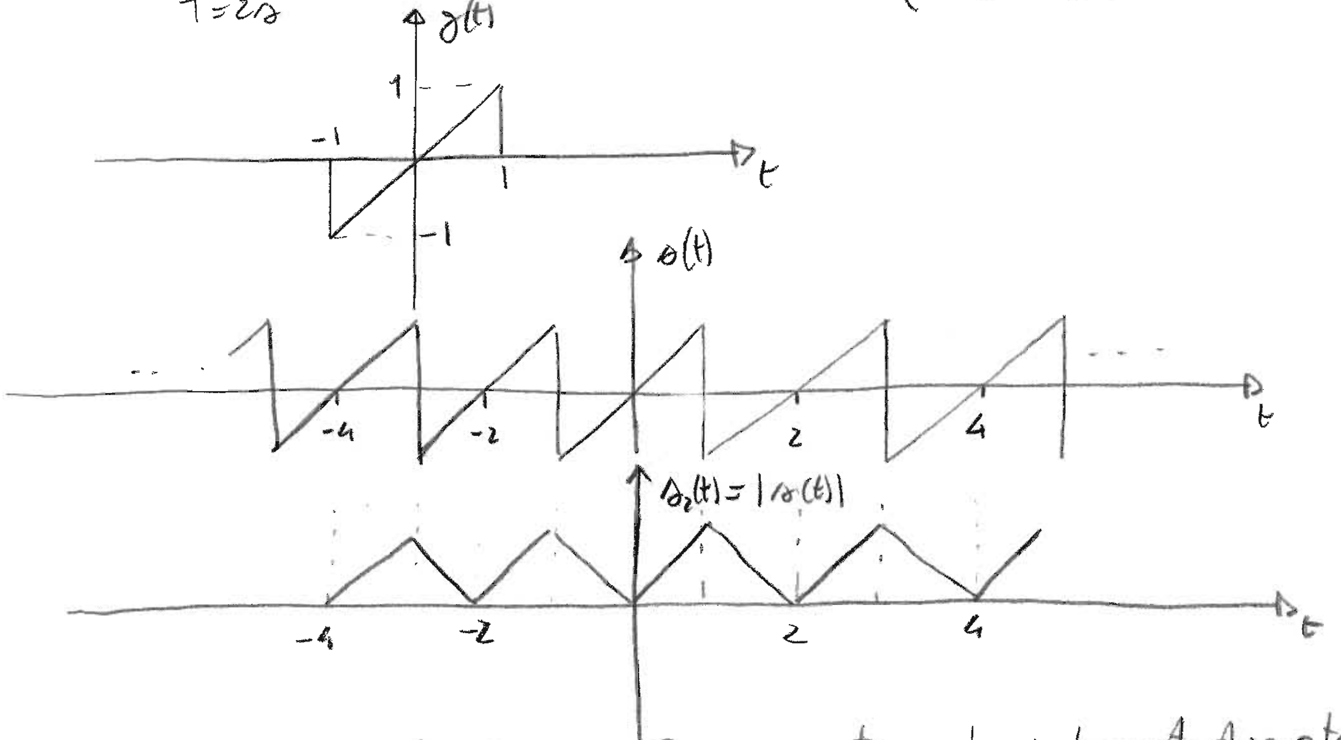
A.1

$$s(t) = g(t) \otimes \sum_{k=-\infty}^{\infty} \delta(t-kT) \qquad g(t) = \begin{cases} t & \text{per } |t|<T/2 \\ 0 & \text{altrove} \end{cases}$$

