Prova d'esame di Dinamica delle Strutture del 21 giugno 2011

Docente: Dott. Ing. Paolo S. VALVO

Matricola dello studente

$M := 400000$

Masse del sistema

$m_1 := 2\% \cdot M = 8000$ $\quad$ $m_2 := 7\% \cdot M = 28000$ $\quad$ $m_3 := 1\% \cdot M = 4000$

$m_1 + m_2 + m_3 = 40000$

Altezze dei piani

$H_1 := 3$ $\quad$ $H_2 := 3$ $\quad$ $H_3 := 3$

Distanze orizzontali tra i piedritti

$B_1 := 3$ $\quad$ $B_2 := 6$ $\quad$ $B_3 := 3$ $\quad$ $B := B_1 + B_2 + B_3 = 12.000$

Lunghezze delle aste inclinate della copertura

$L_1 := \sqrt{\left(\frac{B}{2}\right)^2 + {H_3}^2} = 6.708$ $\quad$ $L_2 := \sqrt{\left(\frac{B}{2}\right)^2 + {H_3}^2} = 6.708$

Angoli di inclinazione delle aste della copertura

$\alpha_1 := \operatorname{atan}\left(2 \cdot \frac{H_3}{B}\right) = 26.565 \cdot deg$ $\quad$ $\alpha_2 := \operatorname{atan}\left(2 \cdot \frac{H_3}{B}\right) = 26.565 \cdot deg$

Modulo di Young del materiale

$E := 210 \cdot 10^9$

Momenti di inerzia dei piedritti

$J_{HE300A} := 18260 \cdot 10^{-8}$ $\quad$ $J_{HE500A} := 86970 \cdot 10^{-8}$

Area delle aste inclinate

$A_{IPE240} := 39.12 \cdot 10^{-4}$

Rigidezze di piano

$$k_{01} := 2 \cdot \frac{3 \cdot E \cdot J_{HE300A}}{{H_1}^3} = 8521333.333$$

$$k_{12} := 2 \cdot \frac{12 \cdot E \cdot J_{HE300A}}{{H_2}^3} = 34085333.333$$

$$k_{02} := 2 \cdot \frac{12 \cdot E \cdot J_{HE500A}}{\left(H_1 + H_2\right)^3} = 20293000.000$$

$$k_{23} := 2 \cdot \frac{E \cdot A_{IPE240}}{L_1} \cdot \left(\cos\left(\alpha_1\right)\right)^2 = 195943953.587$$

$$k_{03} := 2 \cdot \frac{E \cdot A_{IPE240}}{L_1} \cdot \left(\sin\left(\alpha_1\right)\right)^2 = 48985988.397$$

Matrice di rigidezza

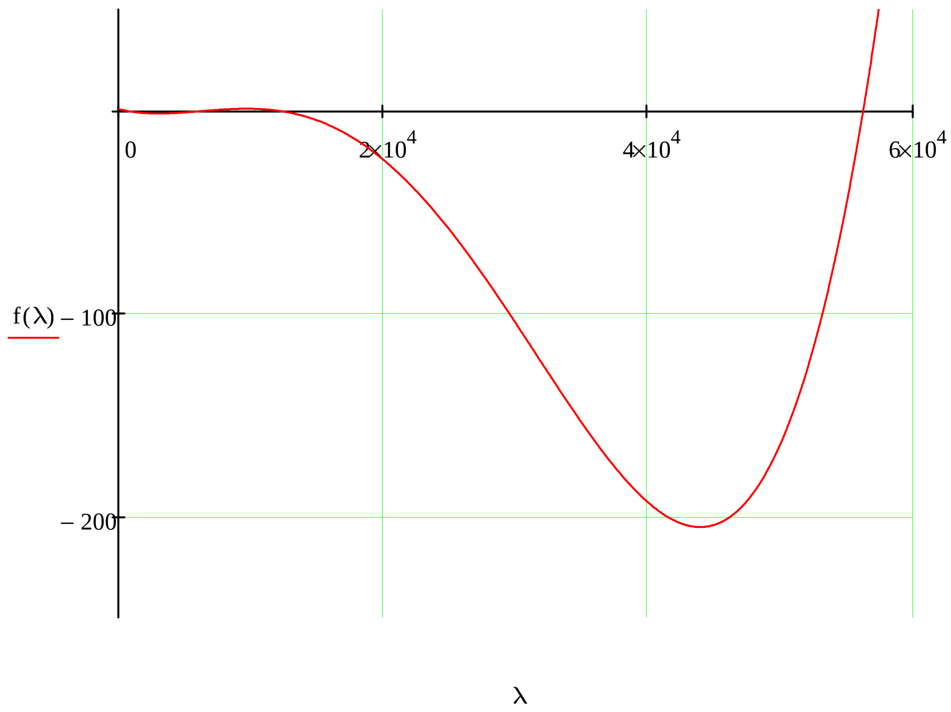
$$K := \begin{pmatrix} k_{01} + k_{12} & -k_{12} & 0 & 0 \\ -k_{12} & k_{02} + k_{12} + k_{23} & -k_{23} & 0 \\ 0 & -k_{23} & k_{23} & 0 \\ 0 & 0 & 0 & k_{03} \end{pmatrix} = \begin{pmatrix} 42606667 & -34085333 & 0 & 0 \\ -34085333 & 250322287 & -195943954 & 0 \\ 0 & -195943954 & 195943954 & 0 \\ 0 & 0 & 0 & 48985988 \end{pmatrix}$$

Matrice di massa

$$M := \begin{pmatrix} m_1 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 \\ 0 & 0 & m_3 & 0 \\ 0 & 0 & 0 & m_3 \end{pmatrix} = \begin{pmatrix} 8000 & 0 & 0 & 0 \\ 0 & 28000 & 0 & 0 \\ 0 & 0 & 4000 & 0 \\ 0 & 0 & 0 & 4000 \end{pmatrix}$$

Ricerca degli autovalori

$$f(\lambda) := \frac{|K - \lambda \cdot M|}{|K|} \qquad |K| = 1.109 \times 10^{31} \qquad |M| = 3.584 \times 10^{15}$$



$$vec\_coeffs := f(\lambda)\ coeffs \rightarrow \begin{pmatrix} 0.99999999999999999999 \\ -0.0016590671235527647706 \\ 3.7920928041070877337e-7 \\ -2.4405845961340488688e-11 \\ 3.2326308634518862083e-16 \end{pmatrix}$$

$$\lambda := polyroots(vec\_coeffs) = \begin{pmatrix} 714.0 \\ 6290.1 \\ 12246.5 \\ 56247.9 \end{pmatrix} \qquad \sqrt{\frac{k_{03}}{m_3}} = 110.664$$

