

Some aspects of viscous effects for geomaterials and modelling

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With the help of PhD works from : Mondher Neifar (97), Cedric Sauzéat (03), Damien P V Bang (04), François Olard (04), Antoine Duttine (05), Brice Delaporte, Alan Ezaoui, Doan T.H.,...

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points on focus

- Viscous effects
- Experimental observations
 - Dry sands, sand-clay mixtures, bituminous mixtures
 - Apparatuses : triaxial, hollow cylinder, C/T test, annular shearing
- Modelling : 3 component model and extension
 - 1Dim & 3Dim

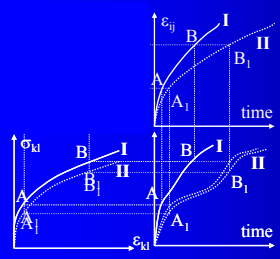
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Definition of viscous effects

- Observed during creep, relaxation, More generally : **change in the stress-strain curve(s) when changing the rate of loading**
- Seems to be present for all? geomaterials even dry sand

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General definition of time effect sensitivity



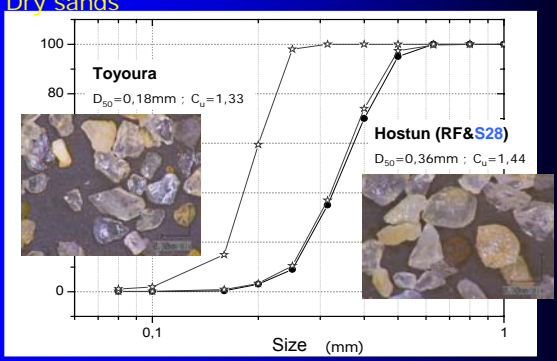
- Time independent
- Time dependant
 - Viscous
 - ~~Ageing~~
- Viscous and non ageing (only rate dependency)
 - time « t » must not appear in the constitutive law

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Experimental investigations

Tested materials

Dry sands



Toyoura
 $D_{50}=0.18\text{mm}$; $C_u=1.33$


Hostun (RF&S28)
 $D_{50}=0.36\text{mm}$; $C_u=1.44$

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Tested materials (2)

Mixture of Hostun sand and clay (Kaolin)

M15
C=15% ; w=4,5%



M30
C=30% ; w=9%

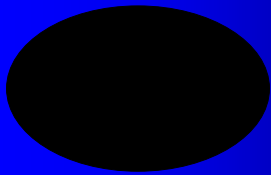


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Tested materials (3)

Bituminous materials : Bitumen, mastics & bituminous mixtures

Bitumens & Mastics up to 60% in Vol of fines



Aggregates : 4 to 7 % of binder





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Two devices for soils

Triaxial test
H=140mm, $\phi_{ext}=80mm$

Hollow cylinder "T4CStaDy"

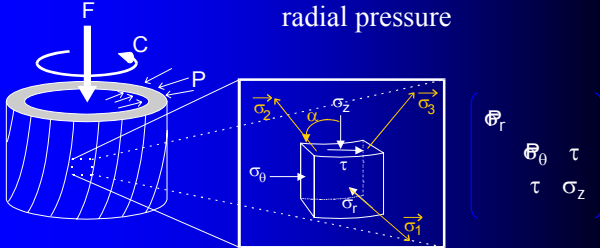
- Local strain measurements from some 10^{-6} to some 10^{-2}
- High stress and strain resolutions
- precise loading conditions
- multi-directional stress path (2&3D)
- Dynamic test: S&P waves

H=120mm, $\phi_{ext}=200mm$, th=20mm
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Hollow cylinder « T4CStaDy »

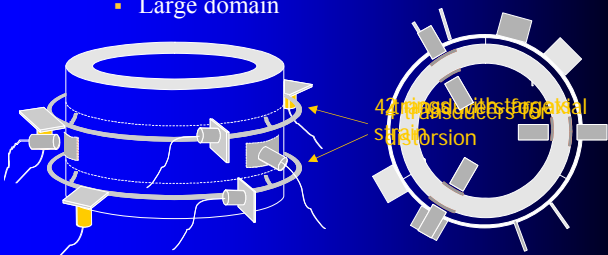
Independent compression, torsion and radial pressure



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Strain measurement system

- Local (moved during the test)
- 14 non contact transducers
- Large domain



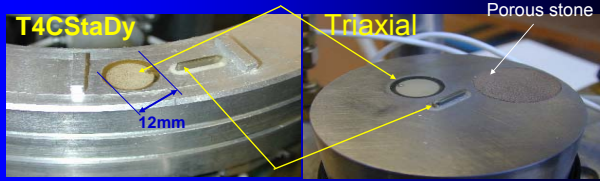
⇒ Strain measurement from some 10^{-6} to some 10^{-2}

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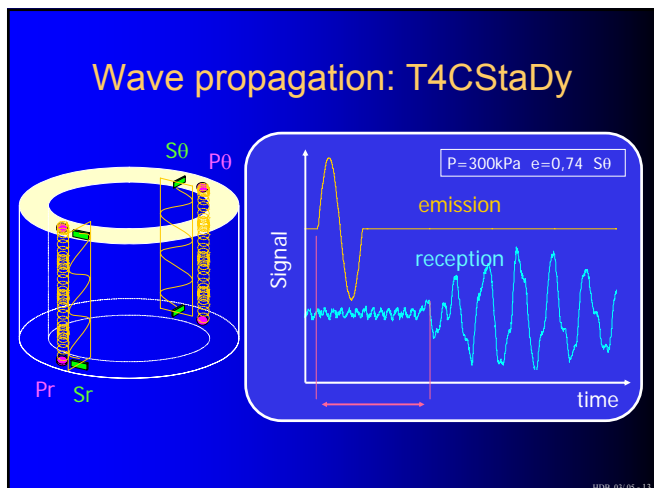
Wave propagation sensors (ISMES type)

Compression wave transducers

Shear wave transducers (bender)



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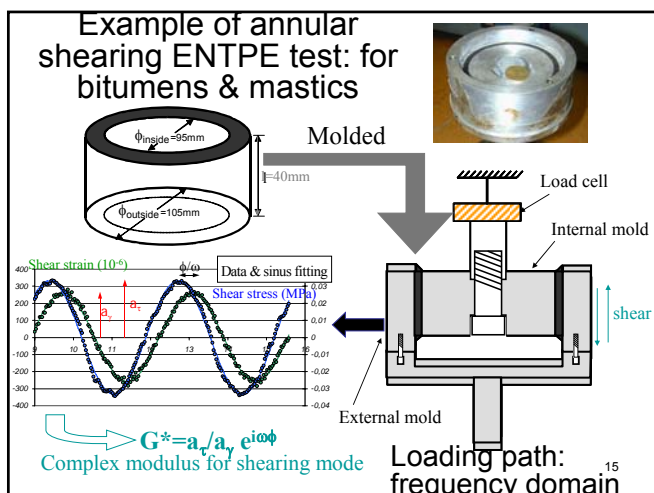
Two devices for bituminous materials