$T = 2s$



La Discussione richiesta è ampia. Qui si riportano alcuni elementi di sintesi

$s(t)$: infinite componenti

andamento modulo coeff. $\propto \frac{1}{n}$

$S_n = -S_{-n} \quad S_n = S_{-n}^* \Rightarrow S_n = jI_n = -jI_{-n}$

$f_n = \frac{n}{2}$ multiple di $f_0 = \frac{1}{2}$

$S_0 = 0$

$s_2(t)$: infinite componenti

andamento modulo coeff. $\propto \frac{1}{n^2}$

$S_n = S_{-n} \quad S_n = S_{-n}^* \Rightarrow S_n = R_n = R_{-n}$

$f_n = \frac{n}{2}$ multiple di $f_0 = \frac{1}{2}$

$S_0 = \frac{1}{2}$

I coeff. S_n della SF di $s(t)$ possono trovarsi come

$$S_n = \frac{1}{2}\int_{-1}^{1} s(t)\, e^{-j\frac{2\pi n t}{2}}\, dt \qquad I_n = \frac{-2}{2}\int_0^1 s(t) \sin 2\pi n \frac{t}{2}\, dt$$

oppure come $S_n = \frac{1}{T} G\left(\frac{n}{T}\right)$ dove $G(f) = \mathcal{F}[g(t)]$

considero $h(t) = g'(t) = -\delta(t+T/2) + rect\left(\frac{t}{T}\right) - \delta(t-T/2)$

$$H(f) = -e^{+j2\pi f T/2} + T\,sinc(fT) - e^{-j2\pi f T/2} =$$
$$= -e^{-j2\pi f} + 2\,sinc(2f) - e^{-j2\pi f} =$$
$$= 2\,sinc(2f) - 2\cos(2\pi f)$$

$$G(f) = \frac{H(f)}{j2\pi f} = \frac{2\,sinc(2f)}{j2\pi f} - \frac{2\cos 2\pi f}{j2\pi f}$$

N.B. $\boxed{H(0)=0}$

$$S_n = \frac{1}{2} G\left(\frac{n}{2}\right) = \frac{2\,sinc(n)}{2\,j\pi n} - \frac{2}{2}\frac{\cos \pi n}{j\pi n} = -j\left(\frac{sinc(n)}{\pi n} - \frac{\cos(\pi n)}{\pi n}\right)$$

per $n=0$ non si ottiene una forma semplice $\Rightarrow$

$$S_0 = \frac{1}{2}\int_{-1}^{1} s(t)\, e^{j2\pi 0 \frac{t}{2}}\, dt = \frac{1}{2}\int_{-1}^{1} s(t)\, dt = 0$$

$$S_1 = -j\left(\cancel{\frac{1}{\pi}}^{\,0} + \frac{1}{\pi}\right) = -\frac{\cancel{2}j}{\pi} \Rightarrow S_{-1} = \frac{\cancel{2}j}{\pi}$$

$$\hat{s}(t) = 0 + \frac{\cancel{2}j}{\pi} e^{-j2\pi\frac{t}{2}} - \frac{\cancel{2}j}{\pi} e^{j2\pi\frac{t}{2}} = \frac{2}{\pi}\sin\pi t$$

ric. con
$n = 0, 1$ e -1

$h(t) = \text{sinc}^2(10t)$

$h(t)$ non è causale $\Rightarrow$ non fisic. realizzabile

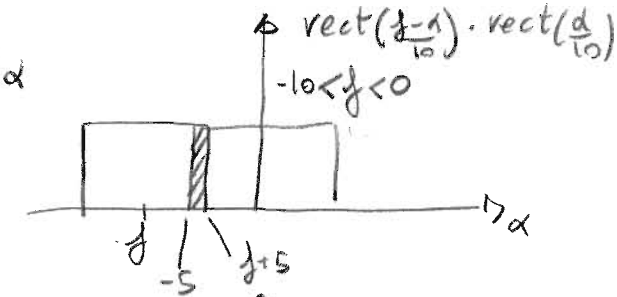
$h(t) = \text{sinc}(10t) \cdot \text{sinc}(10t)$

