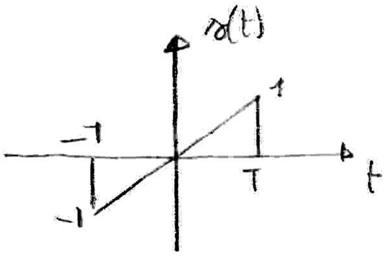
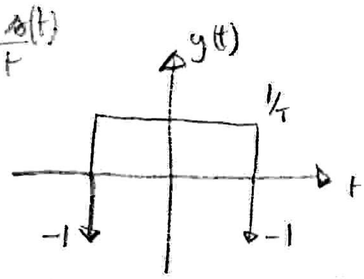


$y(t) = \frac{d\,s(t)}{dt}$



$$y(t) = -\delta(t+T) + \frac{1}{T}\,\mathrm{rect}\left(\frac{t}{2T}\right) - \delta(t-T)$$

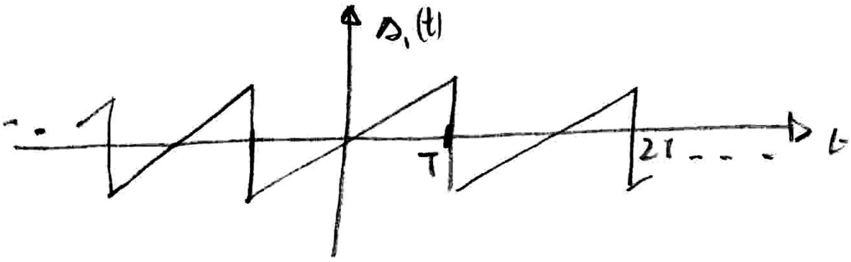
$$Y(f) = -e^{j2\pi f T} + 2\,\mathrm{sinc}(2Tf) - e^{-j2\pi f T} =$$

$$= 2\,\mathrm{sinc}(2Tf) - 2\cos 2\pi f T$$

$Y(0) = 0$

$$S(f) = \frac{Y(f)}{j2\pi f} = -j\left(\frac{\mathrm{sinc}(2Tf)}{\pi f} - \frac{\cos 2\pi f T}{\pi f}\right)$$

$$s_1(t) = \sum_{K=-\infty}^{\infty} s(t - K2T)$$



$$f_0 = \frac{1}{2T} = 0{,}5\ Hz$$

segnale dispari, reale $\quad S_n = j\,I_n \quad S_{-n} = -S_n = -j\,I_n$

$S_0 = 0 \quad |S_n| \propto \frac{1}{n}$

$$S_n = \frac{1}{2T} S\left(\frac{n}{2T}\right) = \frac{1}{2}\left[-j\left(\frac{2\,\mathrm{sinc}(n)}{\pi n} - 2\,\frac{\cos \pi n}{\pi n}\right)\right]$$

$n \neq 0$

$S_0 = 0$

$$S_1 = \frac{1}{2}\left[-j\left(\frac{2}{\pi} + \frac{2}{\pi}\right)\right] = -j\,\frac{2}{\pi} = \frac{2}{\pi}e^{-j\frac{\pi}{2}} \qquad \underset{n\geq 2}{S_n} = j\,\frac{\cos \pi n}{\pi n} = j\,\frac{(-1)^n}{\pi n}$$

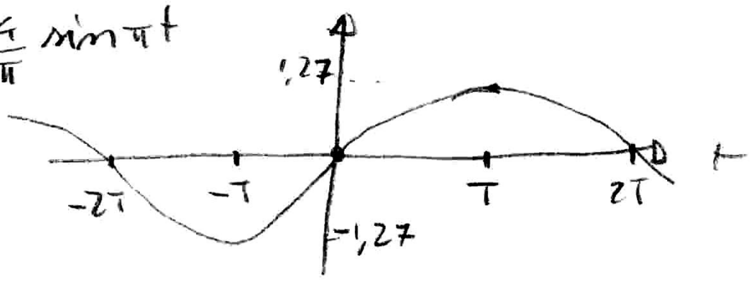
$$S_{-1} = \frac{2}{\pi}e^{j\frac{\pi}{2}}$$

sintesi con le sole componenti $n = 1$ e $n = -1$

$$s_1(t) = S_1\, e^{j2\pi\frac{t}{2T}} + S_{-1}\, e^{-j2\pi\frac{t}{2T}} = |S_1|\, e^{j\left(2\pi\frac{t}{2T} + \angle S_1\right)} + |S_{-1}|\, e^{-j2\pi\frac{t}{2T} + j\angle S_{-1}} =$$

$$= |S_1|\, e^{j\left(2\pi\frac{t}{2T} + \angle S_1\right)} + |S_1|\, e^{-j\left(2\pi\frac{t}{2T} + \angle S_1\right)} = 2|S_1| \cos\left(2\pi\frac{t}{2T} + \angle S_1\right) =$$

$$= \frac{4}{\pi}\cos\left(\pi t - \frac{\pi}{2}\right) = \frac{4}{\pi}\sin \pi t$$