T/C test Focus on small strain
Annular Shearing Rheometer (ASR)

$H=160mm, \phi_{ext}=80mm$

$H=40mm, \phi_{ext}=105mm, th=5mm$

- Local strain measurements from some 10^{-6} to some 10^{-2}
- High stress and strain resolutions
- precise loading conditions
- Temperature control
- Sinusoidal loading up to 10Hz

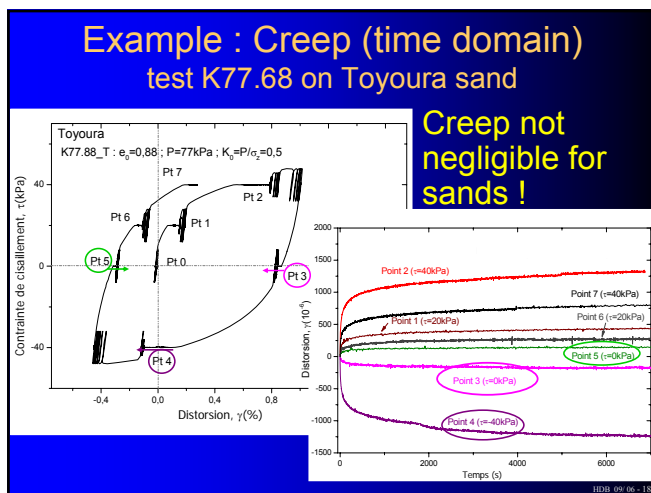
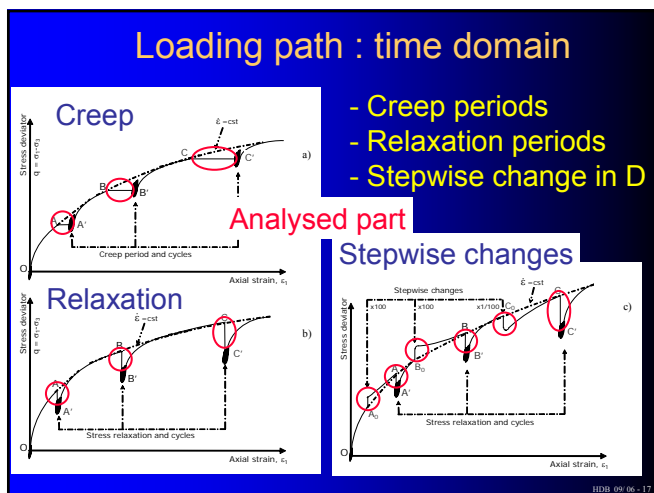


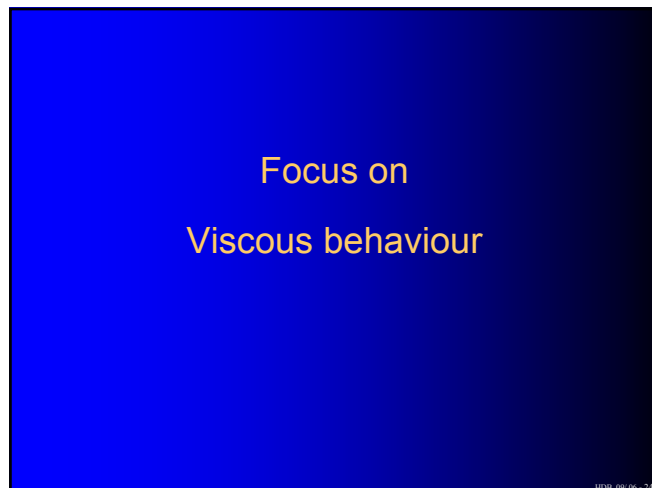
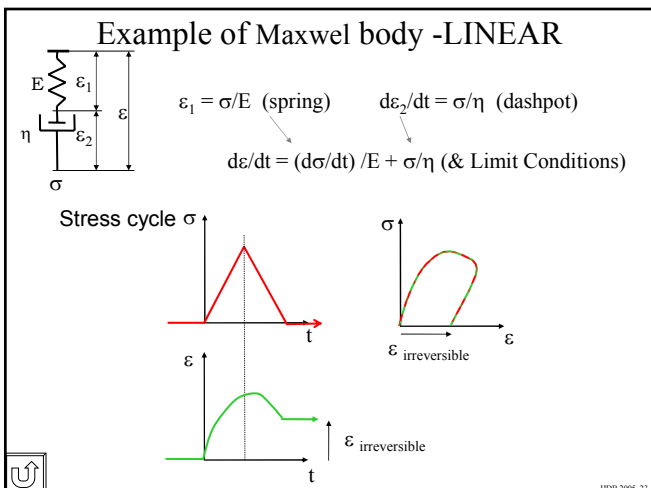
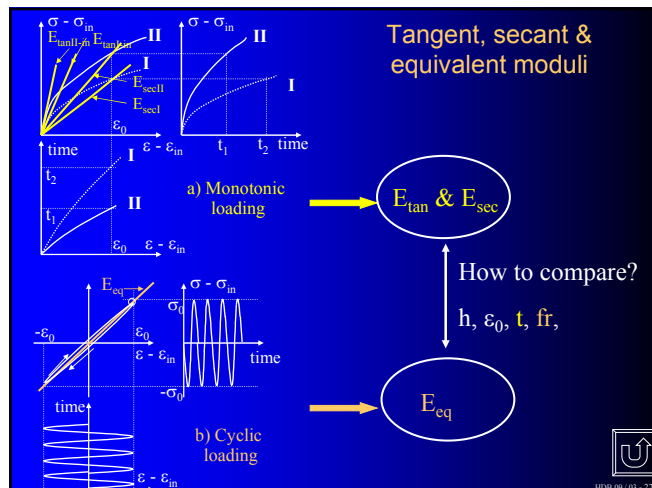
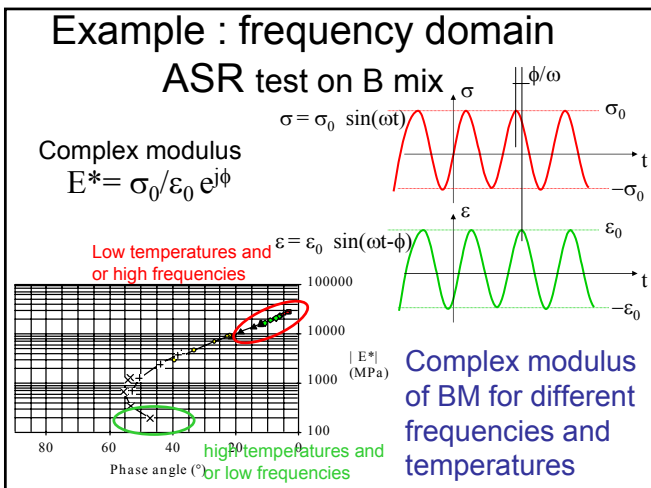
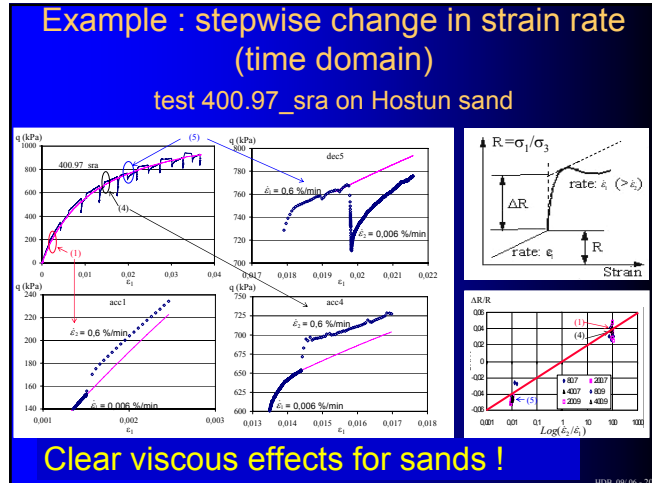
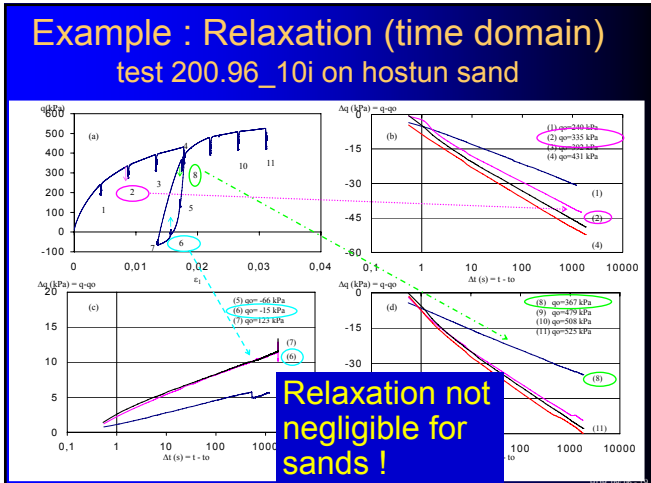
Types of loadings