$H(f) = \mathcal{F}[\text{sinc}(10t)] \otimes \mathcal{F}[\text{sinc}(10t)]$

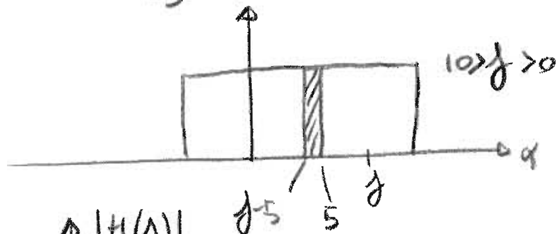
$\mathcal{F}[\text{sinc}(10t)] = \frac{1}{10} \text{rect}\left(\frac{f}{10}\right)$

$H(f) = \frac{1}{10} \text{rect}\left(\frac{f}{10}\right) \otimes \frac{1}{10} \text{rect}\left(\frac{f}{10}\right) =$

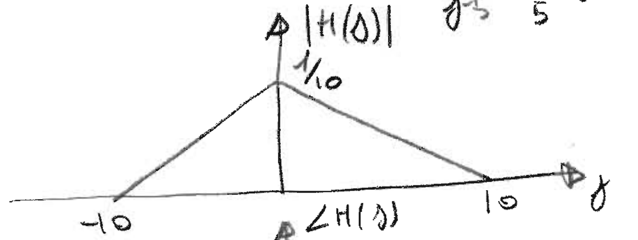
$= \frac{1}{100} \int_{-\infty}^{\infty} \text{rect}\left(\frac{\alpha}{10}\right) \text{rect}\left(\frac{f-\alpha}{10}\right) d\alpha$

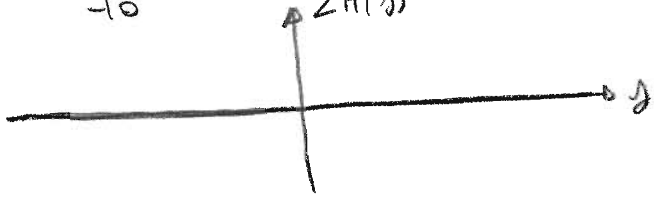


$$H(f) = \begin{cases} 0 & \text{per } f < -10 \\ (f+5+5) \cdot \frac{1}{100} & -10 < f < 0 \\ (-f+5+5) \frac{1}{100} & 0 < f < 10 \\ 0 & f > 10 \end{cases} =$$



$$= \begin{cases} 0 & \text{per } f < -10 \\ \frac{f+10}{100} & 0 < f < -10 \\ \frac{-f+10}{100} & 10 > f > 0 \\ 0 & \text{''} \quad f > 0 \end{cases}$$



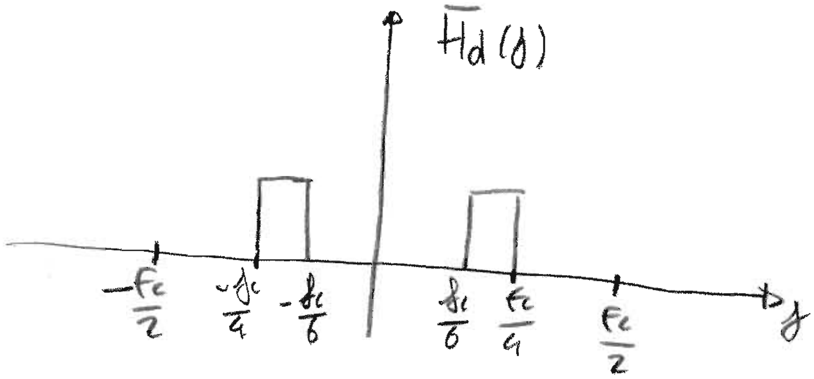


$y(t) = x(t) \otimes h(t)$

$$Y(f) = X(f)\,H(f) = \left(\frac{\delta(f-7,5) + \delta(f+7,5)}{2j} + \frac{\delta(f-10) + \delta(f+10)}{2} \right) H(f) =$$

$$= H(7,5) \frac{\delta(f-7,5)}{2j} + H(-7,5) \frac{\delta(f+7,5)}{2j} = \frac{3}{40} \frac{\delta(f-7,5)}{2j} + \frac{3}{40} \frac{\delta(f+7,5)}{2j} =$$

