# Derivate

Derivate di funzioni elementari

| | |
|---|---|
| $\frac{d}{dt}C = 0$ | $\frac{d}{dt}t^n = nt^{n-1}$ |
| $\frac{d}{dt}\sqrt{bt} = \frac{b}{2\sqrt{bt}}$ | $\frac{d}{dt}sin(\omega t) = \omega\ cos(\omega t)$ |
| $\frac{d}{dt}cos(\omega t) = -\omega\ sin(\omega t)$ | $\frac{d}{dt}tg(\omega t) = \frac{\omega}{cos^2(\omega t)}$ |
| $\frac{d}{dt}e^{bt} = be^{bt}$ | $\frac{d}{dt}ln(bt) = \frac{1}{t}$ |

Regole di derivazione

| | |
|---|---|
| $\frac{d}{dt}[Cf(t)] = C\frac{df(t)}{dt}$ | $\frac{d}{dt}[f(t)+g(t)] = \frac{df(t)}{dt} + \frac{dg(t)}{dt}$ |
| $\frac{d}{dt}[f(t)g(t)] = \frac{df(t)}{dt}g(t) + f(t)\frac{dg(t)}{dt}$ | $\frac{d}{dt}\left[\frac{f(t)}{g(t)}\right] = \frac{\frac{df(t)}{dt}g(t) - f(t)\frac{dg(t)}{dt}}{g^2(t)}$ |
| $\frac{d}{dt}f\big(x(t)\big) = \frac{df}{dx}\frac{dx}{dt}$ | $\frac{dx}{dt} = \frac{1}{\frac{dt}{dx}}$ |

Seguendo le regole precedenti: alcune derivazioni rispetto al tempo di uso comune

| | | | |
|---|---|---|---|
| $\frac{d}{dt}x = \dot{x}$ | $\frac{d}{dt}\dot{x} = \ddot{x}$ | $\frac{d}{dt}x^2 = 2x\dot{x}$ | $\frac{d}{dt}\dot{x}^2 = 2\dot{x}\ddot{x}$ |
| $se \quad x = L\ cos(\theta) \quad allora \quad \dot{x} = -L\ sin(\theta)\dot{\theta} \quad e \quad \ddot{x} = -L\left[cos(\theta)\dot{\theta}^2 + sin(\theta)\ddot{\theta}\right]$ | | | |
| $se \quad y = L\ sin(\theta) \quad allora \quad \dot{y} = L\ cos(\theta)\dot{\theta} \quad e \quad \ddot{y} = L\left[-sin(\theta)\dot{\theta}^2 + cos(\theta)\ddot{\theta}\right]$ | | | |

# Approssimazioni

Le più usate. Se x<<1

| | | | |
|---|---|---|---|
| $sin(x) \approx x$ | $cos(x) \approx 1\left(-\frac{x^2}{2}\right)$ | $tg(x) \approx x$ | $(1+x)^\alpha \approx 1 + \alpha x$ |

Più in generale, se Δf e Δx sono "piccoli"

$$\Delta f = \frac{df}{dx}\Delta x$$

# Integrali

Regola d'integrazione - Linearità

| |
|---|
| $$\int [A\,f(t) + B\,g(t)]dt = A\int f(t)dt + B\int g(t)dt$$ |

Integrali indefiniti di alcune funzioni utili

| | |
|---|---|
| $\int dt = t + C$ | $\int t^n dt = \frac{1}{n+1}t^{n+1} + C$ |
| $\int \frac{1}{t}dt = ln(t) + C$ | $\int e^{bt}dt = \frac{1}{b}e^{bt} + C$ |
| $\int sin(\omega t)dt = -\frac{1}{\omega}cos(\omega t) + C$ | $\int cos(\omega t)dt = \frac{1}{\omega}sin(\omega t) + C$ |
| $\int \frac{dt}{(t^2 - b^2)} = \frac{1}{2b}ln\left(\frac{b-t}{b+t}\right) + C$ | |

Integrali definiti calcolabili facilmente per sostituzione

| integrale di partenza | sostituzione | integrale ottenuto | soluzione |
|---|---|---|---|
| $\int_a^b \frac{dt}{(Ct + D)}$ | $u = Ct + D$ | $\frac{1}{C}\int_{Ca+D}^{Cb+D}\frac{du}{u}$ | $\frac{1}{C}\Big[ln(u)\Big]_{Ca+D}^{Cb+D}$ |
| $\int_a^b (Ct + D)^n dt$ | $u = Ct + D$ | $\frac{1}{C}\int_{Ca+D}^{Cb+D} u^n du$ | $\frac{1}{C(n+1)}\Big[u^{n+1}\Big]_{Ca+D}^{Cb+D}$ |
| $\int_a^b \frac{t\,dt}{(Ct^2 + D)}$ | $u = Ct^2 + D$ | $\frac{1}{2C}\int_{Ca^2+D}^{Cb^2+D}\frac{du}{u}$ | $\frac{1}{2C}\Big[ln(u)\Big]_{Ca^2+D}^{Cb^2+D}$ |
| $\int_a^b (Ct^2 + D)^n\,t\,dt$ | $u = Ct^2 + D$ | $\frac{1}{2C}\int_{Ca^2+D}^{Cb^2+D} u^n du$ | $\frac{1}{2C(n+1)}\Big[u^{n+1}\Big]_{Ca^2+D}^{Cb^2+D}$ |

# Equazione differenziale a variabili separabili

| equazione | condizioni iniziali | soluzione |
|---|---|---|
| $\frac{dx}{dt} = f(x)g(t)$ | $x(t_0) = x_0$ | $\int_{x_0}^{x}\frac{dx}{f(x)} = \int_{t_0}^{t} g(t)dt$ |

# Alcune identità trigonometriche

| | | |
|---|---|---|
| $sin^2(\alpha) + cos^2(\alpha) = 1$ | $sin(\alpha) = \pm\sqrt{1 - cos^2(\alpha)}$ | $cos(\alpha) = \pm\sqrt{1 - sin^2(\alpha)}$ |
| $tg(\alpha) = \frac{sin(\alpha)}{cos(\alpha)}$ | $sin(\alpha) = \pm\frac{tg(\alpha)}{\sqrt{1 + tg^2(\alpha)}}$ | $cos(\alpha) = \pm\frac{1}{\sqrt{1 + tg^2(\alpha)}}$ |

| | |
|---|---|
| $sin\big(arctg(x)\big) = \frac{x}{\sqrt{1 + x^2}}$ | $cos\big(arctg(x)\big) = \frac{1}{\sqrt{1 + x^2}}$ |
| $sin(\alpha \pm \beta) = sin(\alpha)cos(\beta) \pm cos(\alpha)sin(\beta)$ | $sin(2\alpha) = 2sin(\alpha)cos(\alpha)$ |
| $cos(\alpha \pm \beta) = cos(\alpha)cos(\beta) \mp sin(\alpha)sin(\beta)$ | $cos(2\alpha) = cos^2(\alpha) - sin^2(\alpha)$ |
| $tg(\alpha \pm \beta) = \frac{tg(\alpha) \pm tg(\beta)}{1 \mp tg(\alpha)tg(\beta)}$ | $tg(2\alpha) = \frac{2\,tg(\alpha)}{1 - tg^2(\alpha)}$ |
| $sin\left(\frac{\alpha}{2}\right) = \pm\sqrt{\frac{1 - cos(\alpha)}{2}}$ | $cos\left(\frac{\alpha}{2}\right) = \pm\sqrt{\frac{1 + cos(\alpha)}{2}}$ |

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