- Monotonic loading (constant rate, ...)
- Cyclic loadings (sinusoidal, saw-tooth, ...)
- Small strain level \rightarrow large strain level
- Creep, relaxation, stepwise changes

Rational analysis needed to interpret the data : non linearity & irreversibility \rightarrow viscous /non viscous

A Linear Behaviour can be irreversible !!!





Interpretation and modelling 3 components model framework and extension

Model with 3 components....

Non viscous **Viscous**

$\epsilon = \epsilon^{nv} + \epsilon^v$ or $d\epsilon = d\epsilon^{nv} + d\epsilon^v$ or $D = D^{nv} + D^v$

or more!

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EP type body

Non viscous behaviour

$\sigma' = N^{nv}(h, \text{dir} D) D$
or $D = M^{nv}(h, \text{dir} \sigma') \sigma'$

Stress rate \longleftrightarrow strain rate (D)

Elasticity
plasticity
elastoplasticity
hypoelasticity
hypoplasticity
interpolation type
....

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V type body

Purely viscous behaviour

$\sigma = H(h, D)$
or $D = N(h, \sigma)$

Newtonian linear
Newtonian non linear
Parabolic creep
Viscous evanescent
TESRA
....

Stain rate (D)
independent on stress rate
depends only of history & σ

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The 3 component model can describe the observed behaviour

→ Determination of EP_i and V_i

first step small strain domain

Small strain domain
Linear behaviour

Soils : depend on materials

- sand → elastic
- clay, soft rocks → visco-elastic

Bituminous materials : always viscous

→ visco-elastic

Small strain domain

Asymptotic behaviour in the small strain domain (linear)

Di Benedetto et al. S&F 99

Simplification (SAB) Simple Asymptotic Body for soils and soft rocks

For bituminous materials : frequency analysis : complex modulus

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Modulus in the linear domain (small strain)

In the considered range :

- Very sensitive
- Slightly sensitive
- Very small sensitivity (elastic)

Does not mean that the material is non viscous !

Di Benedetto et al 03

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Very small sensitivity

Experimental findings for sands (many tests)

Cycles in the small strain domain (after creep or relaxation)

- in different directions
- at different rates

For tested sands and sand/clay mixtures

→ No viscous effect

« quasi » elastic behaviour in a wide range

→ 3D Hypoelastic model (DBGS)

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Slightly sensitive Small cyclic loading tests on soft rock

« SAB »

$K_0 = 3380 \text{ Mpa}$
 $K = 61455 \text{ Mpa}$
 $\eta = 2 \text{ 100 000 Mpa.s}$

Only at very small strain amplitude

Di Benedetto et al 97

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Very sensitive (bituminous mixtures)

Sinusoidal vibration : Complex modulus

- When considering sinusoidal loading the response is also sinusoidal for LVE (Linear ViscoElastic) materials. But there is a phase lag between stress and strain (late)

$\sigma = \sigma_0 \sin(\omega t)$

$\varepsilon = \varepsilon_0 \sin(\omega t - \phi)$

Complex modulus (2)

Imposed loading : $\sigma = \sigma_0 \sin(\omega t)$

When Changing pulsation(ω) or frequency (f): ε_0 and ϕ change

$\omega_1 \rightarrow \varepsilon_{01}$ and ϕ_1

$\omega_2 \rightarrow \varepsilon_{02}$ and ϕ_2

Introduction of complex numbers:

$\sigma = \sigma_0 \sin(\omega t) = \text{Im}(\sigma_0 e^{j\omega t}) = \text{Im}(\sigma^+)$

$\varepsilon = \varepsilon_0 \sin(\omega t - \phi) = \text{Im}(\varepsilon_0 e^{j(\omega t - \phi)}) = \text{Im}(\varepsilon^+)$

$j^2 = -1$

Complex modulus $E^* = \sigma_0 / \varepsilon_0 e^{j\phi}$

Complex modulus (3)

- Complex modulus
 $E^* = \sigma_0 / \varepsilon_0 e^{i\phi} = |E^*| e^{i\phi}$
- Norm is the ratio of the amplitude: $\sigma_0 / \varepsilon_0 (\omega)$
- ϕ is the phase lag between σ & $\varepsilon (\omega)$
- Complex modulus
 $E^* = E_1 + jE_2$

Complex modulus (4)

- Complex modulus
 $E^* = E_1 + jE_2$
- Complex modulus
 $E^* = \sigma_0 / \varepsilon_0 e^{i\phi} = |E^*| e^{i\phi}$

Complex modulus (5)

- Example for bituminous mixes (BBSG 50/70)

Find a rheological model for bituminous materials ?

