$

$$E_x = \sum_{n=-\infty}^{\infty} |e^{-n}|^2 = \sum_{n=0}^{\infty} e^{-2n} = \sum_{n=0}^{\infty} (e^{-2})^n = \frac{1}{1-e^{-2}}$$

Rm

$$\boxed{\sum_{n=0}^{\infty} a^n = \frac{1}{1-a}}$$

$$u[n] \triangleq \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$



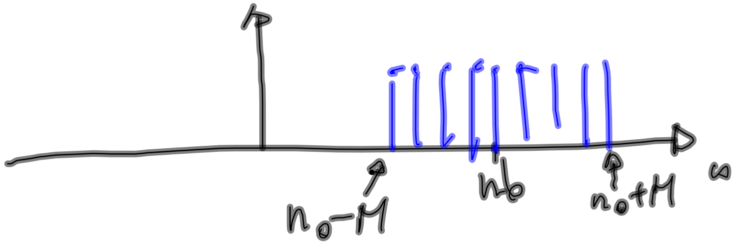
Segn. esp. complesso

$$x[n] = A e^{j\frac{2\pi n}{N_0}}$$

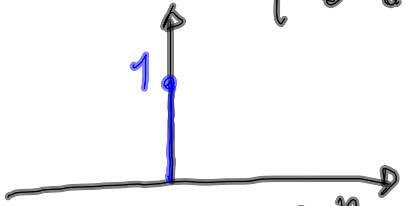
$$P_x = \frac{1}{N_0} \sum_{n=0}^{N_0-1} A^2 \underbrace{\left|e^{j\frac{2\pi n}{N_0}}\right|^2}_{=1} = A^2$$

Impulso rettangolare

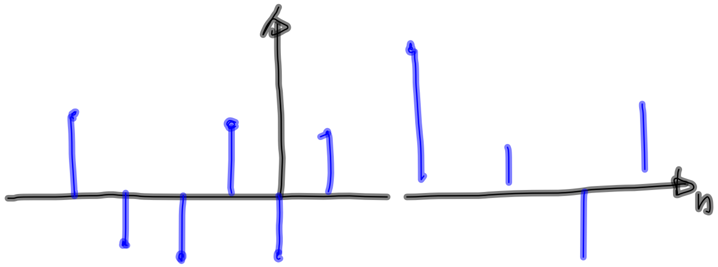
$$x[n] = \begin{cases} 1 & |n-n_0| \le M \\ 0 & |n-n_0| > M \end{cases}$$



$$\delta[n] = \begin{cases} 1 & \text{per } n=0 \\ 0 & \text{altrove} \end{cases}$$



$$x[n] = \sum_{k=-\infty}^{\infty} a_k \, \delta[k-n] \qquad a_k = x[k]$$



[redacted]

$s(t) \in \mathbb{R}$

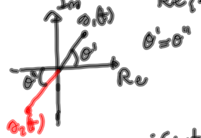
$s(t) \in \mathbb{C}$

$$s(t) = \mathrm{Re}\{s(t)\} + j\,\mathrm{Im}\{s(t)\} =$$
$$= A(t)\, e^{j\theta(t)}$$

$$A(t) = |s(t)| = \sqrt{\mathrm{Re}\{s(t)\}^2 + \mathrm{Im}\{s(t)\}^2} =$$
$$= \sqrt{s(t)\, s^*(t)}$$

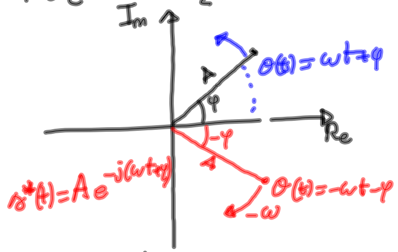
$$\theta(t) = \mathrm{atg}\,\frac{\mathrm{Im}\{s(t)\}}{\mathrm{Re}\{s(t)\}}$$

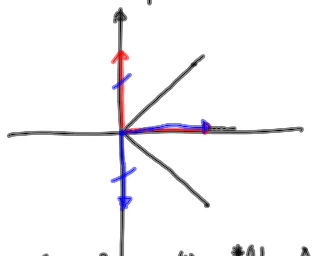
⚠ I e IV quadrante
altrimenti aggiungere π



$$s(t) = A\, e^{j(\omega t + \varphi)} = A\cos(\omega t + \varphi) + jA\sin(\omega t + \varphi)$$

$$\mathrm{Re}\{s(t)\} = \frac{s(t) + s^*(t)}{2} = A\cos(\omega t + \varphi)$$





$$\mathrm{Im}\{s(t)\} = \frac{s(t) - s^*(t)}{2j} = A\sin(\omega t + \varphi)$$

$$s_1(t) = A_1\cos(\omega_1 t + \varphi_1) = \frac{A_1}{2} e^{j(\omega_1 t + \varphi_1)} + \frac{A_1}{2} e^{-j(\omega_1 t + \varphi_1)}$$