$$= \frac{3}{40} \sin(5\pi t)$$



$$h_d[n] = \frac{1}{f_c}\int_{-\frac{f_c}{2}}^{\frac{f_c}{2}} \overline{H}_d(f)\, e^{j2\pi n f T}\, df = \frac{1}{f_c}\int_{-\frac{f_c}{4}}^{-\frac{f_c}{6}} e^{j2\pi n f T}\, df\, +$$

$$+ \frac{1}{f_c}\int_{\frac{f_c}{6}}^{\frac{f_c}{4}} e^{j2\pi n f T}\, df = \frac{1}{\cancel{f_c}}\left[\frac{1}{j2\pi n \cancel{T}}\left(e^{-j2\pi n \cancel{T}\frac{\cancel{f_c}}{\cancel{6}\,3}} - e^{-j2\pi n \cancel{T}\frac{\cancel{f_c}}{\cancel{4}\,2}}\right) + \right.$$

$$\left. + \frac{1}{j2\pi n \cancel{T}}\left(e^{j2\pi n \cancel{T}\frac{\cancel{f_c}}{\cancel{4}\,2}} - e^{j2\pi n \cancel{T}\frac{\cancel{f_c}}{\cancel{6}\,3}}\right)\right] =$$

$$= \frac{e^{-j\frac{\pi n}{3}} - e^{j\frac{\pi n}{3}}}{j2\pi n} + \frac{e^{j\frac{\pi n}{2}} - e^{-j\frac{\pi n}{2}}}{j2\pi n} = \frac{-2j\sin\frac{\pi n}{3}}{j2\pi n} + \frac{2j\sin\frac{\pi n}{2}}{j2\pi n} =$$

$$= 2\cdot\frac{\sin\frac{\pi n}{2}}{\frac{\pi n}{2}} - 3\,\frac{\sin\frac{\pi n}{3}}{\frac{\pi n}{3}} = 2\,\mathrm{sinc}\left(\frac{n}{2}\right) - 3\,\mathrm{sinc}\left(\frac{n}{3}\right)$$

Bisogna troncare e traslare al fine di rendere finita e causale la risposta

$$h[n] = \left[2\,\mathrm{sinc}\left(\frac{n-3}{2}\right) - 3\,\mathrm{sinc}\left(\frac{n-3}{3}\right)\right]\left[u[n] - u[n-7]\right]$$

$$x[n] = \cos\frac{2\pi n}{5} = \cos\frac{2\pi n T}{5T}$$

visto che ho due fasori alla stessa freq. e tali per cui $\tilde{X}_1 = \tilde{X}_{-1}^*$

calcolo $y[n] = \frac{\overline{H}\left(\frac{1}{5T}\right)}{2} e^{j\frac{2\pi n}{5}} + \frac{\overline{H}\left(-\frac{1}{5T}\right)}{2} e^{-j\frac{2\pi n}{5}}$

per trovare $\overline{H}(\frac{1}{5T})$ e $\overline{H}(-\frac{1}{5T})$ calcolo $\overline{H}(f)$ in $f=\frac{1}{5T}$ partendo dai coeff. della $h[n]$

L'alternativa è trovare la forma completa di $\overline{H}(f)$: in questo caso tale operazione è complessa

$$h[0] = 2\,\mathrm{sinc}(\tfrac{3}{2}) - 3\,\mathrm{sinc}(-1) = 2\,\mathrm{sinc}(-\tfrac{3}{2}) \cong -0{,}42$$

$$h[1] = 2\,\mathrm{sinc}(-1) - 3\,\mathrm{sinc}(-\tfrac{2}{3}) = -3\,\mathrm{sin}(-\tfrac{2}{3}) \cong 1{,}24$$

$$h[2] = 2\,\mathrm{sinc}(-\tfrac{1}{2}) - 3\,\mathrm{sinc}(-\tfrac{1}{3}) \cong -1{,}21$$