- Find a function $E(\omega)$ fitting with the data for a wide range of ω and T (practical cases)
- Example : choice of SAB & Burger models for mixes

No way to fit on the whole range → model not adapted

Adapted V body for BM: 2P1D

- Parabolic creep dashpot: $F(t) = \delta t^k$
 δ & k constants, $0 < k < 1$
- 2S2P1D (2 Springs, 2 Parabolic elements, 1 Dashpot)
 For bitumens and mixes

V=2P1D
2 Parabolic 1 Dashpot
Di Benedetto et al 04

E^* for 2S2P1D model

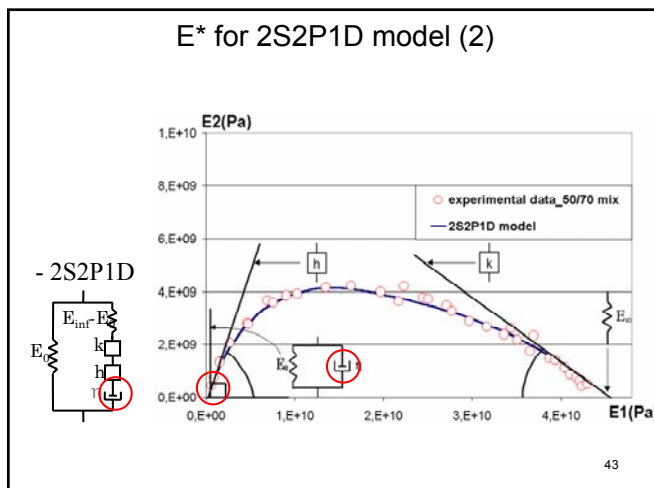
- 2S2P1D (2003) For bitumens and mixes

$$E^*(i\omega\tau) = E_0 + \frac{E_{inf} - E_0}{1 + \delta(j\omega\tau)^{-k} + (j\omega\tau)^{-h} + (j\omega\beta\tau)^{-1}}$$

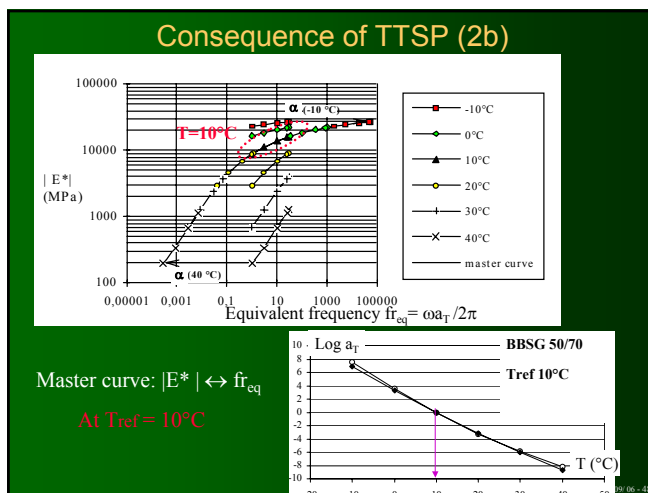
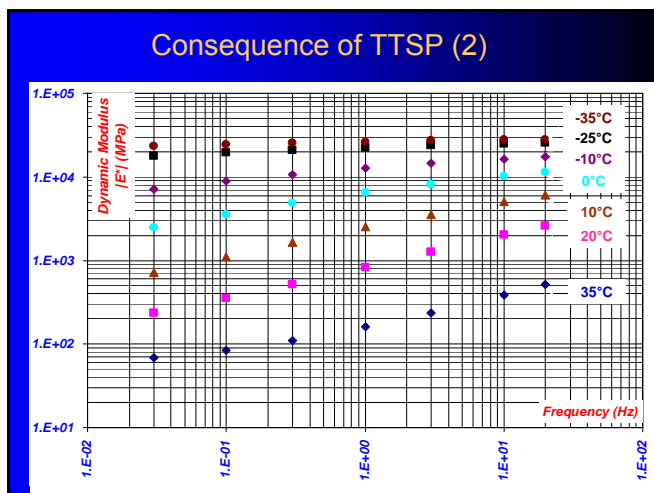
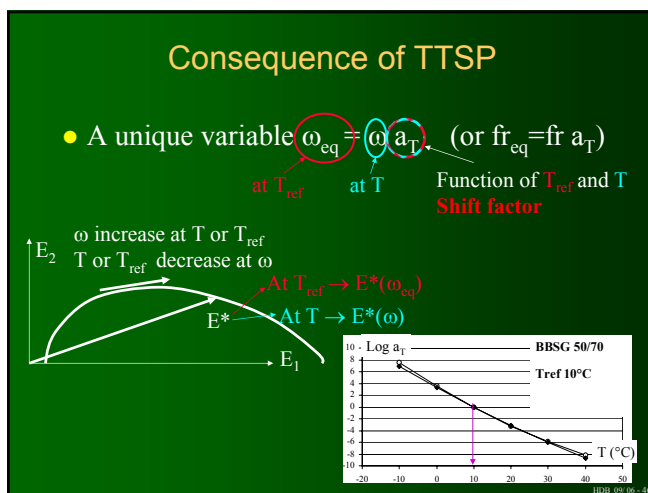
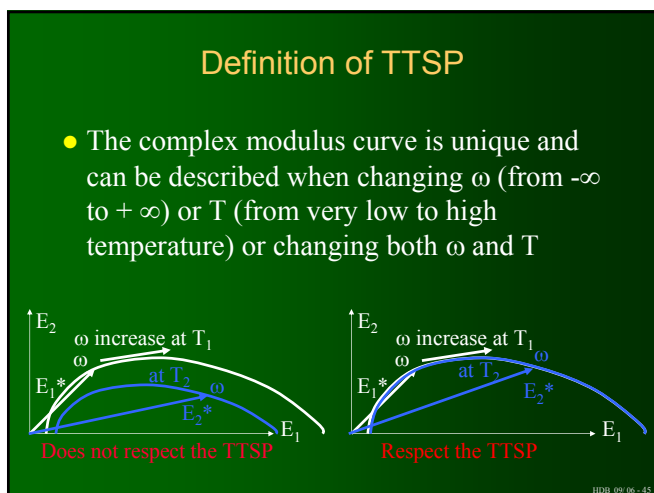
$$\beta = \eta\tau / (E_{inf} - E_0)$$