Pulsazioni, frequenze e periodi propri

$$\omega := \sqrt{\lambda} = \begin{pmatrix} 26.720 \\ 79.310 \\ 110.664 \\ 237.166 \end{pmatrix} \qquad f := \frac{\omega}{2 \cdot \pi} = \begin{pmatrix} 4.253 \\ 12.623 \\ 17.613 \\ 37.746 \end{pmatrix} \qquad T := \frac{1}{f} = \begin{pmatrix} 0.235 \\ 0.079 \\ 0.057 \\ 0.026 \end{pmatrix}$$

Ricerca degli autovettori

Primo autovettore

$w := 1 \qquad x := 1 \qquad y := 1 \qquad z := 1$

Given

$$\left[(K - \lambda_1 \cdot M)\cdot\begin{pmatrix} w \\ x \\ y \\ z \end{pmatrix}\right]_1 = 0 \qquad \left[(K - \lambda_1 \cdot M)\cdot\begin{pmatrix} w \\ x \\ y \\ z \end{pmatrix}\right]_2 = 0 \qquad \left[(K - \lambda_1 \cdot M)\cdot\begin{pmatrix} w \\ x \\ y \\ z \end{pmatrix}\right]_4 = 0$$

$$w^2 + x^2 + y^2 + z^2 = 1$$

$$a_1 := \text{Find}(w, x, y, z) \qquad a_1 = \begin{pmatrix} 0.544 \\ 0.589 \\ 0.598 \\ 0.000 \end{pmatrix}$$

Secondo autovettore

$w := 1 \qquad x := 1 \qquad y := 1 \qquad z := 1$

Given

$$\left[(K - \lambda_2 \cdot M)\cdot\begin{pmatrix} w \\ x \\ y \\ z \end{pmatrix}\right]_1 = 0 \qquad \left[(K - \lambda_2 \cdot M)\cdot\begin{pmatrix} w \\ x \\ y \\ z \end{pmatrix}\right]_2 = 0 \qquad \left[(K - \lambda_2 \cdot M)\cdot\begin{pmatrix} w \\ x \\ y \\ z \end{pmatrix}\right]_4 = 0$$

$$w^2 + x^2 + y^2 + z^2 = 1$$

$$a_2 := \text{Find}(w, x, y, z) \qquad a_2 = \begin{pmatrix} 0.945 \\ -0.214 \\ -0.246 \\ 0.000 \end{pmatrix}$$

Terzo autovettore

$w := 1 \qquad x := 1 \qquad y := 1 \qquad z := 1$

Given

$$\left[\left(K - \lambda_3 \cdot M\right) \cdot \begin{pmatrix} w \\ x \\ y \\ z \end{pmatrix}\right]_1 = 0 \qquad \left[\left(K - \lambda_3 \cdot M\right) \cdot \begin{pmatrix} w \\ x \\ y \\ z \end{pmatrix}\right]_2 = 0 \qquad \left[\left(K - \lambda_3 \cdot M\right) \cdot \begin{pmatrix} w \\ x \\ y \\ z \end{pmatrix}\right]_3 = 0$$

$$w^2 + x^2 + y^2 + z^2 = 1$$

$$a_3 := \mathrm{Find}(w, x, y, z) \qquad a_3 = \begin{pmatrix} 0.000 \\ 0.000 \\ 0.000 \\ 1.000 \end{pmatrix}$$

Quarto autovettore

$w := 1 \qquad x := 1 \qquad y := 1 \qquad z := 1$

Given

$$\left[\left(K - \lambda_4 \cdot M\right) \cdot \begin{pmatrix} w \\ x \\ y \\ z \end{pmatrix}\right]_1 = 0 \qquad \left[\left(K - \lambda_4 \cdot M\right) \cdot \begin{pmatrix} w \\ x \\ y \\ z \end{pmatrix}\right]_2 = 0 \qquad \left[\left(K - \lambda_4 \cdot M\right) \cdot \begin{pmatrix} w \\ x \\ y \\ z \end{pmatrix}\right]_4 = 0$$

$$w^2 + x^2 + y^2 + z^2 = 1$$

$$a_4 := \mathrm{Find}(w, x, y, z) \qquad a_4 = \begin{pmatrix} 0.012 \\ -0.147 \\ 0.989 \\ 0.000 \end{pmatrix}$$

Autovettori di norma unitaria

$$a_1 = \begin{pmatrix} 0.544 \\ 0.589 \\ 0.598 \\ 0.000 \end{pmatrix} \quad a_2 = \begin{pmatrix} 0.945 \\ -0.214 \\ -0.246 \\ 0.000 \end{pmatrix} \quad a_3 = \begin{pmatrix} 0.000 \\ 0.000 \\ 0.000 \\ 1.000 \end{pmatrix} \quad a_4 = \begin{pmatrix} 0.012 \\ -0.147 \\ 0.989 \\ 0.000 \end{pmatrix}$$

Normalizzazione rispetto alla matrice di massa

$$\mu_1 := a_1^T \cdot M \cdot a_1 \quad \mu_2 := a_2^T \cdot M \cdot a_2 \quad \mu_3 := a_3^T \cdot M \cdot a_3 \quad \mu_4 := a_4^T \cdot M \cdot a_4$$

$$\mu_1 = 13507.875 \quad \mu_2 = 8674.638 \quad \mu_3 = 4000.000 \quad \mu_4 = 4516.614$$

$$\phi_1 := \frac{a_1}{\sqrt{\mu_1}} \quad \phi_2 := \frac{a_2}{\sqrt{\mu_2}} \quad \phi_3 := \frac{a_3}{\sqrt{\mu_3}} \quad \phi_4 := \frac{a_4}{\sqrt{\mu_4}}$$

$$\phi_1 = \begin{pmatrix} 0.004681 \\ 0.005067 \\ 0.005142 \\ 0.000000 \end{pmatrix} \quad \phi_2 = \begin{pmatrix} 0.010151 \\ -0.002297 \\ -0.002636 \\ 0.000000 \end{pmatrix} \quad \phi_3 = \begin{pmatrix} 0.000000 \\ 0.000000 \\ 0.000000 \\ 0.015811 \end{pmatrix} \quad \phi_4 = \begin{pmatrix} 0.000183 \\ -0.002182 \\ 0.014718 \\ 0.000000 \end{pmatrix}$$