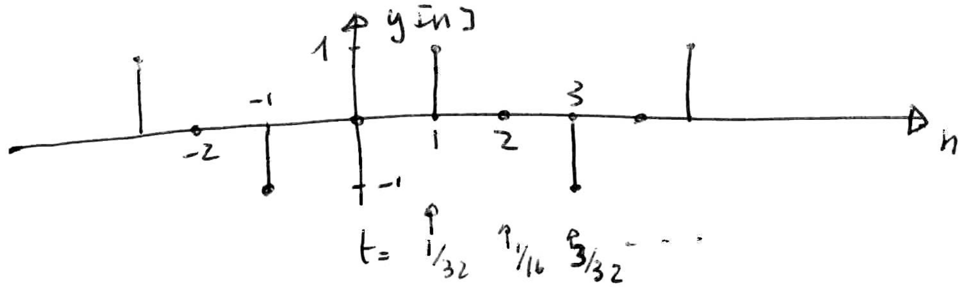


$y(t) = \sin(16\pi t)$

$f = 8\ Hz \qquad f_c \geq 16\ Hz$

la f_c richiesta è quindi $f_c = 32\ Hz \Rightarrow T_c = \frac{1}{32}\,s$

$y[n] = y(nT) = \sin\left(\frac{\pi n}{2}\right)$



<u>TDF</u> $\quad N_0 = 4$

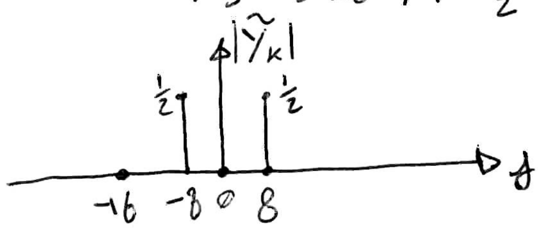
$$\tilde{Y}_k = \frac{1}{4}\sum_{n=0}^{3} y[n]\, e^{-j2\pi n\frac{k}{4}} = \frac{1}{4}\left(e^{-j2\pi\frac{k}{4}} - e^{-j2\pi 6\frac{k}{4}}\right) =$$

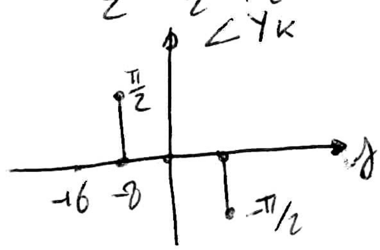
$$= \frac{1}{4}\, e^{-j2\pi 2\frac{k}{4}}\left[e^{j2\pi\frac{k}{4}} - e^{-j2\pi\frac{k}{4}}\right] = j\frac{1}{2}\sin\left(2\pi\frac{k}{4}\right) e^{-j4\pi\frac{k}{4}} =$$

$$= \frac{j}{2}\sin\left(\frac{\pi k}{2}\right) e^{-j\pi k}$$

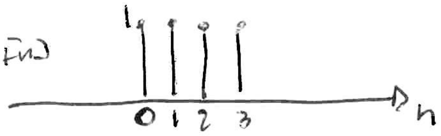
$\tilde{Y}_0 = 0 \qquad \tilde{Y}_1 = -\frac{j}{2} = \frac{1}{2}e^{-j\frac{\pi}{2}} \qquad \tilde{Y}_2 = \tilde{Y}_{2-N_0} = \tilde{Y}_{-2} = 0$

$\tilde{Y}_3 = \tilde{Y}_{3-N_0} = \tilde{Y}_{-1} = \frac{j}{2}(-1)\, e^{j\pi} = \frac{j}{2} = \frac{1}{2}e^{j\frac{\pi}{2}}$





Finestra $w[n]$



$$\overline{W}(f) = \sum_{n=0}^{3} e^{-j2\pi n f T} = \sum_{n=0}^{3} \left(e^{-j2\pi f T}\right)^n = \frac{1-e^{-j8\pi f T}}{1-e^{-j2\pi f T}} =$$

$$= \frac{e^{-j4\pi f T}}{e^{-j\pi f T}} \; \frac{e^{j4\pi f T}-e^{-j4\pi f T}}{e^{j\pi f T}-e^{-j\pi f T}} = \frac{2j\sin(4\pi f T)}{2j\sin(\pi f T)}\, e^{-j3\pi f T}$$