$E_{00}, E_0, \delta, \tau, \eta, h$ & k constants, $0 < k$ & $h < 1$

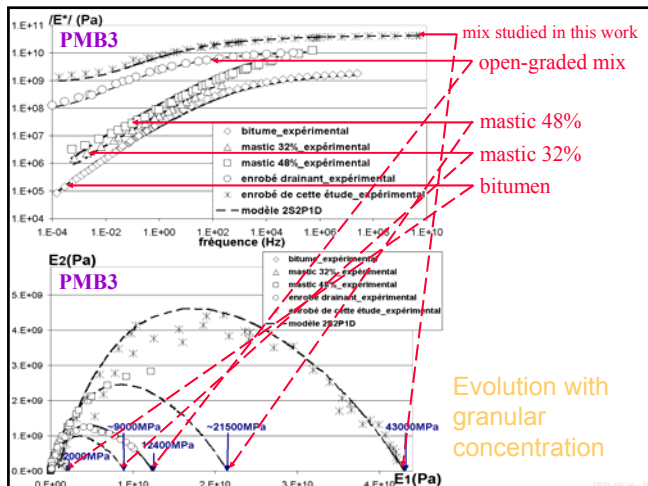
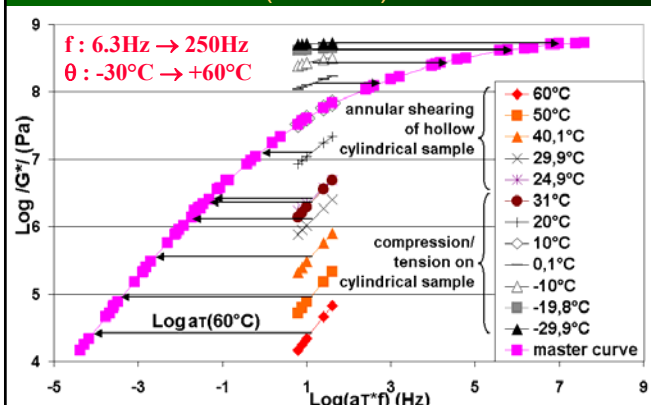
→ No simple analytical expression in the time domain



Time-temperature superposition principle (TTSP):
Thermorhologically simple behaviour

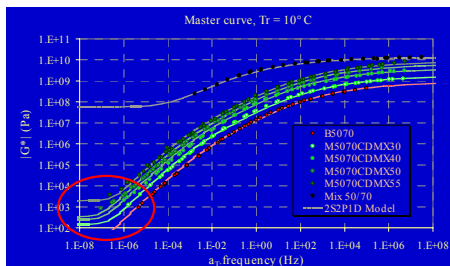


Construction of the 50/70 master curve at 10°C (bitumen)



Simulation of mastics U100µ

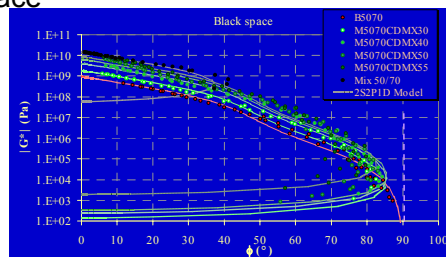
Norm $|G^*|$ of the complex modulus in master curve



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Simulation of mastics U100µ

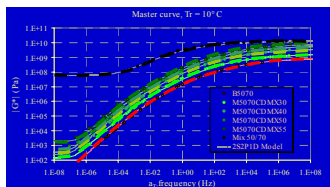
Complex modulus represented in Black space



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2S2P1D Parameters

$$G^*(\omega) = G_0 + \frac{G_\infty - G_0}{1 + \delta(i\omega\tau)^k + (i\omega\tau)^\beta + (i\omega\tau)^\beta}$$



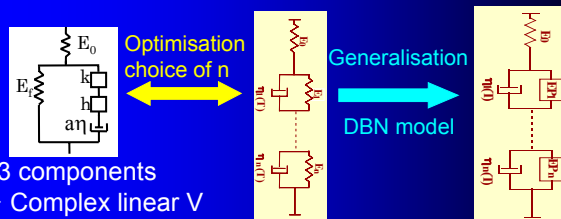
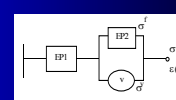
4 constant parameters:
k, h, δ , β
Only 3 parameters function of the filler concentration:
 G_0 , G_∞ , τ_0

Material	G_0 (Pa)	G (Pa)	k	h	δ	$\tau_0 = \tau(10^\circ\text{C})$	β
B5070	0	9.50E+08	0.21	0.55	2.3	9.18E-05	450
MS070U100µ30	150	1.85E+08	0.21	0.55	2.3	1.40E-04	450
MS070U100µ40	250	4.10E+09	0.21	0.55	2.3	1.73E-04	450
MS070U100µ50	350	6.30E+09	0.21	0.55	2.3	1.83E-04	450
MS070U100µ55	2000	8.80E+09	0.21	0.55	2.3	2.18E-04	450
Mix 50/70	6.00E+07	1.40E+10	0.21	0.55	2.3	7.00E-02	450

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Choice to describe non linearities and irreversibilities

- Soils: 3 components model
- Bituminous materials



If 3 components
→ Complex linear V

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Optimisation process: small strain

→ E_i & η_i

$$E^* = \left(\frac{1}{E_0} + \sum_{j=1}^n \frac{1}{E_j + i\eta_j(T)\omega} \right)^{-1}$$

Optimisation

$$E^*(i\omega\tau) = E_0 + \frac{E_\infty - E_0}{1 + \delta(i\omega\tau)^{-k} + (i\omega\tau)^{-h} + (i\omega\beta\tau)^{-l}}$$

Time-Temperature principle (W.L.F.)

$\eta_j(T) = \eta_j(T_0) a_T \rightarrow \tau(T) = \tau_0(T_0) a_T$

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E 50/70 BB: optimisation VEL

3 components 2S2P1D

DBN n=20

For pure bitumen

3D & Small strain domain VEL

Sands : elastic

Bituminous materials : always viscous

Complex young's modulus & Poisson's ratio

LVE Theory

• Tension/compression

Axial stress $\sigma_1(t) = \sigma_{01} \sin(\omega t + \phi)$

Axial strain $\varepsilon_1(t) = \varepsilon_{01} \sin(\omega t)$

Radial strain $\varepsilon_2(t) = -\varepsilon_{02} \sin(\omega t + \phi_\nu)$

Complex Young's modulus $E^* = (\sigma_{01}/\varepsilon_{01}) e^{i\phi}$

Poisson's ratio $\nu^* = (\varepsilon_{01}/\varepsilon_{02}) e^{i\phi_\nu}$

→ 3D isotropic approach

Simulation for a Mastic (50/70 bitumen with 32% of fines)

3 components 2S2P1D

$\nu_{00} = 0,5$

$\nu_{00} = 0,37$

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nu 50/70 BB: modelling and optimisation

Hypothesis : Isotropy

3 components 2S2P1D

2 constants

DBN n=20

$$\nu^* - \nu_0 = \frac{1}{1 + \delta(i\omega\tau)^k + (i\omega\tau)^h + (i\omega\beta\tau)^l}$$

$$\nu^*_{DBN3D LVE} = \sum_{i=1}^n \frac{\nu_i^*}{E_i^*} = \sum_{i=1}^n \frac{1}{E_i^*}$$