Condizioni di ortogonalità

$\phi_1^T \cdot M \cdot \phi_1 = 1.000$ $\quad \phi_1^T \cdot M \cdot \phi_2 = -0.000$ $\quad \phi_1^T \cdot M \cdot \phi_3 = 0.000$ $\quad \phi_1^T \cdot M \cdot \phi_4 = 0.000$

$\phi_2^T \cdot M \cdot \phi_1 = -0.000$ $\quad \phi_2^T \cdot M \cdot \phi_2 = 1.000$ $\quad \phi_2^T \cdot M \cdot \phi_3 = 0.000$ $\quad \phi_2^T \cdot M \cdot \phi_4 = 0.000$

$\phi_3^T \cdot M \cdot \phi_1 = 0.000$ $\quad \phi_3^T \cdot M \cdot \phi_2 = 0.000$ $\quad \phi_3^T \cdot M \cdot \phi_3 = 1.000$ $\quad \phi_3^T \cdot M \cdot \phi_4 = 0.000$

$\phi_4^T \cdot M \cdot \phi_1 = 0.000$ $\quad \phi_4^T \cdot M \cdot \phi_2 = 0.000$ $\quad \phi_4^T \cdot M \cdot \phi_3 = 0.000$ $\quad \phi_4^T \cdot M \cdot \phi_4 = 1.000$

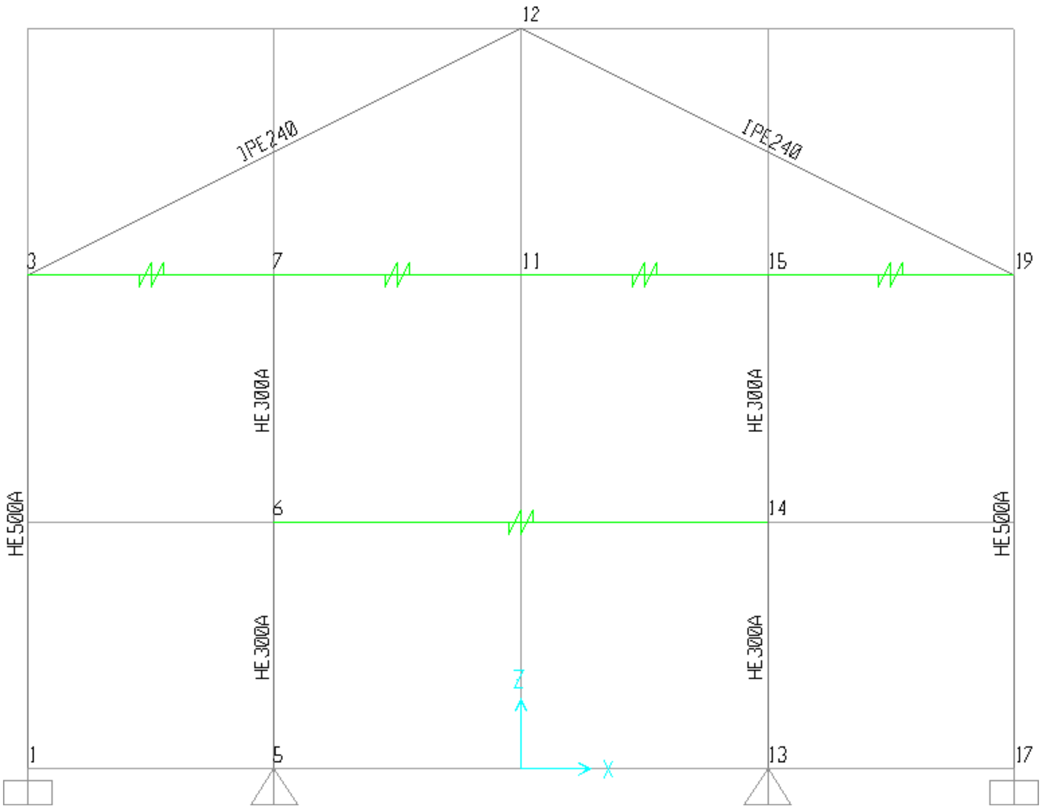
$\phi_1^T \cdot K \cdot \phi_1 = 713.953$ $\quad \phi_1^T \cdot K \cdot \phi_2 = -0.000$ $\quad \phi_1^T \cdot K \cdot \phi_3 = 0.000$ $\quad \phi_1^T \cdot K \cdot \phi_4 = 0.000$

$\phi_2^T \cdot K \cdot \phi_1 = -5.252\times 1$ $\quad \phi_2^T \cdot K \cdot \phi_2 = 6290.081$ $\quad \phi_2^T \cdot K \cdot \phi_3 = 0.000$ $\quad \phi_2^T \cdot K \cdot \phi_4 = 0.000$

