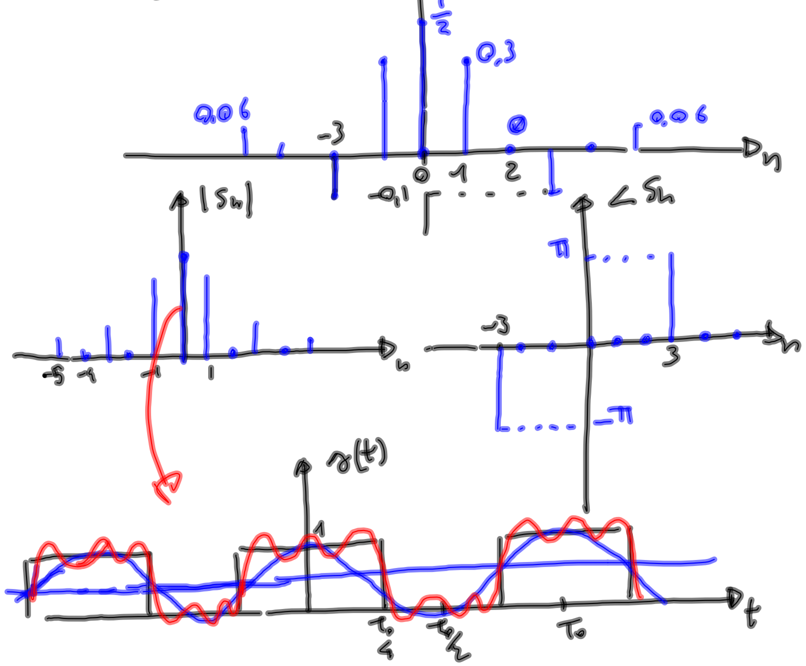
$$s(t)=\sum_{n=-\infty}^{\infty} S_n\, e^{j2\pi n \frac{t}{T_0}} \qquad s(t)=s(t+T_0)\ \forall t$$

$$s(t)\in\mathbb{R} \iff S_n = S_{-n}^{*}$$

$$\begin{cases} |S_n| = |S_{-n}| \\ \angle S_n = -\angle S_{-n} \end{cases}$$

<u>onda quadra</u>

$$S_n = \frac{1}{2}\,\mathrm{sinc}\left(\frac{n}{2}\right)$$



$$|S_n| \to 0 \qquad \frac{1}{n}$$

$s(t) \in \mathbb{C}$

$s(t)$ pari $\quad s(t) = s(-t)$

$$S_n = \frac{1}{T_0} \int_{[T_0]} s(t) e^{-j\frac{2\pi n t}{T_0}} dt =$$

$$= \frac{1}{T_0} \int_{[T_0]} s(t) \cos\frac{2\pi n t}{T_0} dt - \frac{1}{T_0} j \cancel{\int_{[T_0]} s(t) \sin\frac{2\pi n t}{T_0}} dt =$$

integrate tra $-\frac{T_0}{2}$ e $\frac{T_0}{2}$

$$S_n = \frac{2}{T_0} \int_0^{\frac{T_0}{2}} s(t) \cos\frac{2\pi n t}{T_0} dt$$

$$S_n = S_{-n}$$

- $s(t)$ dispari $\quad s(t) = -s(-t)$

$$S_n = -\frac{2j}{T_0} \int_0^{\frac{T_0}{2}} s(t) \sin\frac{2\pi n t}{T_0} dt$$

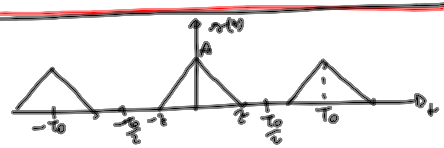
$$S_{-n} = -S_n$$

- $s(t) = s(-t) \Longleftrightarrow S_n = S_{-n}$
- $s(t) = -s(-t) \Longleftrightarrow S_n = -S_{-n}$

$$\begin{cases} s(t) = s(-t) \\ s(t) \in \mathbb{R} \end{cases} \Leftrightarrow \begin{cases} S_n = S_{-n} \\ S_n = S_{-n}^{*} \end{cases}$$

$$\begin{cases} R_n = R_{-n} & I_n = I_{-n} \\ R_n = R_{-n} & I_n = -I_{-n} \end{cases} \Rightarrow \quad I_n = 0, \quad R_n = \frac{2}{T_0}\int_0^{T_0/2} s(t)\cos\frac{2\pi n t}{T_0}\,dt$$

$$\begin{cases} s(t) = -s(-t) \\ s(t) \in \mathbb{R} \end{cases} \Rightarrow \quad R_n = 0, \quad I_n = -\frac{2}{T_0}\int_0^{T_0/2} s(t)\sin\frac{2\pi n t}{T_0}\,dt$$