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Determination of EP1, EP2 & V behaviour

small strain & « fast » loading for sands

Small cycles → EP1

Recall of experimental findings

Linear domain ($\epsilon < \sim 10^{-5}$)

Anisotropy → M^e hypoelastic [DBGS]

Symmetry of M^e

$$d\epsilon^{nv} = M^e \cdot d\sigma$$

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Hypoelastic model DBGS (Di Benedetto, Geoffroy, Sauzéat)

$$d\epsilon = M^e \cdot d\sigma \quad M^e = \frac{1}{F(e)} (S_v \Sigma + {}^t\Sigma S_v)$$

$$\Sigma = \begin{pmatrix} 1/\sigma_1^m & 0 & 0 & 0 \\ 0 & 1/\sigma_2^m & 0 & 0 \\ 0 & 0 & 1/\sigma_3^m & 0 \\ 0 & 0 & 0 & 1/\sigma_1^{m/2}\sigma_3^{m/2} \end{pmatrix} \quad S_v = \begin{pmatrix} 1 & -v_0 & -v_0 & 0 \\ -v_0 & 1 & -v_0 & 0 \\ -v_0 & -v_0 & 1 & 0 \\ 0 & 0 & 0 & 1+v_0 \end{pmatrix}$$

In the principal axes of stress (12 and 23 directions not written) (from isotropic initial state)

⇒ v_0 and m : constants

⇒ $F(e)$: function of the void ratio e

Not developed in this presentation

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Determination of EP_i and V_i up to large strain domain

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Model For Bituminous mixtures DBN law (Di Benedetto, Neifar)

Instantaneous strain/stress & brittle

Simple V bodies

Each EP body behaves as a non cohesive granular material

$$\Delta\sigma_j = f_n(\Delta\epsilon_j)$$

Not developed in this presentation

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V body: introduces viscous effects

Needed for intermediate and pure viscous evanescent behaviours

Not for "classical" isotach behaviour

Di Benedetto et al 02

$\sigma^v_{isotach}(h, \dot{\epsilon}^{vp})$

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Viscous Evanescent (VE) : new type of body

Test at constant $\dot{\epsilon}^{vp}$ rate

And superposition during history

$$\sigma^v_{(t)} = \sigma^v_{isotach}(t) \int_{\chi=0}^h \sigma^v_{isotach}(\chi) \cdot g'_{decay}(\epsilon^{vp}_{(t)} - \epsilon^{vp}_{(\chi)}) \cdot d\epsilon^{vp}_{(\chi)}$$

Classical isotach behaviour

Correction term Time history of ϵ^v

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Models proposed following the geomaterials

Type of model

- New isotach : $\sigma^v \sim \sigma^f$ (cf. viscous coefficient β)
- TESRA
- Pure VE
- General TESRA
- VE

Type of materials

- Low plasticity Clays
- Soft rocks
- Bituminous materials
-
- Clean sands
- Cement mixed soils (& ageing)
- Low plasticity Clays
- Chiba Gravels
- Soft rocks

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Creep or relaxation or change in D : V

⇒ Strain non negligible but still small ($< \eta \cdot 10^{-3}$)

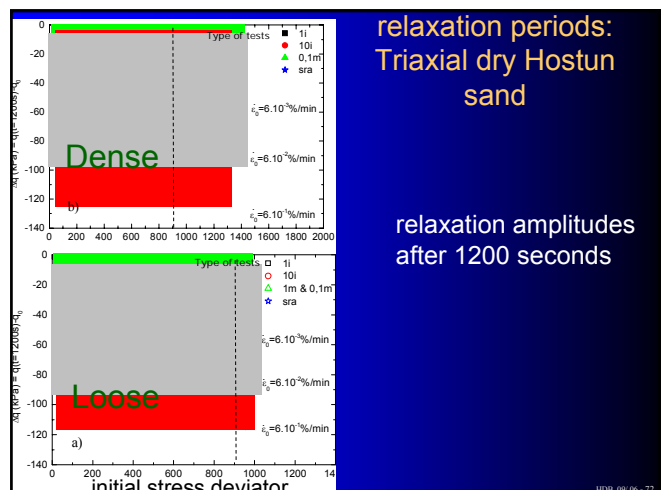
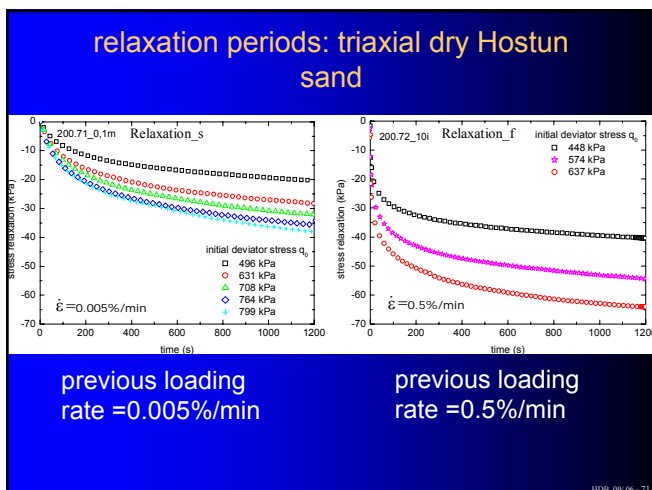
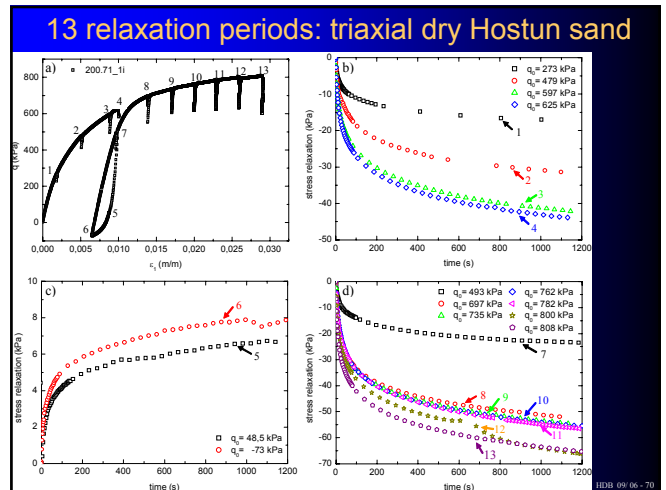
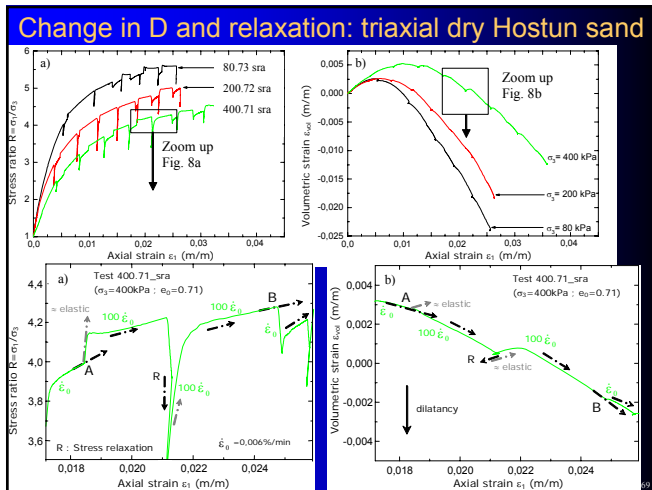
Tangent asymptotic model