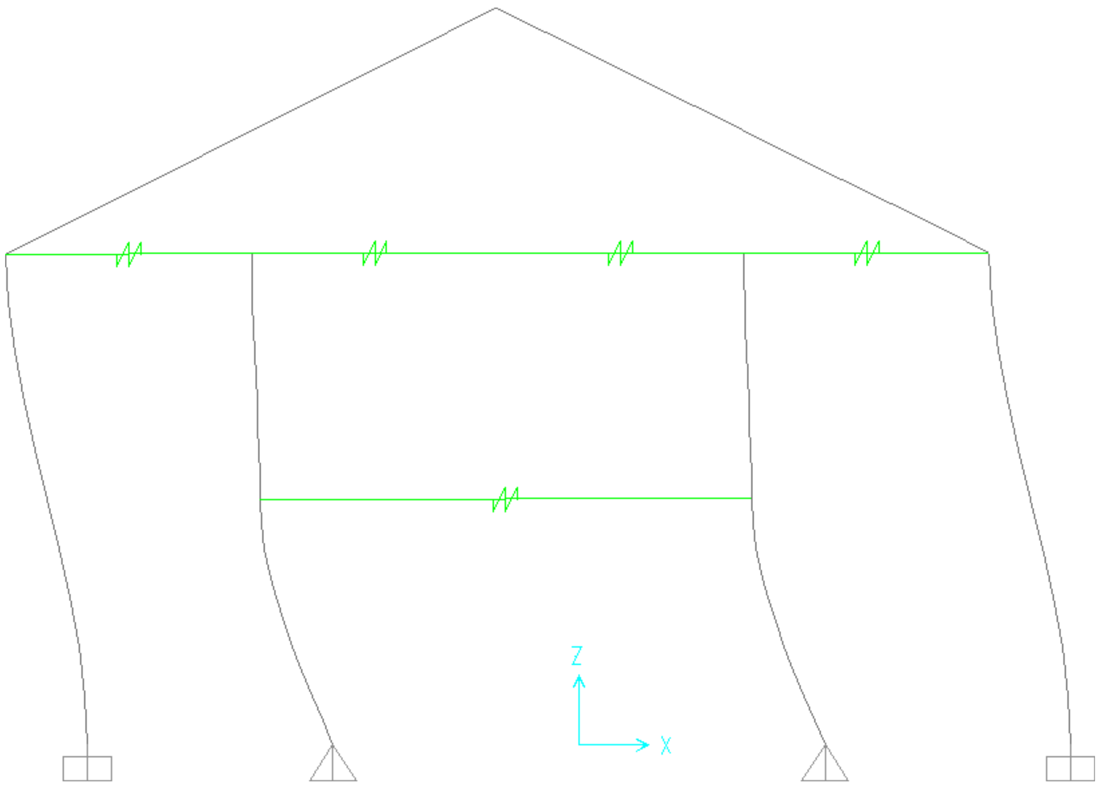
$\phi_3^T \cdot K \cdot \phi_1 = 0.000$ $\quad \phi_3^T \cdot K \cdot \phi_2 = 0.000$ $\quad \phi_3^T \cdot K \cdot \phi_3 = 12246.497$ $\quad \phi_3^T \cdot K \cdot \phi_4 = 0.000$

$\phi_4^T \cdot K \cdot \phi_1 = 0.000$ $\quad \phi_4^T \cdot K \cdot \phi_2 = 0.000$ $\quad \phi_4^T \cdot K \cdot \phi_3 = 0.000$ $\quad \phi_4^T \cdot K \cdot \phi_4 = 56247.869$

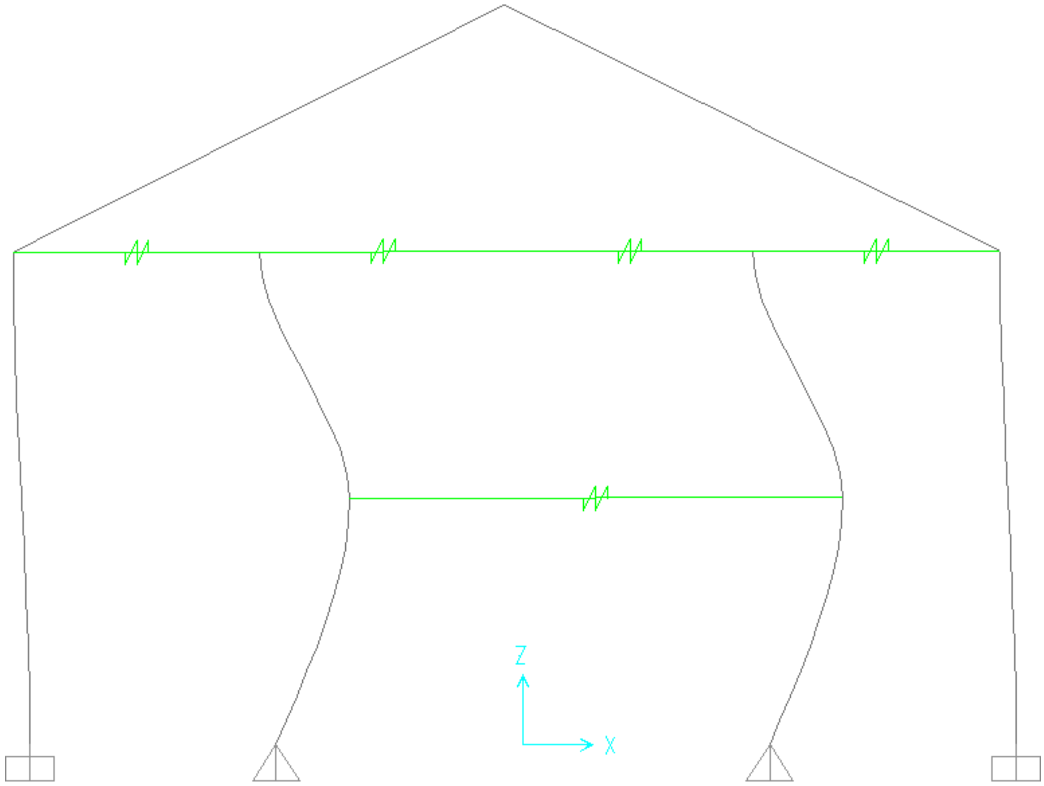
Prova d'esame del 21 giugno 2011 – Risultati analisi FEM



