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MODELLING OF INTERFACIAL FRACTURE OF LAYERED STRUCTURES

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ABSTRACT

The paper analyses the problem of an interfacial crack propagating between two elastic layers, generally different the one from the other for thickness and material. The layers are modelled as shear-deformable laminated beams of finite length, connected one another by an interface of negligible thickness, and subjected to general loads at their ends. The problem is formulated through a set of six differential equations. In the simplest case of a linearly elastic interface, the problem is analytically solved by adopting the interfacial stresses as principal unknowns. Thus, explicit expressions for the interfacial stresses, internal forces, displacements, energy release rate, and mode mixity angle are determined.

KEYWORDS

Layered structures, interfacial fracture, delamination, mixed-mode fracture, fracture mode partition, interfacial stresses, energy release rate.

INTRODUCTION

Interfacial fracture is a major failure mode for a wide class of layered structures [1]. Examples include the delamination of composite laminates [2], the decohesion of thin films from substrates, the cracking of the adhesive in adhesively bonded joints [3]. Within the framework of fracture mechanics, crack-growth criteria can be formulated in terms of the energy release rate, G. However, since interfacial fracture usually occurs under mixed-mode conditions, it is also necessary to determine the contributions, G_{I} and G_{II}, respectively related to fracture modes I (opening) and II (sliding) [4]. In order to model in detail the exchange of stresses between the separating layers, an interface layer can be explicitly introduced in the mechanical model. Depending on the adopted interfacial constitutive law, various effects such as anisotropy, plasticity, viscosity, damage and so on can be taken into account [5].

This paper analyses the problem of an interfacial crack propagating between two elastic layers, generally different the one from the other for thickness and material. By extending a modelling approach already adopted for the study of composite delamination [6, 7], the two layers are modelled as shear-deformable laminated beams of finite length, connected one another by an interface of negligible thickness, and subjected to general loads at their ends. The problem is formulated through a set of six differential equations, along with suitable boundary conditions. In the simplest case of a linearly elastic interface, the governing differential equations are uncoupled and analytically solved by adopting the interfacial stresses as principal unknowns. Thus, explicit expressions for the interfacial stresses, internal forces, displacements, energy release rate, and mode mixity angle are determined.

The obtained solution can be used to deal with a wide gamut of interfacial fracture problems, ranging from laboratory test specimens to real structural components.

LAYERED STRUCTURE MODELLING

Mechanical model

Consider a layered structural element AB of length L, thickness H, and width W, obtained by connecting two layers of thicknesses H_1 and H_2 through an interface of thickness $t \ll H$ (Fig. 1). An interfacial crack of length a runs from the end section A to an intermediate section 0, so that the length of the unbroken part, from section 0 to B, is $b = L - a$.