$s(t) = s(-t)$

$$S_n = \frac{2}{T_0}\int_0^{T_0/2} s(t)\cos\frac{2\pi n t}{T_0}\,dt$$
(R_n)

$$s(t) = \begin{cases} 0 & \tau < t \le \frac{T_0}{2} \\ A\left(1-\frac{t}{\tau}\right) \end{cases}$$

$s(t) = a + bt$
$s(0) = A = a$
$s(\tau) = a + b\tau = 0$
$b = -\frac{A}{\tau}$

$$R_n = \frac{2}{T_0}\int_0^{\tau} A\left(1-\frac{t}{\tau}\right)\cos\frac{2\pi n t}{T_0}\,dt =$$

$$= \frac{2}{T_0}\int_0^{\tau} A\cos\frac{2\pi n t}{T_0}\,dt - \frac{2A}{T_0\tau}\int_0^{\tau} t\cos\frac{2\pi n t}{T_0}\,dt =$$

$$= \frac{2A}{T_0}\frac{T_0}{2\pi n}\sin\frac{2\pi n t}{T_0}\Big|_0^{\tau} - \frac{2A}{T_0\tau}\left(t\sin\frac{2\pi n t}{T_0}\Big|_0^{\tau} -\right.$$

$$\left. - \int_0^{\tau}\sin\frac{2\pi n t}{T_0}\,dt\right)\frac{T_0}{2\pi n} = \frac{2A}{T_0}\frac{T_0}{2\pi n}\sin\frac{2\pi n \tau}{T_0} -$$

$$- \frac{2A}{T_0\tau}\frac{T_0}{2\pi n}\left(\cancel{\tau}\sin\frac{2\pi n\tau}{T_0} + \frac{T_0}{2\pi n}\cos\frac{2\pi n t}{T_0}\Big|_0^{\tau}\right) =$$

$$= \cancel{\frac{2A}{T_0}\frac{T_0}{2\pi n}\sin\frac{2\pi n\tau}{T_0}} - \cancel{\frac{2A}{T_0}\frac{T_0}{2\pi n}\sin\frac{2\pi n\tau}{T_0}} -$$

$$- \frac{2A}{T_0\tau}\frac{(T_0)^2}{(2\pi n)^2}\left(\cos\frac{2\pi n\tau}{T_0} - 1\right) =$$

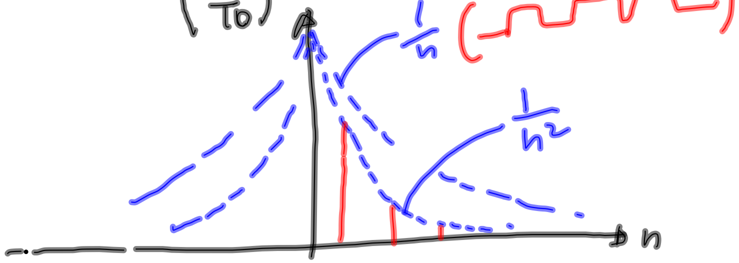
$\cos 2\alpha = 1 - 2\sin^2\alpha$

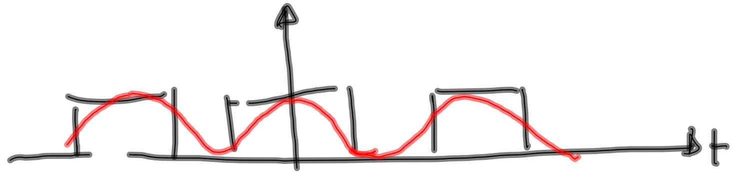
$$= \cancel{-}\frac{2A}{T_0\tau}\frac{T_0^2}{(2\pi n)^2}\left(\cancel{-}2\sin^2\left(\frac{\pi n\tau}{T_0}\right)\right) =$$

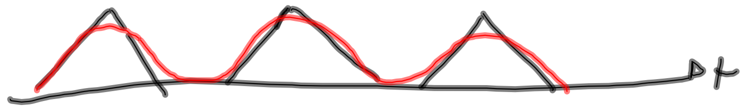
$\text{sinc}(\alpha) = \frac{\sin\pi\alpha}{\pi\alpha}$

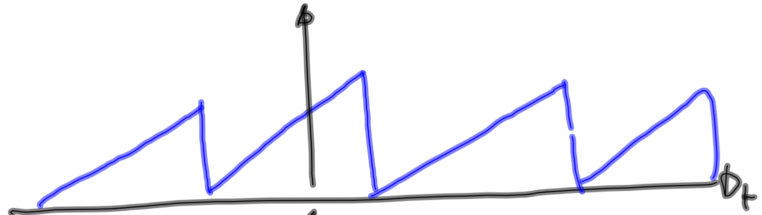
$$= \frac{2A\tau}{4T_0}\frac{1}{\left(\frac{\pi n\tau}{T_0}\right)^2}$$

