Probabilistic assessment of geotechnical objects by means of MONTE CARLO METHOD

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What is the principal objective?

If you think that safety is expensive, you should try to have an accident!
To be brief…

GEOTECHNICAL OBJECT = STRUCTURE

Definition derived from EN 1990:2002

Structure = organised combination of connected parts, including fill placed during execution of the construction works, designed to carry loads and provide adequate rigidity

- Foundations
- Anchorages
- Retaining structures
- Embankments
- Slopes
- Excavations
CONTENTS OF LECTURE

I. THEORETICAL PART

1. Introduction to structural reliability
2. Reliability & failure probability of structures
3. Estimation of failure probability by means of Monte Carlo method

II. COMPUTER EXERCISES

1. Introduction to AntHill computer code
2. Stability of slope excavated in a clay layer
3. Variability of load capacity of a ground anchor
4. Reliability of concrete friction pile
5. Reliability of gravity retaining wall
6. Reliability of cantilever retaining wall
PART I
Introduction to structural reliability
I. Structural reliability 1/10

Failures of structures

Structures unfortunately fail ...

*Failure of a structure* = insufficient load bearing capacity or inadequate serviceability of a structure or structural element

*Failure* = exceedance of limit state
Design situations*

Failure can take place in one of the three situations:

- Persistent design situations (situation of normal use)
- Transient design situations** (construction, repair, demolition)
- Accidental design situations (fire, explosion, impact)

** Only the transient design situations are mentioned in Eurocode 7
# I. Structural reliability 3/10

## The risk of death as a result of structural failure*

<table>
<thead>
<tr>
<th>Activity</th>
<th>Approximate death rate $\times 10^{-9}$ deaths/hr exposure</th>
<th>Estimated typical exposure (hr/year)</th>
<th>Typical risk of death $\times 10^{-6}$/ year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Construction work</td>
<td>70...200</td>
<td>2200</td>
<td>150...400</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>20</td>
<td>2000</td>
<td>40</td>
</tr>
<tr>
<td>Coal mining (UK)</td>
<td>210</td>
<td>1500</td>
<td>300</td>
</tr>
<tr>
<td><strong>Building fires</strong></td>
<td><strong>1...3</strong></td>
<td><strong>8000</strong></td>
<td><strong>8...24</strong></td>
</tr>
<tr>
<td>Air travel</td>
<td>1200</td>
<td>20</td>
<td>24</td>
</tr>
<tr>
<td>Car travel</td>
<td>700</td>
<td>300</td>
<td>200</td>
</tr>
<tr>
<td>Train travel</td>
<td>80</td>
<td>200</td>
<td>15</td>
</tr>
<tr>
<td>Structural failures</td>
<td>0,02</td>
<td>6000</td>
<td>0,1</td>
</tr>
</tbody>
</table>

I. Structural reliability 4/10

Why should we be concerned about structural reliability

- **Individuals:** involuntary of risk due to structural failures

  The risk levels for buildings and bridges are usually associated with *involuntary risk* and are much lower than the risk associated with voluntary activities (travel, mountain climbing, deep see fishing)

- **Society:** failure results in decrease of confidence in stability and continuity of one's surroundings

  Society is interested in structural reliability only in the sense that a structural failure with significant consequences shatters confidence in the stability and continuity of one’s surroundings

- **Engineers:** the need to apply novel structures and novel construction methods generates interest in safety

  Design, construction, and use of *new or particularly hazardous* systems should be of particular interest in their safety (new and unique bridge, new off-shore structure, NPP, chemical plant, liquefied gas depot)
I. Structural reliability 5/10

Human errors cause up to 95% of failures

<table>
<thead>
<tr>
<th>Phase</th>
<th>Percentage of cases (493 cases)</th>
<th>Percentage of total cost damage (493 cases)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Design</td>
<td>37</td>
<td>43</td>
</tr>
<tr>
<td>Construction</td>
<td>35</td>
<td>20</td>
</tr>
<tr>
<td>Design &amp; construction</td>
<td>18</td>
<td>22</td>
</tr>
<tr>
<td>Occupation</td>
<td>5</td>
<td>11</td>
</tr>
<tr>
<td>Others</td>
<td>5</td>
<td>4</td>
</tr>
</tbody>
</table>

I. Structural reliability 6/10

Failures of FORMULA 1 cars

TOP SPEED
MASS
AERODYNAMIC PROPERTIES

CHANCE OF
FAIL-SAFE BEHAVIOUR
(RELIABILITY)
I. Structural reliability 7/10

Structural reliability theory: *arguments in favor*

- Analysis of structures, say, EC7
  - Time-independent analysis of error-free structures

- Assessment of
  - Incorporation of possibility of
    - Consideration of rather than individual components
    - Consideration of (accidental actions)
I. Structural reliability 8/10

Structural reliability theory: arguments against

- The need to study probability calculus and statistics
- The need to collect statistical data on structures and actions (loads)
- The need to move outside the “safe and customary“ area ruled by design codes of practice
- Do you know the answer on the question “How safe is safe enough?” ?
I. Structural reliability 9/10

The need to bridge a gap: how to join quickly?
I. Structural reliability 10/10

The textbook

PROBABILISTIC ASSESSMENT OF STRUCTURES USING MONTE CARLO SIMULATION
Background, Exercises, Software
(P. Marek, J. Brozzetti and M. Guštar, P.Tikalski, editors)

Publisher: ITAM CAS CZ Academy of Sciences of the Czech Republic, Prague, 2001.