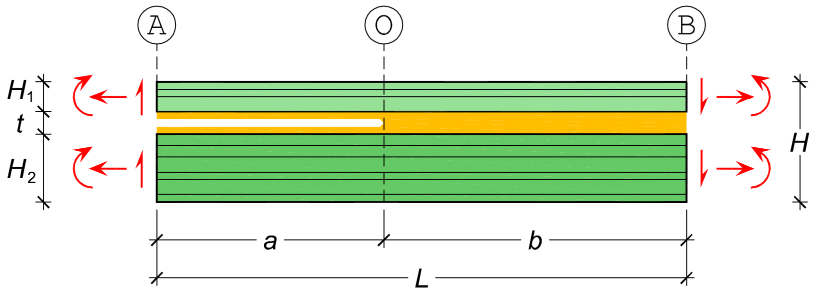

Fig. 1: Layered structural element

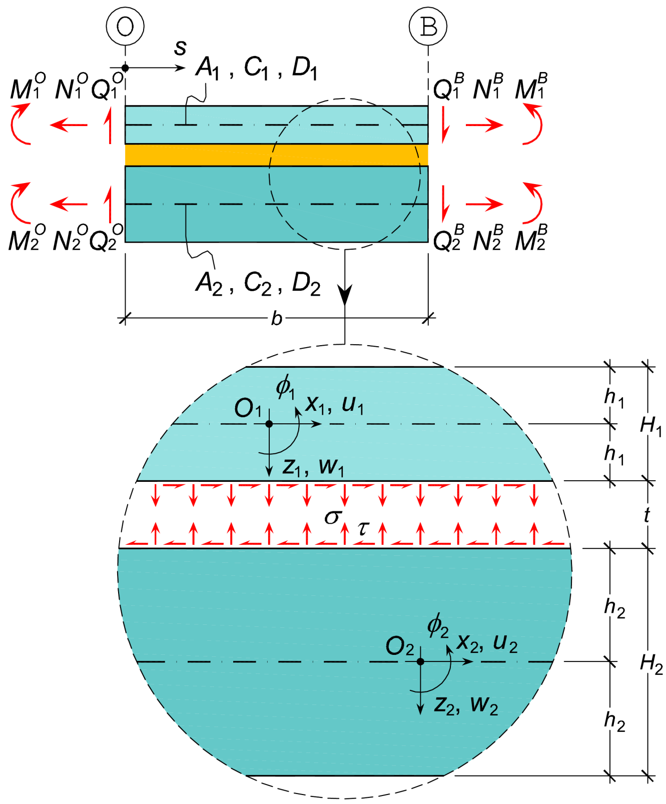

Fig. 2: Mechanical model

A mechanical model of the abovementioned element can be developed by supposing the two connected layers behave as elastic, shear-deformable laminated beams. Since the parts between sections A and 0 can be studied as cantilever beams, in the model it will suffice to consider the segment 0B of the layered element (Fig. 2). An abscissa s is introduced to specify the position of the beams' cross sections. Two local reference systems, $O_1x_1z_1$ and $O_2x_2z_2$, are defined with the origins placed on the centrelines of the upper and lower beams, respectively. Accordingly, u_α and w_α are the beams' mid-plane displacements along the axial and transverse directions, respectively, and ϕ_α are the cross-section rotations, positive if counter-clockwise (here and in the following the upper and lower beams are identified by subscripts $\alpha = 1, 2$, respectively). Moreover, let $h = H / 2$, $h_1 = H_1 / 2$, and $h_2 = H_2 / 2$.

According to Timoshenko's beam theory,

$$\varepsilon_\alpha(s) = \frac{du_\alpha}{ds}, \qquad \gamma_\alpha(s) = \frac{dw_\alpha}{ds} + \phi_\alpha(s), \qquad \kappa_\alpha(s) = \frac{d\phi_\alpha}{ds} \tag{1}$$

respectively are the axial strain, shear strain, and curvature. Correspondingly,

$$N_\alpha(s) = W\, A_\alpha \varepsilon_\alpha(s), \qquad Q_\alpha(s) = W\, C_\alpha \gamma_\alpha(s), \qquad M_\alpha(s) = W\, D_\alpha \kappa_\alpha(s) \tag{2}$$

respectively are the axial force, shear force, and bending moment, and A_α, C_α and D_α respectively denote the beams' extension, shear and bending stiffnesses (per unit width) [8]. Also, it is convenient to define the beams' extension, shear, and bending compliances,

$$a_\alpha = \frac{1}{A_\alpha}, \qquad c_\alpha = \frac{1}{C_\alpha}, \qquad d_\alpha = \frac{1}{D_\alpha}. \tag{3}$$

Differential problem

The equilibrium equations for the connected beams are

$$\frac{dN_\alpha}{ds} + n_\alpha = 0, \qquad \frac{dQ_\alpha}{ds} + q_\alpha = 0, \qquad \frac{dM_\alpha}{ds} + m_\alpha - Q_\alpha = 0, \tag{4}$$

where

$$n_1 = -n_2 = W\,\tau, \qquad q_1 = -q_2 = W\,\sigma, \qquad m_\alpha = W\,\tau\, h_\alpha \tag{5}$$

are distributed loads and couples, and σ and τ respectively are the normal and tangential stresses exchanged through the interface. By substituting equations (1–3) and (5) into (4), the set of governing differential equations for the problem is obtained,

$$\begin{aligned} &\frac{d^2u_1}{ds^2} = -a_1\,\tau, \qquad \frac{d^2w_1}{ds^2} + \frac{d\phi_1}{ds} = -c_1\,\sigma, \qquad \frac{d^3\phi_1}{ds^3} = -d_1\,(\sigma + h_1 \frac{d\tau}{ds}), \\ &\frac{d^2u_2}{ds^2} = a_2\,\tau, \qquad \frac{d^2w_2}{ds^2} + \frac{d\phi_2}{ds} = c_2\,\sigma, \qquad \frac{d^3\phi_2}{ds^3} = d_2\,(\sigma - h_2 \frac{d\tau}{ds}), \end{aligned} \tag{6}$$

where the interfacial stresses,

$$\sigma = \sigma(\Delta u, \Delta w), \qquad \tau = \tau(\Delta u, \Delta w), \tag{7}$$

are functions of the axial and transverse relative displacements at the interface,

$$\Delta u = u_2 - u_1 - \phi_2\, h_2 - \phi_1\, h_1, \qquad \Delta w = w_2 - w_1. \tag{8}$$

The layered element OB is supposed to be in equilibrium under a given system of in-plane concentrated loads, acting at the ends of the upper and lower beams.