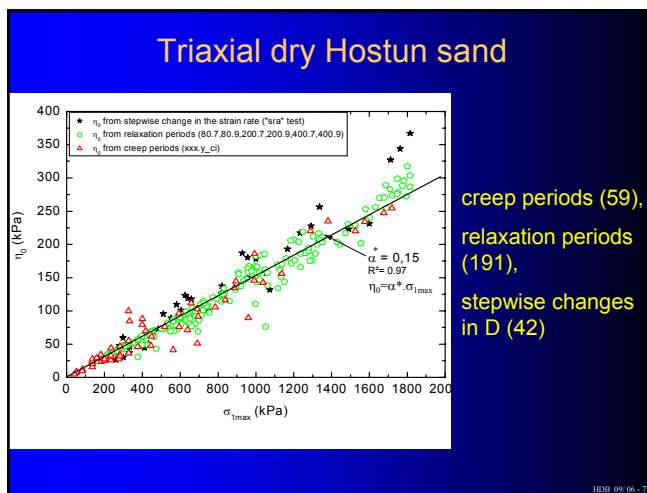
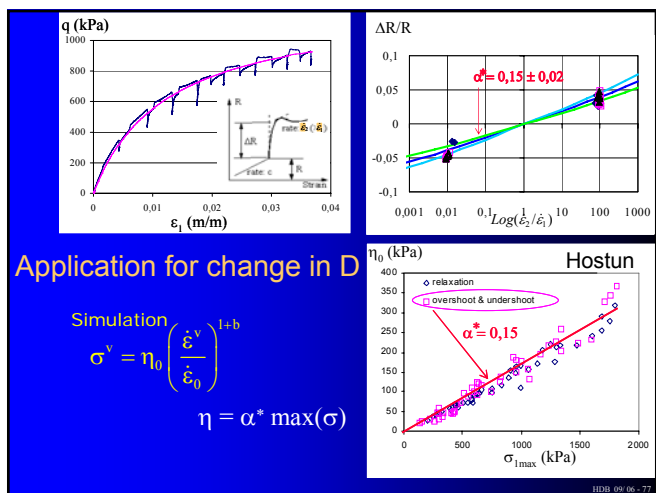
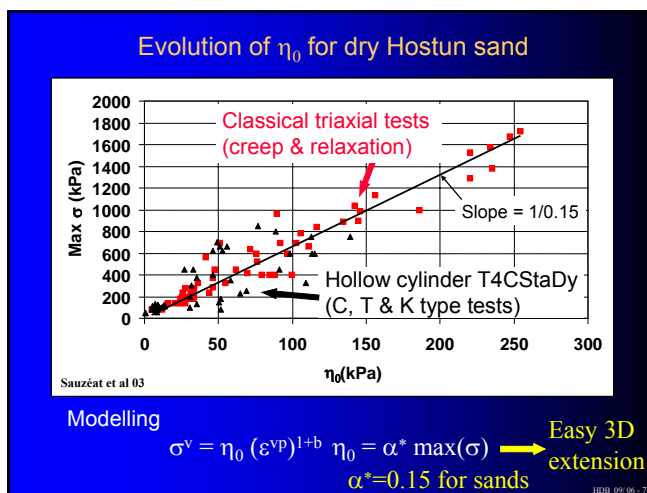
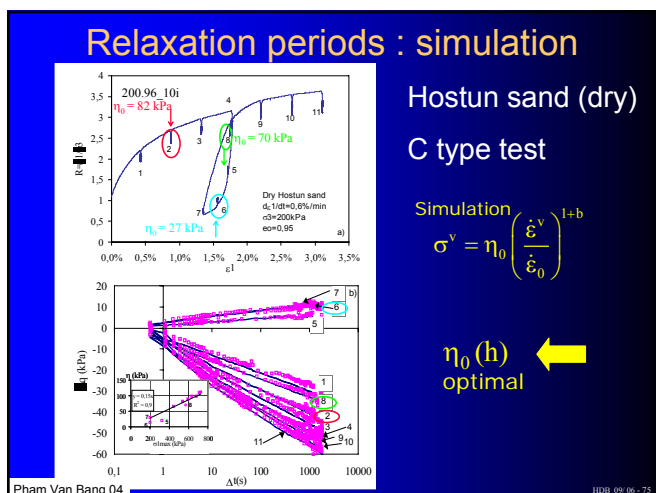
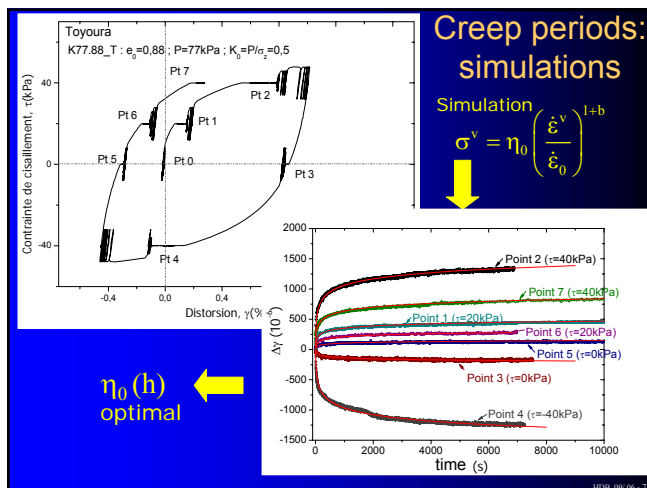
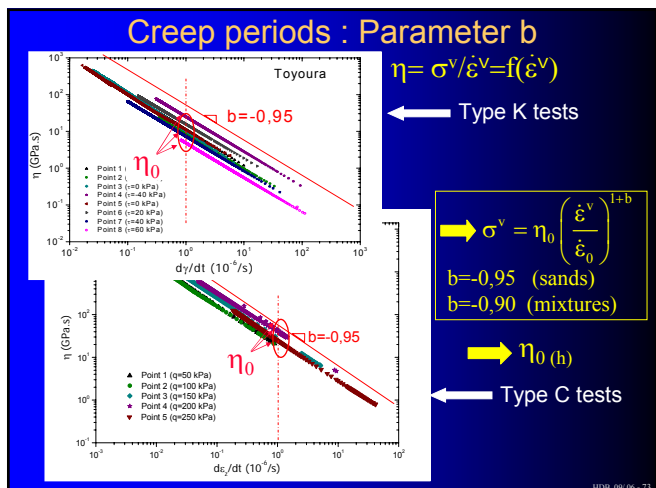
⇒ Negligible evanescent effects

$$\sigma^v = \eta_0 \left(\frac{\dot{\epsilon}^v}{\dot{\epsilon}_0} \right)^{1+b}$$