$$h[3] = 1$$

$h[4] = h[2]$ infatti è pari rispetto a $n=3$

$$h[5] = h[1]$$

$$h[6] = h[0]$$

$$\overline{H}(f) = \sum_{n=0}^{6} h[n]\, e^{-j2\pi n f T}$$

$$\overline{H}\left(\tfrac{1}{5T}\right) = \sum_{n=0}^{6} h[n]\, e^{-j\frac{2\pi n}{5}} = -0{,}42 + 1{,}24\, e^{-j\frac{2\pi}{5}} - 1{,}21\, e^{-j\frac{4\pi}{5}} + e^{-j\frac{6\pi}{5}} + \\ -0{,}42\, e^{-j\frac{12\pi}{5}} + 1{,}24\, e^{-j\frac{10\pi}{5}} - 1{,}21\, e^{-j\frac{8\pi}{5}} =$$

$$= e^{-j\frac{6\pi}{5}} \left(-0{,}84 \cos\tfrac{6\pi}{5} + 2{,}48 \cos\tfrac{4\pi}{5} - 2{,}42 \cos\tfrac{2\pi}{5} + 1\right) =$$

$$\cong -1{,}08\, e^{-j\frac{6\pi}{5}} = 1{,}08\, e^{-j\frac{\pi}{5}}$$

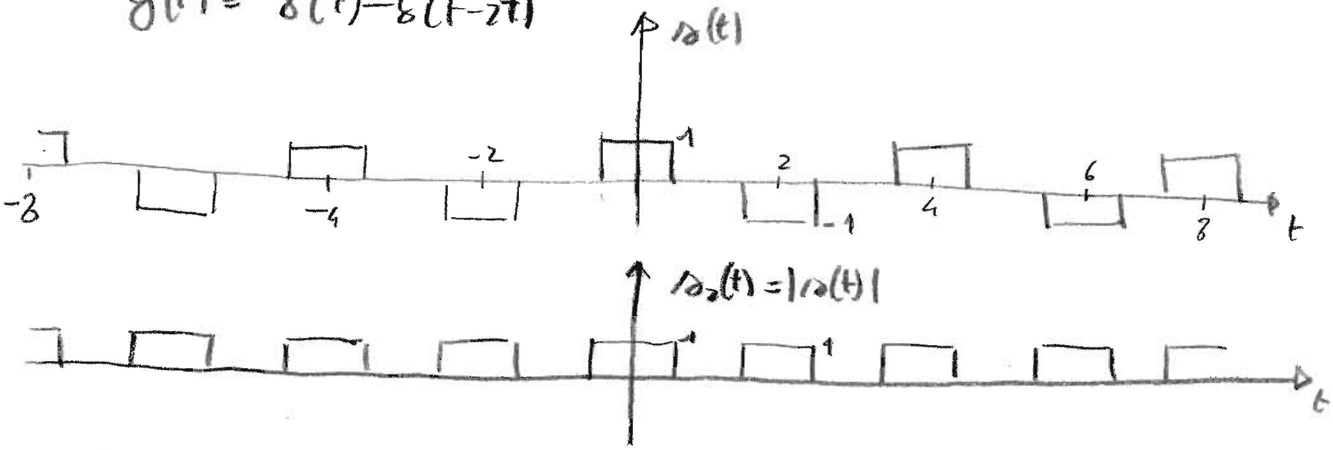
$$\overline{H}\left(-\tfrac{1}{5T}\right) = 1{,}08\, e^{j\frac{\pi}{5}}$$

$$y[n] = \frac{1{,}08}{2}\, e^{-j\frac{\pi}{5}} e^{j\frac{2\pi n}{5}} + \frac{1{,}08}{2}\, e^{j\frac{\pi}{5}} e^{-j\frac{2\pi n}{5}} =$$

$$= 1{,}08 \cos\left(\frac{2\pi n}{5} - \frac{\pi}{5}\right)$$

$$s(t) = rect\left(\frac{t}{\tau}\right) \otimes \sum_{K=-\infty}^{\infty} g(t-K4\tau) \qquad T = 1s$$ D.1.