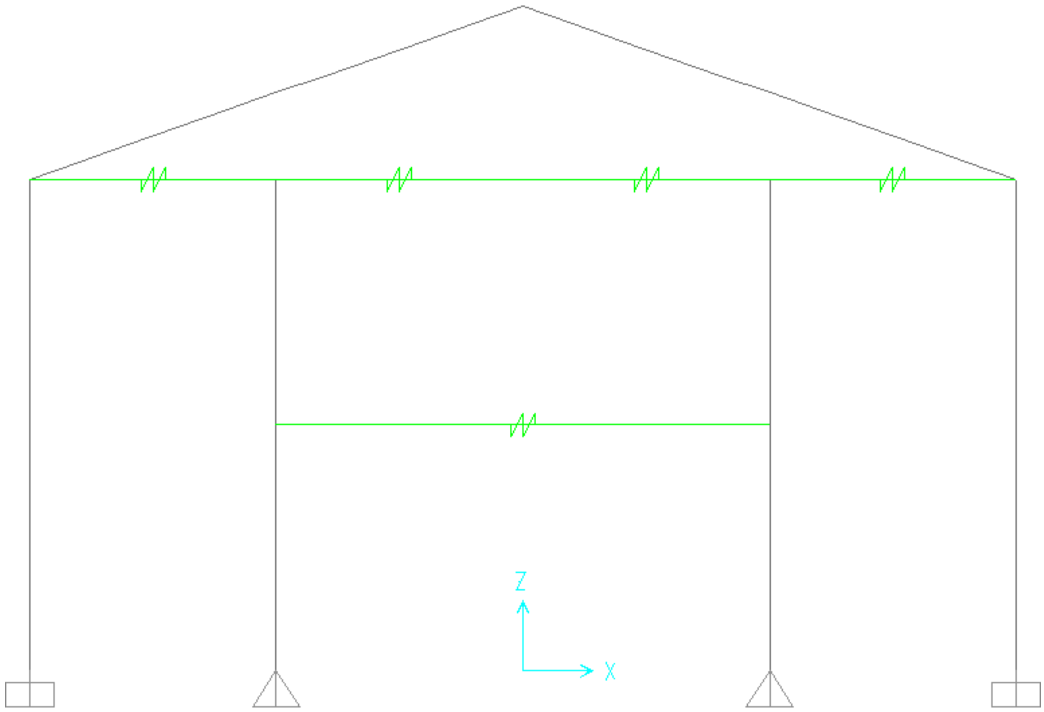
Configurazione di riferimento



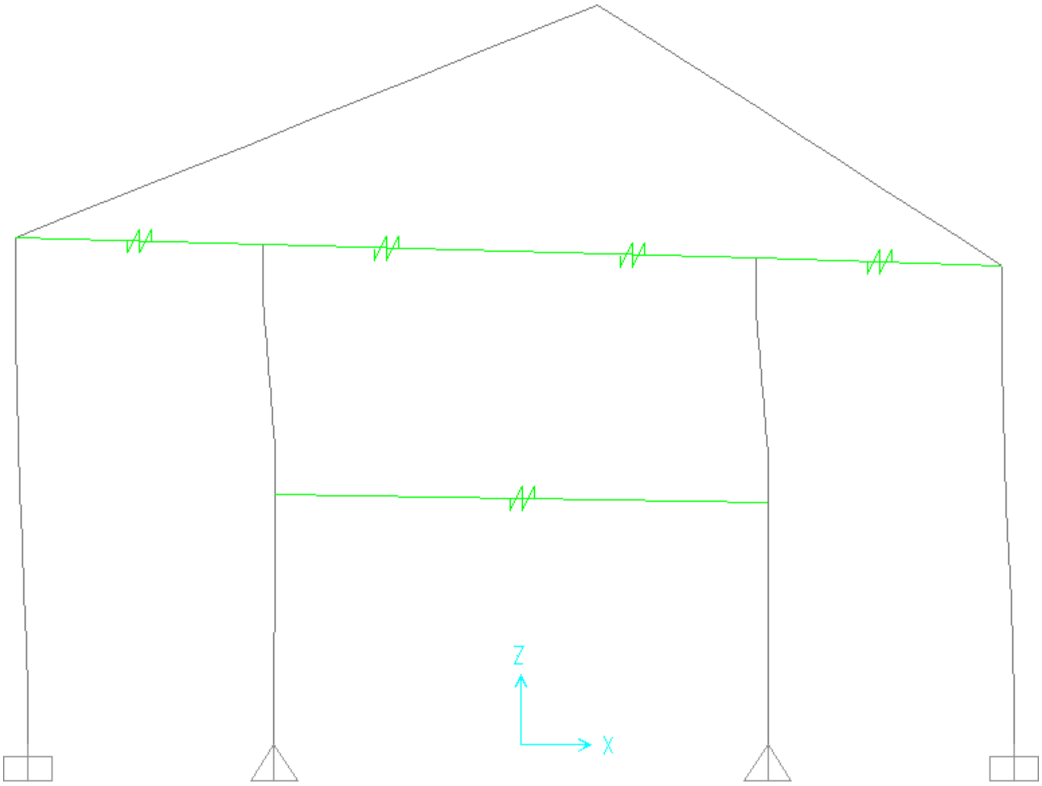
Forma modale – Modo 1 (f_1 = 3.920 Hz)



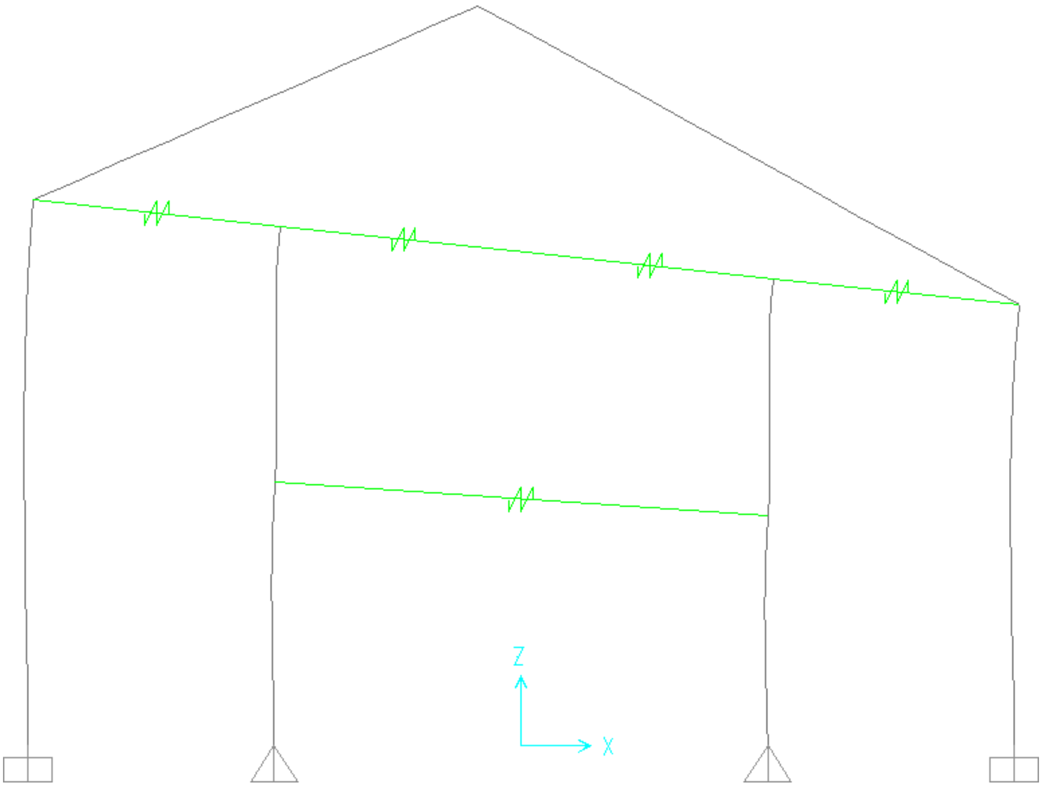
Forma modale – Modo 2 (f_2 = 11.049 Hz)