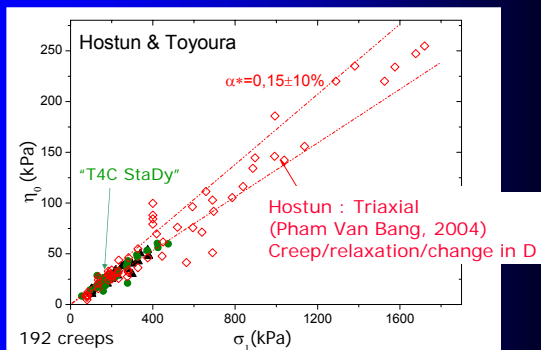
Tangent : different if loading or unloading

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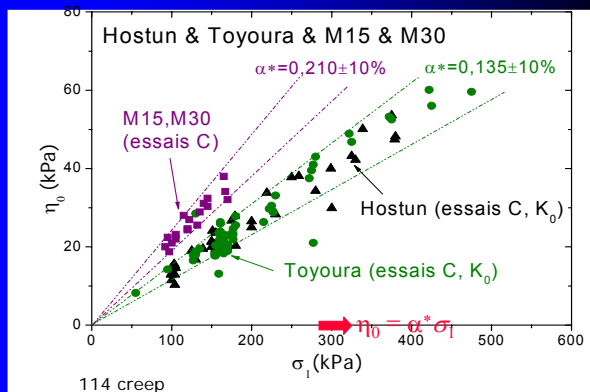


Dry Hostun and Toyoura



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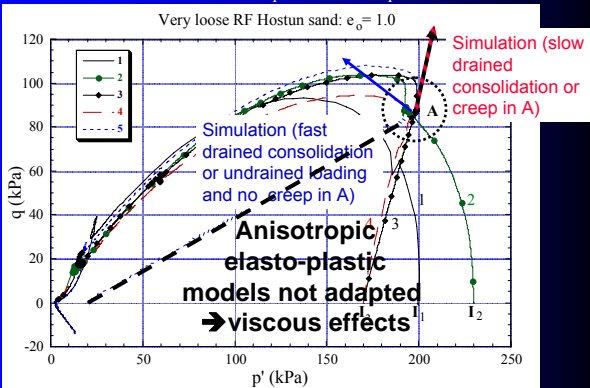
Hostun and Toyoura and mixtures



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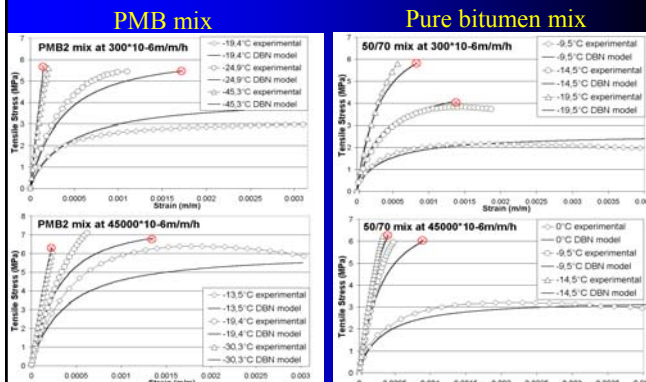
Undrained tests : simulations

Undrained triaxial tests after isotropic and anisotropic consolidation

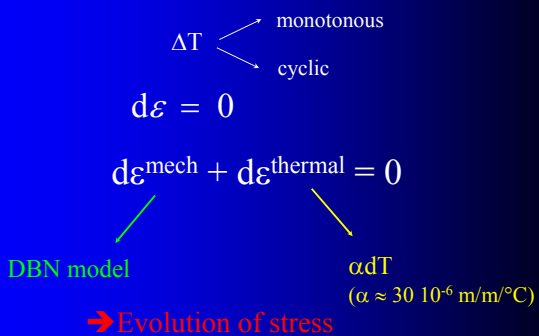


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Simulation of constant strain rate traction tests

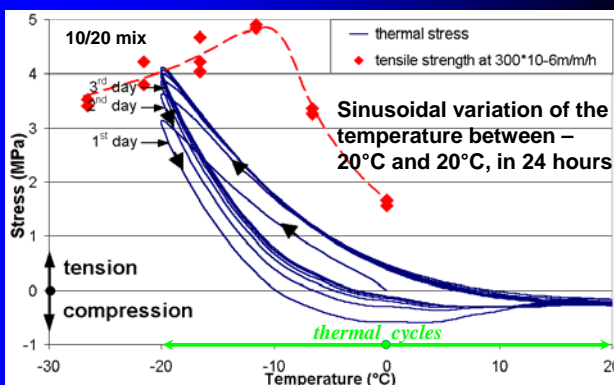


Modelling TSRS Test (Thermo-mechanical behaviour)

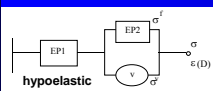


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Simulations of cyclic TSRST



3 D formalism

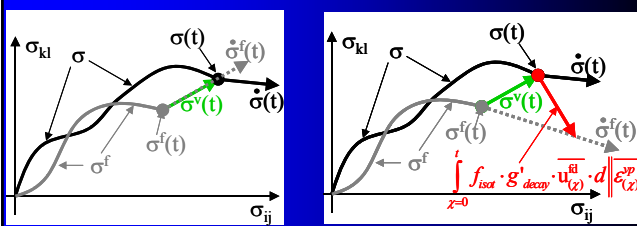


- EP2 body : use 3D expression
- V body only one scalar equation and a mapping rule
 - Same equation 1D with:

$$\sigma^v \rightarrow \|\sigma^v\| \text{ and } \dot{\epsilon}^{vp} \rightarrow \|\dot{\epsilon}^{vp}\|$$
 - Mapping rule : direction of $d(\sigma^f)$

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Mapping rule



Isotach case

σ^v and $d\sigma^f$ have the same direction

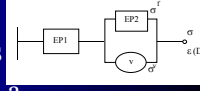
With viscous evanescent property

More complex expression history dependent

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Conclusion

- Investigation on sands, sand/clay mixtures and bituminous materials
- Similarity of behaviour for these materials
- For certain loading conditions : linear behaviour
- Viscous effects non negligible (small or very important) act from small to large strain domain
- Non linearities and irreversibilities
- 3 components model appears to be a powerful formalism which can be extended



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