$$g(t) = \delta(t) - \delta(t-2\tau)$$



La discussione richiesta è ampia. Qui si riportano alcuni elementi di sintesi

$s(t)$: infinite componenti
andamento modulo coeff. $\propto \frac{1}{n}$
$S_n = S_{-n} \quad S_n = S_{-n}^* \Rightarrow S_n = R_n = R_{-n}$
$f_K = \frac{K}{4}$ multiple di $f_0 = 0{,}25\ Hz$
$S_0 = 0$

$s_2(t)$: periodo dimezzato rispetto a $s(t)$ $\quad f_K = \frac{K}{2}$ multiple di $f_0 = 0{,}5\ Hz$
$S_0 = \frac{1}{2}$
l'ampiezza di S_1 sarà inferiore a quella di S_1 nel caso $s(t)$
andamento modulo coeff. $\propto \frac{1}{n}$
$S_n = S_{-n} \quad S_n = S_{-n}^* \Rightarrow S_n = R_n = R_{-n}$

I coeff. S_n di $s(t)$ si possono trovare dalla formula

$$S_n = \frac{1}{4}\int_0^4 s(t)\, e^{-j\frac{2\pi n t}{4}}\, dt \qquad S_n = 2\int_0^2 s(t)\, e^{-j\frac{2\pi n t}{4}}\, dt$$

oppure utilizzando la relazione tra Serie di Fourier e TCF del segnale aperiodico, ottenuto da un periodo del segnale $s(t)$

$$s_1(t) = \text{rect}\left(\frac{t}{T}\right) \otimes \left(\delta(t) - \delta(t-2T)\right) =$$

$$= \text{rect}\left(\frac{t}{T}\right) - \text{rect}\left(\frac{t-2T}{T}\right)$$

$$S_1(f) = T\,\text{sinc}(fT) - T\,\text{sinc}(fT)\, e^{-j2\pi f 2T} =$$

$$= T\,\text{sinc}(fT)\left(1 - e^{-j4\pi f T}\right) =$$

$$= \text{sinc}(f)\, e^{-j2\pi f}\left(e^{j2\pi f} - e^{-j2\pi f}\right) =$$

$$= \text{sinc}(f)\, e^{-j2\pi f}\, 2j \sin 2\pi f$$

$$S_{2,n} = \frac{1}{4} S_1\left(\frac{n}{4}\right) = \frac{1}{2}\, j \sin\left(2\pi \frac{n}{4}\right) \text{sinc}\left(\frac{n}{4}\right) e^{-j\frac{\pi n}{2}} =$$

$$= \frac{j}{2}\, \text{sinc}\left(\frac{n}{4}\right) \sin\left(\frac{\pi n}{2}\right) e^{-j\frac{\pi n}{2}}$$

$S_{2,0} = 0$ → ok

$$S_{2,1} = \frac{j}{2}\, \text{sinc}\left(\frac{1}{4}\right) e^{-j\frac{\pi}{2}} = \frac{1}{2}\, \frac{\sin\left(\frac{\pi}{4}\right)}{\pi/4} = \frac{1}{2}\, \frac{\sqrt{2}}{2}\, \frac{4}{\pi} = \frac{\sqrt{2}}{\pi}$$

$$S_{2,-1} = \frac{\sqrt{2}}{\pi}$$

$$\hat{s}(t) = 0 + \frac{\sqrt{2}}{\pi} e^{j\frac{2\pi t}{4}} + \frac{\sqrt{2}}{\pi} e^{-j\frac{2\pi t}{4}} =$$

$$= 2\frac{\sqrt{2}}{\pi} \cos\left(\frac{\pi t}{2}\right)$$

ricostruito con
$n = 0, 1, -1$