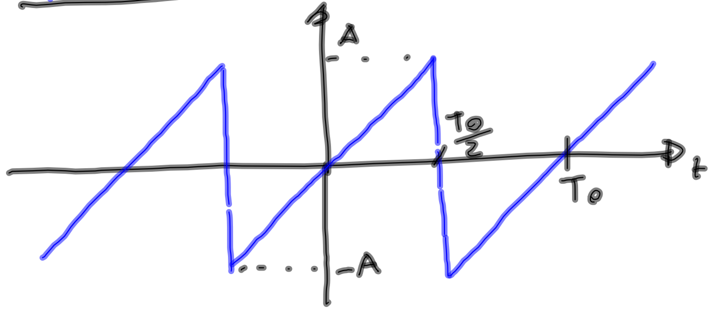
$$= \frac{\cancel{4} A \tau}{4 T_0} \frac{1}{\left(\frac{\pi n \tau}{T_0}\right)^2} \sin^2 \frac{\pi n \tau}{T_0} = A \frac{\tau}{T_0} \mathrm{sinc}^2\left(\frac{n\tau}{T_0}\right)$$





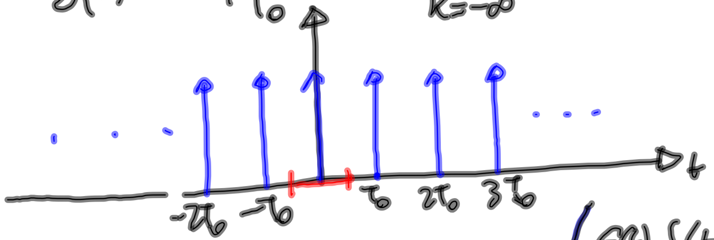






$\delta(t)$

$$s(t) = \mathrm{rep}_{T_0}(\delta(t)) = \sum_{k=-\infty}^{+\infty} \delta(t-kT_0)$$

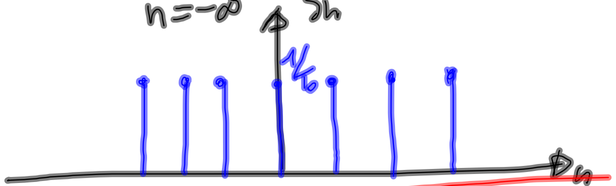


$$\int f(t)\,\delta(t-t_0)\,dt = f(t_0)$$

$$S_n = \frac{1}{T_0}\int_{[T_0]} s(t)\, e^{-j\frac{2\pi n t}{T_0}}\, dt =$$

$$= \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} \delta(t)\,\underbrace{e^{-j\frac{2\pi n t}{T_0}}}_{f(t)}\, dt = \frac{1}{T_0}$$

$$s(t) = \sum_{n=-\infty}^{\infty} S_n\, e^{j\frac{2\pi n t}{T_0}} = \sum_{n=-\infty}^{\infty} \frac{1}{T_0}\, e^{j\frac{2\pi n t}{T_0}}$$



N.B.

$$\boxed{\sum_{n=-\infty}^{\infty} \delta(t-nT_0) = \sum_{n=-\infty}^{\infty} \frac{1}{T_0}\, e^{j\frac{2\pi n t}{T_0}}}$$

$$s(t) = \sum_{n=-\infty}^{\infty} S_n \, e^{j\frac{2\pi n t}{T_0}}$$

$$\int_{-\infty}^{+\infty} S(f)\, df \; e^{j2\pi f t}$$

$$\boxed{f = \lim_{T_0 \to \infty} \frac{n1}{T_0}}$$

$$S(f)\, df$$

$S(f)$ trasformata continua di Fourier di $s(t)$

$$S_n = \frac{1}{T_0} \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} s(t)\, e^{-j\frac{2\pi n t}{T_0}}\, dt$$

$$\lim_{T_0 \to \infty} \qquad S_n = \frac{1}{T_0} \int_{-\infty}^{\infty} s(t)\, e^{-j2\pi f t}\, dt$$

$$S(f) \triangleq \int_{-\infty}^{\infty} s(t)\, e^{-j2\pi f t}\, dt$$

$$s_p(t) = \sum_{n=-\infty}^{\infty} S_n \, e^{j\frac{2\pi n t}{T_0}} = \qquad S_n = f_0 S\left(\frac{n}{T_0}\right)$$

$$= \sum_{n=-\infty}^{\infty} S\left(\frac{n}{T_0}\right) f_0 \, e^{j\frac{2\pi n t}{T_0}}$$

$$\lim_{T_0 \to \infty} \qquad f_0 \to \Delta f \qquad s(t) = \int_{-\infty}^{+\infty} S(f)\, e^{-j2\pi f t}\, df$$

$$\frac{n}{T_0} \to f$$

$$\sum \to \int$$

$$S(f) = |S(f)|\, e^{j\theta(f)}$$