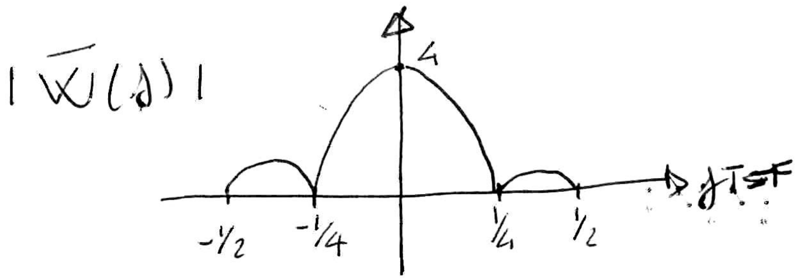
$$\overline{Y}(f) = \frac{\delta(f+f_0)-\delta(f-f_0)}{2j} = \frac{1}{2j}\left[\delta\left(f+\frac{1}{4}\right)-\delta\left(f-\frac{1}{4}\right)\right]$$

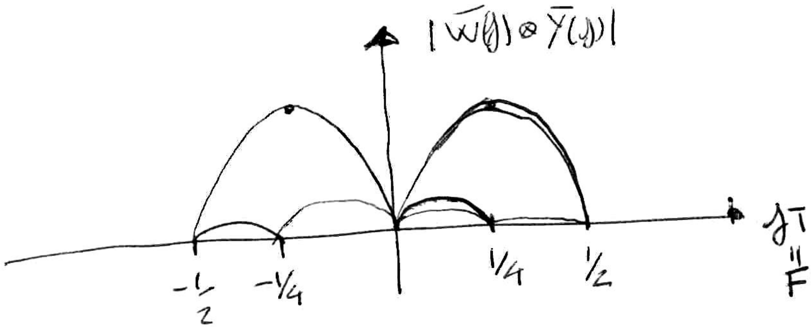
per $f \in \left[-\frac{1}{2T}, \frac{1}{2T}\right]$

$$\overline{Z}(f) = \overline{W}(f) \otimes \overline{Y}(f)$$

convoluzione circolare, perché sia $\overline{W}(f)$ che $\overline{Y}(f)$ sono periodiche di periodo $\frac{1}{T}$

$$\overline{Z}(f) = \frac{1}{1/T}\int_{-\frac{1}{2T}}^{\frac{1}{2T}} \overline{W}(\alpha)\, \overline{Y}(f-\alpha)\, d\alpha$$





I massimi si hanno in corrispondenza di $F=\frac{1}{4T}$ e $F=-\frac{1}{4T}$ ovvero $f = 8$ Hz e $f=-8$ Hz

si annulla per $F=0$ $F=\frac{1}{2}$ e $F=-\frac{1}{2}$

Es. 3

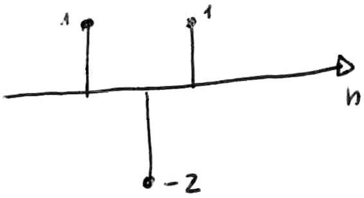
$y[n] = x[n-2] - x[n-3] + 0{,}4\, y[n-1] - 0{,}81\, y[n-2]$

$$Y(z)\left[1 - 0{,}4 z^{-1} + 0{,}81 z^{-2}\right] = X(z)\left[z^{-2} - z^{-3}\right]$$

$$H(z) = \frac{z^{-2} - z^{-3}}{1 - 0{,}4 z^{-1} + 0{,}81 z^{-2}} = \frac{z-1}{z\,(z^2 - 0{,}4z + 0{,}81)}$$

zeri $z = 1$

poli $z_{p_1} = 0$ $\quad z_{p_2} = 0{,}2 + 0{,}2775 i$ $\quad z_{p_3} = z_{p_2}^*$



$$h[n] = \delta[n] - 2\delta[n-1] + \delta[n-2]$$

$$H(z) = 1 - 2z^{-1} + z^{-2} = \frac{z^2 - 2z + 1}{z^2}$$

zeri $z_1 = z_2 = 1$

poli $z_{p_1} = z_{p_2} = \emptyset$

3) l'uscita è un oscillazione il cui periodo è 1/3 rispetto a quello di partenza
nel caso fosse passa alto, si aspetteremmo contributi più significativi nei passaggi da $\emptyset$ a 1 e da 1 a $\emptyset$
mancano oscillazioni alla frequenza del segnale e manca il valor medio
quindi si tratta di un passa banda