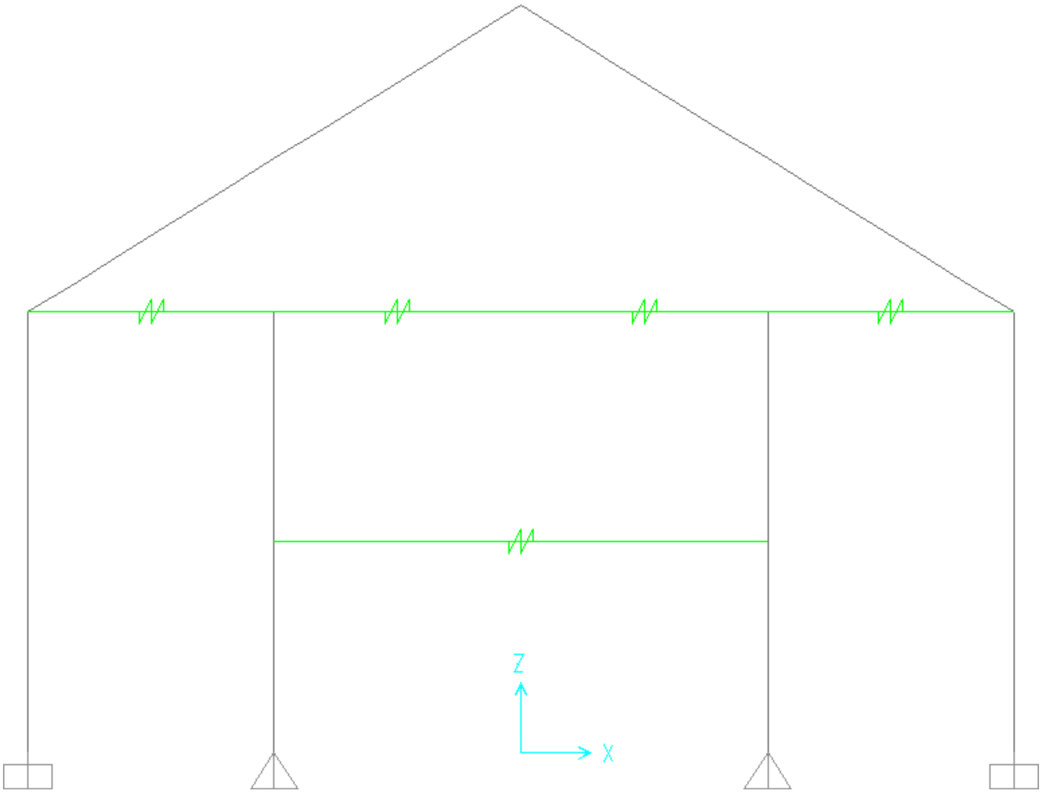
Forma modale – Modo 3 (f_3 = 16.957 Hz)



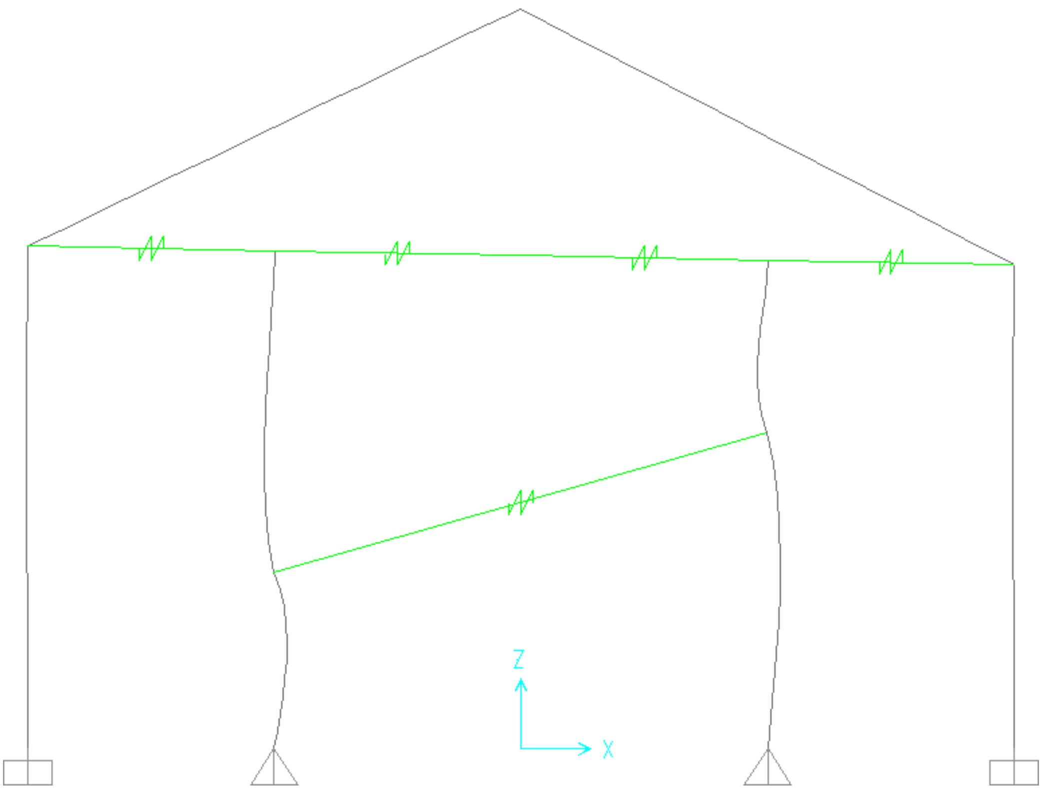
Forma modale – Modo 4 (f_4 = 35.344 Hz)