For more information visit the website: http://www.noise.cz/SBRA
PART II
Reliability & failure probability of structures
II. Reliability & failure probability 1/10

Basic definition

**Reliability** = 1 – failure probability

\[ P_s = 1 - P_f \]

Structure can either fail or survive:

\[ P_s + P_f = 1 \]

- *Failure*: an insufficient load-bearing capacity or inadequate serviceability of a structure or structural element.
- *Limit state*: a state beyond which the structure no longer satisfies the design performance requirements.

Reliability is usually not calculated!

<table>
<thead>
<tr>
<th>SURVIVAL OF STRUCTURE</th>
<th>FAILURE OF STRUCTURE</th>
</tr>
</thead>
<tbody>
<tr>
<td>(fulfillment of specified requirements)</td>
<td>(exceedance of irreversible or reversible limit state)</td>
</tr>
<tr>
<td>$P_s = P(\text{survival})$</td>
<td>$P_f = P(\text{failure})$</td>
</tr>
</tbody>
</table>

$P(\text{survival}) + P(\text{failure}) = 1$

$P_s + P_f = 1$

<table>
<thead>
<tr>
<th>Structure 1</th>
<th>$P_{s1} = 0,9995$</th>
<th>$P_{f1} = 0,0005$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Structure 2</td>
<td>$P_{s2} = 0,999$</td>
<td>$P_{f2} = 0,001$</td>
</tr>
<tr>
<td>Difference</td>
<td>0,05%</td>
<td>200%</td>
</tr>
</tbody>
</table>
II. Reliability & failure probability  3/10

How safe is safe enough?

<table>
<thead>
<tr>
<th>Limit state</th>
<th>Tolerable failure probability (design working life)</th>
<th>Tolerable failure probability (one year)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ultimate</td>
<td>$723 \times 10^{-7}$</td>
<td>$13 \times 10^{-7}$</td>
</tr>
<tr>
<td>Fatigue</td>
<td>$0.0668...723 \times 10^{-7*}$</td>
<td>~ ---</td>
</tr>
<tr>
<td>Serviceability</td>
<td>$0.0668$</td>
<td>$0.00135$</td>
</tr>
</tbody>
</table>

* Depends on degree of inspectability, reparability, and damage tolerance.

The way to failure probability

1. Statistical data on structural parameters and loads
2. Processing the statistical data
3. Mechanical model of structure
4. Estimation of failure probability $P$

25.1; 33.2; 21.7; 30.3; ...
The role of data

- Statistical data reflects the ubiquitous *uncertainty* in structural parameters and loads.
- Statistical data is used to fit *probability distributions* of the structural parameters and loads.
- Statistical data *determine* the value of the failure probability $P_f$ in the end.

- Data on *material properties*
- Data on *geometrical quantities*
- Data on *direct actions (loads)* and *indirect actions*
- Data on *model uncertainties*
II. Reliability & failure probability  6/10

Processing the data

Estimation of distribution parameters:

\[ \mu = \bar{x} \]

\[ \sigma^2 = s^2 \]

Fitting a probability distribution

\[ \bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i \]

\[ s = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2} \]

\[ \delta = \frac{s}{\bar{x}} \times 100\% \]
II. Reliability & failure probability  7/10

Vector of basic variables

$$X = (X_1, X_2, \ldots, X_n); \quad x = (x_1, x_2, \ldots, x_n)$$

random variables particular values

Visualisation of $X$ and $x$

The case $n = 3$

$$X = (X_1, X_2, X_3)$$

three-dimensional space

$$x = (x_1, x_2, x_3)$$

point in the space
II. Reliability & failure probability  8/10

Probability density function (PDF)

\[ f(x) = f(x_1) \cdot f(x_2) \cdot \ldots \cdot f(x_n) \]

in the case that basic variables \( X_i \) are independent
II. Reliability & failure probability  9/10

It is nothing more than uncertainty propagation!

\[ P = P( \text{Random Safety Margin} \geq 0) = P( g(X) \geq 0 ) \]

- **Limit state function**: a function \( g \) of the basic variables, which characterizes a limit state when \( g(x_1, x_2, \ldots, x_n) = 0 \); \( g > 0 \) identifies with the desired state and \( g < 0 \) with the undesired state [state beyond the limit state].

