



VISTA LA FIGURA, IN OGNI MOMENTO SI HA

$$l = \frac{L}{2} \sin \frac{\theta}{2} \quad d = \frac{L}{2} \cos \frac{\theta}{2} \quad b = \frac{L}{2} \sin \theta$$

$$F_k = K \left( l - \frac{L}{4} \right) \quad I_0 = \frac{1}{3} mL^2 \quad \text{E LA II EQ. CARD. E'}$$

$$mgb - F_k d = I_0 \ddot{\theta} \quad \text{CIOE'}$$

$$mg \frac{L}{2} \sin \theta - KL \left( \sin \frac{\theta}{2} - \frac{1}{4} \right) \frac{L}{2} \cos \frac{\theta}{2} = \frac{1}{3} mL^2 \ddot{\theta} \quad (1)$$

NELLA POSIZIONE DI EQUILIBRIO SI HA

$$\theta = \theta_0 \quad \sin \theta_0 = \frac{\sqrt{3}}{2} \quad \cos \frac{\theta_0}{2} = \frac{\sqrt{3}}{2} \quad \sin \frac{\theta_0}{2} = \frac{1}{2} \quad \ddot{\theta} = 0$$

QUINDI

$$\frac{mg \sqrt{3}}{2} - KL \left( \frac{1}{4} \right) \frac{1}{2} \frac{\sqrt{3}}{2} = 0 \Rightarrow K = \frac{4mg}{L} \quad (2)$$

INSERENDO LA (2) NELLA (1) SI OTTIENE

$$m \frac{g}{2} \sin \theta - m \frac{Lg}{2} \left( \sin \frac{\theta}{2} - \frac{1}{4} \right) \cos \frac{\theta}{2} = m \frac{L}{3} \ddot{\theta}$$

SE ORA SI PONE  $\theta = \theta_0 + \delta$  SI HA  $\ddot{\theta} = \ddot{\delta}$  ED INOLTRE SI POSSONO SVILUPPARE LE F TRIGONOMETRICHE DIMENTICANDO I TERMINI IN  $\delta^2$  VISTO CHE  $\delta \ll 1$

$$\sin \theta = \sin \theta_0 \cos \delta + \cos \theta_0 \sin \delta \approx \frac{\sqrt{3}}{2} + \frac{\delta}{2}$$

$$\sin \frac{\theta}{2} = \sin \frac{\theta_0}{2} \cos \frac{\delta}{2} + \cos \frac{\theta_0}{2} \sin \frac{\delta}{2} \approx \frac{1}{2} + \frac{\sqrt{3}}{4} \delta$$

$$\cos \frac{\theta}{2} = \cos \frac{\theta_0}{2} \cos \frac{\delta}{2} - \sin \frac{\theta_0}{2} \sin \frac{\delta}{2} \approx \frac{\sqrt{3}}{2} - \frac{\delta}{4} \quad \text{ED ALLORA...}$$

$$\frac{g}{2} \left( \frac{\sqrt{3}}{2} + \frac{\delta}{2} \right) - 2g \left( \frac{1}{4} + \frac{\sqrt{3}}{4} \delta \right) \left( \frac{\sqrt{3}}{2} - \frac{\delta}{4} \right) = \frac{L}{3} \ddot{\delta} \quad \text{II ORDINE}$$

$$\frac{g}{2} \left( \frac{\sqrt{3}}{2} + \frac{\delta}{2} \right) - 2g \left( \frac{\sqrt{3}}{8} - \frac{\delta}{16} + \frac{3\delta}{8} - \frac{\sqrt{3}}{16} \delta^2 \right) = \frac{L}{3} \ddot{\delta}$$

$$\cancel{\frac{g}{2} \left( \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} \right)} + g \left( \frac{1}{4} + \frac{1}{8} - \frac{3}{4} \right) \delta = \frac{L}{3} \ddot{\delta}$$

$$-\frac{3}{8} g \delta = \frac{L}{3} \ddot{\delta} \Rightarrow \ddot{\delta} + \frac{9g}{8L} \delta = 0 \quad \text{QUINDI}$$

$$\omega = \frac{3}{2} \sqrt{\frac{g}{2L}}$$