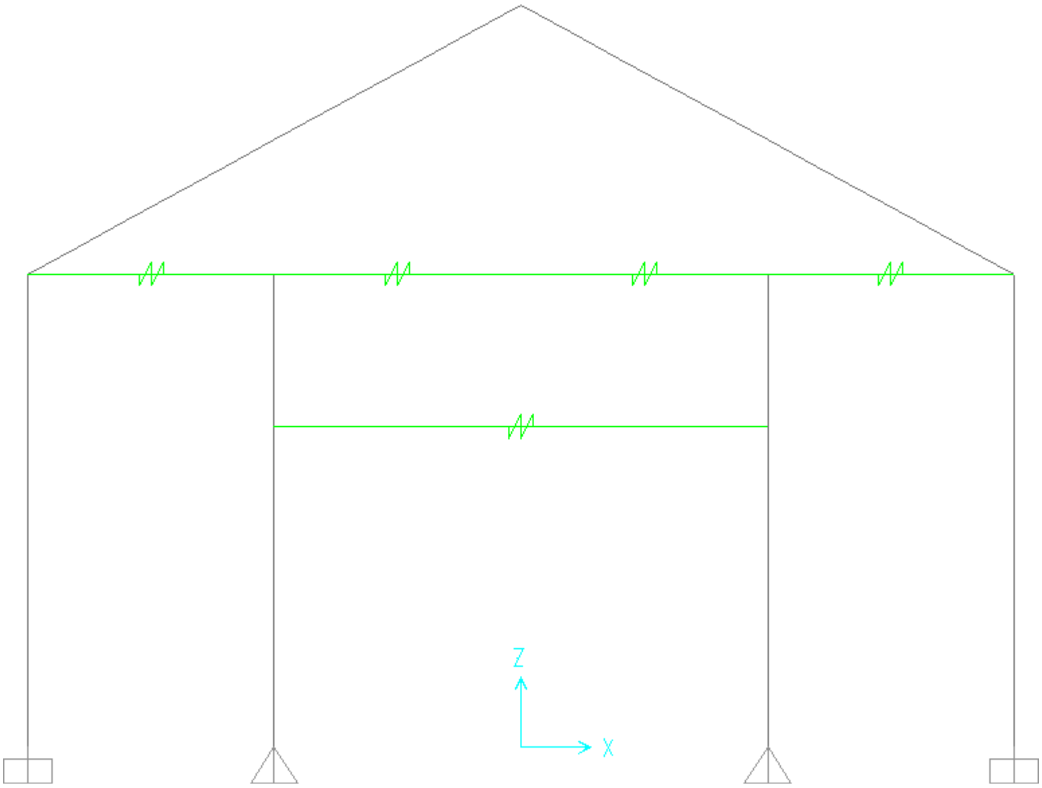
Forma modale – Modo 5 (f_5 = 45.455 Hz)



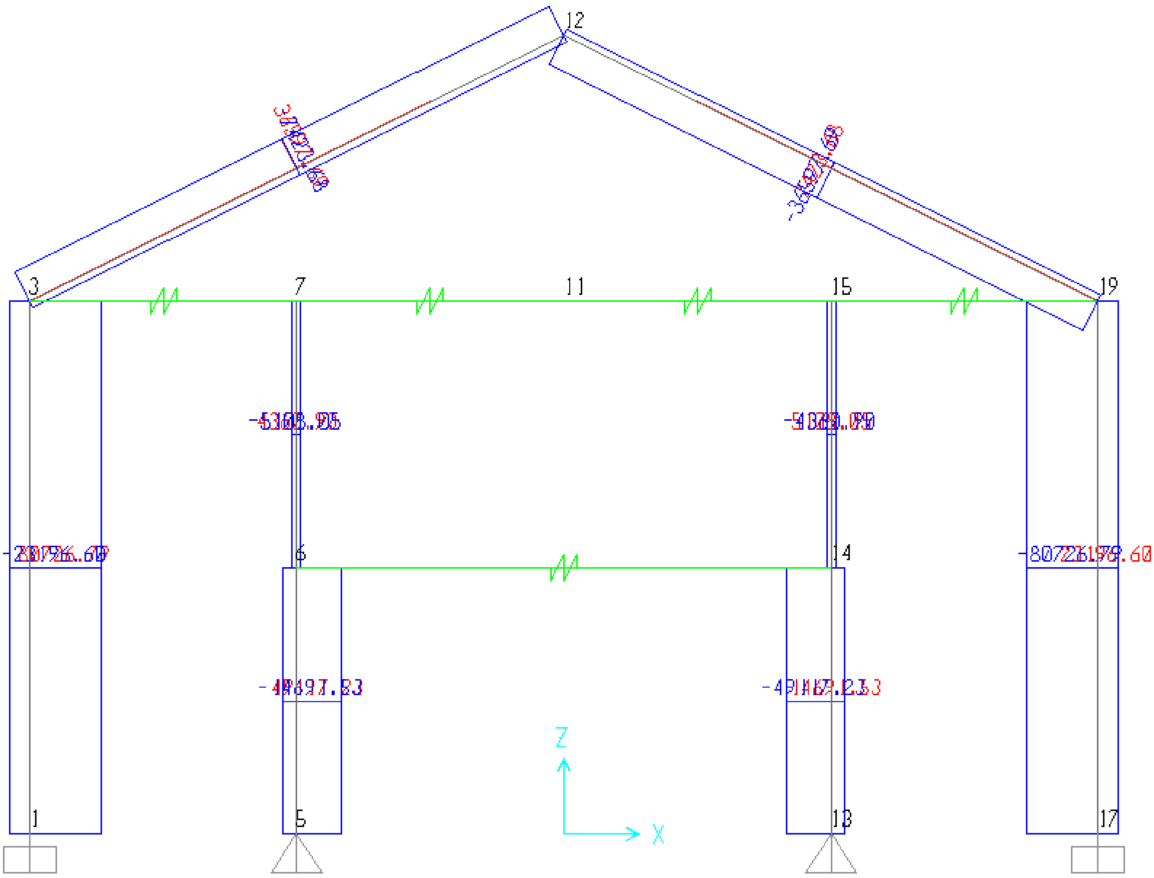
Forma modale – Modo 6 (f_6 = 49.136 Hz)