- **Basic variable** \( X_i \): a part of a specified set of variables, \( X_1, X_2, \ldots, X_n \), representing physical quantities which characterize actions and environmental influences, material properties including soil properties, and geometrical quantities.
The problem is an integral, not the reliability itself!

\[ P = P(g(X) = 0) = \int_{D_f} \ldots \int f_X(x) \, dx \]

\[ D_f = \{x \mid g(x) \leq 0\} \]

The role of model

Failure domain

Joint PDF

\[ x_1, x_2, x_3, \ldots \]

The role of data
PART III
Monte Carlo method
The power of Monte Carlo method

MECHANICS
ANALYSIS OF STRUCTURES

STOCHASTICS
PROBABILITY CALCULUS

LEVEL LINES:
ENERGY OF MISSILE IMPACT

IMPACT VELOCITY, M/S

IMPACT MASS, KG

200 600 1000 1400

30 50 70 90
Where can it be useful?

- Structural Reliability
- Structural aspects of risk analysis
- Solving special problems, e.g., sensitivity analysis
IIII. Monte Carlo method 3/10

Only one small “detail” is necessary to run the business!

1. Generator of random numbers is in your {pocket calculator}

\[
0 \quad u = 0.321577 \quad 1
\]

\[
\begin{align*}
  u_1 &= 0.405255 \\
  u_2 &= 0.764611 \\
  u_3 &= 0.041139 \\
  u_4 &= 0.013642 \\
  \vdots
\end{align*}
\]
Only one small “detail” is necessary to run the business!

2. Generating values from the uniform distribution $U(0, 1)$
III. Monte Carlo method  5/10

Only one small “detail” is necessary to run the business!

3. Generalization to the multidimensional case is beneficial

Evaluating multidimensional integrals & solving integral equations

Solving systems of differential equations

Solving systems of linear equations

III. Monte Carlo method 6/10

How to evaluate multiple integral?

\[ P_f = P(g(X) \leq 0) = \int_{D_f} \ldots \int f_X(x) \, dx \]

- Exact analytical methods
- Classical methods of numerical integration
- Approximate analytical methods (FORM/SORM methods)
- **SIMULATION (MONTE CARLO) METHODS:**
  - Direct Monte Carlo method
  - Variance reduction techniques
  - Methods utilising knowledge on mechanical model (response surface method, directional simulation)
Failure probability is a mean of random variable

\[ I(x) = \begin{cases} 
1 & \text{if } g(x) \leq 0 \text{ (failure) } \\
0 & \text{if } g(x) > 0 \text{ (survival) }
\end{cases} \]

Vector of random arguments \( X \)

Probability mass function of \( I(X) \)

\[ P = P(I(X) = 1) = \int \ldots \int I(x)f_X(x)\,dx = E(I(X)) \]
Estimate of failure probability

**Intuitive definition**

\[ P_{fe} = \frac{\text{Number of failures } N_f}{\text{Number of trials } N} \]

**Formal definition**

\[ P_{fe} = \frac{1}{N} \sum_{j=1}^{N} I(x_j) \]

\[ E(I(X)) = P \]
Generating values of basic variables: inverse transform

\[ y = F_{X_i}(x_i) \]

\[ x_i = F^{-1}_{X_i}(y_j) \]
### Generating values of basic variables: in general

<table>
<thead>
<tr>
<th>Extreme value distributions</th>
<th>Inverse transform method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exponential, <em>Pareto-</em>-, <em>Raleigh-</em>-, *Cauchy-*distributions</td>
<td>Inverse transform method</td>
</tr>
<tr>
<td>Normal distribution</td>
<td>Composition method, other methods</td>
</tr>
<tr>
<td>Gamma distribution</td>
<td>Acceptance-rejection method</td>
</tr>
<tr>
<td>Beta distribution</td>
<td>Composition method, other methods</td>
</tr>
<tr>
<td>Lognormal distribution</td>
<td>Simple transformation from normal</td>
</tr>
<tr>
<td>Truncated distributions</td>
<td>Acceptance-rejection method</td>
</tr>
</tbody>
</table>

**Generating individual values of basic variables**

<table>
<thead>
<tr>
<th>Multi-normal distribution</th>
<th>Special method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-normal vectors with correlated components</td>
<td>Methods based on <em>USUALLY HIDDEN</em> from normal</td>
</tr>
<tr>
<td>General case of dependence</td>
<td>Multivariate transformation method, multivariate acceptance-rejection method</td>
</tr>
</tbody>
</table>

For more visit, e.g., [http://random.mat.sbg.ac.at/literature/](http://random.mat.sbg.ac.at/literature/)
Monte Carlo method

The end of theoretical part