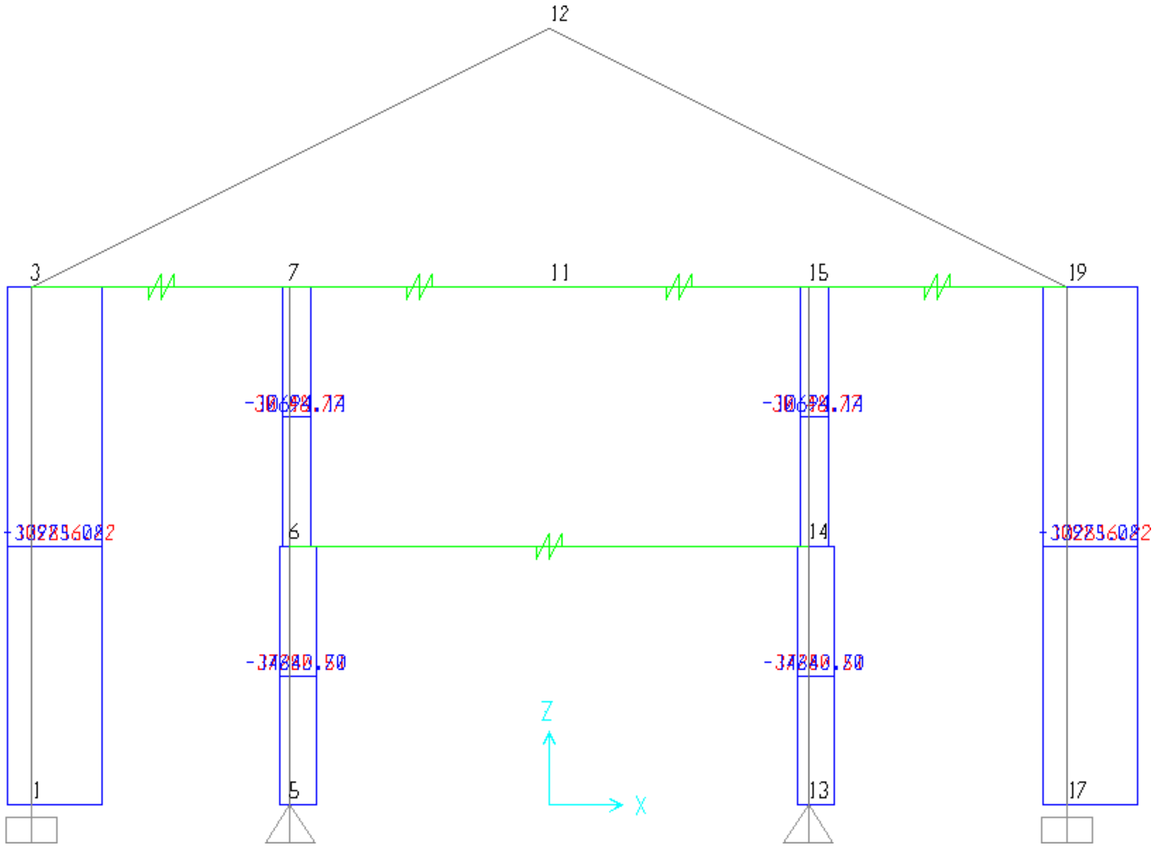
Forma modale – Modo 7 (f_7 = 98.744 Hz)



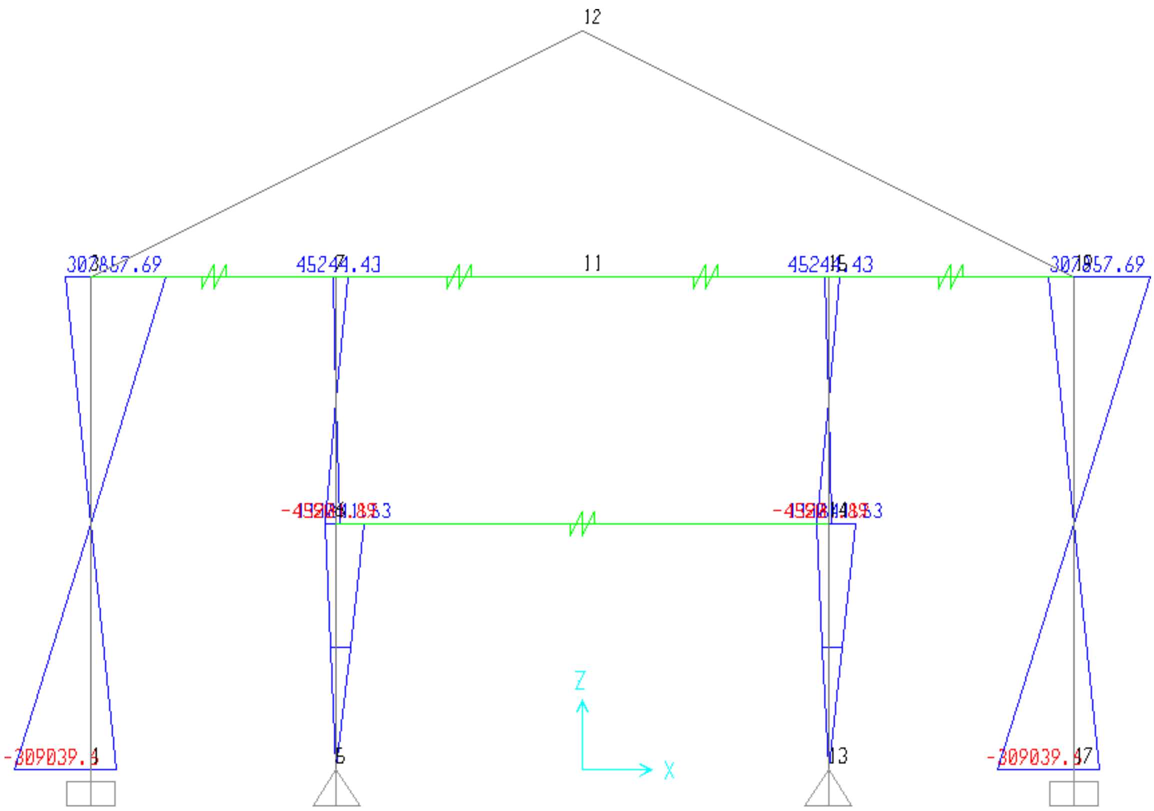
Forma modale – Modo 8 (f_8 = 103.460 Hz)



Forza normale – Inviluppo



Forza di taglio – Inviluppo



Momento flettente – Inviluppo