Consequently, the following boundary conditions apply:

$$\begin{aligned}&N_1\big|_{s=0} = N_1^O, \quad Q_1\big|_{s=0} = Q_1^O, \quad M_1\big|_{s=0} = M_1^O; \quad N_1\big|_{s=b} = N_1^B, \quad Q_1\big|_{s=b} = Q_1^B, \quad M_1\big|_{s=b} = M_1^B;\\ &N_2\big|_{s=0} = N_2^O, \quad Q_2\big|_{s=0} = Q_2^O, \quad M_2\big|_{s=0} = M_2^O; \quad N_2\big|_{s=b} = N_2^B, \quad Q_2\big|_{s=b} = Q_2^B, \quad M_2\big|_{s=b} = M_2^B;\end{aligned} \tag{9}$$

where, because of global equilibrium,

$$\begin{aligned}&N_2^B = N_1^O + N_2^O - N_1^B, \qquad Q_2^B = Q_1^O + Q_2^O - Q_1^B,\\ &M_2^B = M_1^O + M_2^O - M_1^B + (Q_1^O + Q_2^O)\, b + (N_1^B - N_1^O)\, h.\end{aligned} \tag{10}$$

SOLUTION STRATEGY

Change of variables

In many applications, the interface can be thought of as a continuous distribution of elastic-brittle springs. In the linearly elastic behaviour range, the interfacial stresses are

$$\sigma = k_z\, \Delta w, \qquad \tau = k_x\, \Delta u, \tag{11}$$

where k_z and k_x are the interface's elastic constants. Following a solution strategy already adopted for the study of composite delamination [7], the interfacial stresses are conveniently assumed as main unknowns. Equations (8) are substituted into (11), which are then differentiated with respect to s four and three times, respectively. Next, from equations (6) the following differential equation set is obtained:

$$\begin{aligned}&\frac{d^4\sigma}{ds^4} - k_z(c_1 + c_2)\frac{d^2\sigma}{ds^2} + k_z(d_1 + d_2)\,\sigma + k_z(d_1h_1 - d_2h_2)\frac{d\tau}{ds} = 0,\\ &\frac{d^3\tau}{ds^3} - k_x(a_1 + a_2 + d_1h_1^2 + d_2h_2^2)\frac{d\tau}{ds} - k_x(d_1h_1 - d_2h_2)\,\sigma = 0.\end{aligned} \tag{11}$$

Two cases have to be considered in solving the problem:
a) $d_1h_1 = d_2h_2$, the 'balanced' case, including the case of identical connected beams;
b) $d_1h_1 \neq d_2h_2$, the 'unbalanced' or general case.

In the balanced case, equations (11) are uncoupled and can be solved separately to obtain the normal and tangential stresses. Details, here omitted for brevity, are postponed to an extended paper. In the general case, uncoupling of equations (11) is obtained by solving the first equation for $d\tau/ds$ and substituting the result into the second equation. A sixth-order, linear homogeneous differential equation for the normal interfacial stress is obtained,

$$\frac{d^6\sigma}{ds^6} + \hat{b}\frac{d^4\sigma}{ds^4} + \hat{c}\frac{d^2\sigma}{ds^2} + \hat{d}\,\sigma = 0\,, \tag{12}$$

where

$$\begin{aligned}&\hat{b} = -k_x(a_1 + a_2 + d_1h_1^2 + d_2h_2^2) - k_z(c_1 + c_2),\\ &\hat{c} = k_xk_z(a_1 + a_2 + d_1h_1^2 + d_2h_2^2)(c_1 + c_2) + k_z(d_1 + d_2),\\ &\hat{d} = -k_xk_z[(a_1 + a_2)(d_1 + d_2) + d_1d_2(h_1 + h_2)^2].\end{aligned} \tag{13}$$

Interfacial stresses

By solving equation (12), and substituting the result into equations (11), the general solution for the normal and tangential interfacial stresses is obtained,

$$\sigma(s) = \sum_{i=1}^{6} F_i \exp(\lambda_i\, s),$$
$$\tau(s) = -\frac{1}{\mathrm{d}_1 h_1 - \mathrm{d}_2 h_2}\left\{\sum_{i=1}^{6} F_i \left[\frac{\lambda_i^3}{k_z} - (\mathrm{c}_1 + \mathrm{c}_2)\,\lambda_i + (\mathrm{d}_1 + \mathrm{d}_2)\,\frac{1}{\lambda_i}\right] \exp(\lambda_i\, s) + F_7\right\}, \quad (14)$$

where F_1, F_2, …, F_7 are integration constants to be determined by imposing the boundary conditions, and λ_1, λ_2, …, λ_6 are the roots of the characteristic equation,

$$\lambda^6 + \hat{b}\lambda^4 + \hat{c}\lambda^2 + \hat{d} = 0\,. \quad (15)$$

Internal forces

The internal forces can be deduced by substituting the interfacial stresses (14) into equations (4) and (5), and integrating with respect to s. Thus, six new integration constants, F_8, F_9, …, F_{13}, appear. Then, by substituting the internal forces into equations (1) and (2), and integrating with respect to s, the expressions of the displacements, involving six more constants, F_{14}, F_{15}, …, F_{19}, are obtained. To sum up, there are 19 integration constants to be determined by imposing the boundary conditions (9). Although these equations consist of only 9 independent equations, by introducing the expressions of the interfacial stresses and displacements into equations (11), it can be proved the independent constants are only 12, three of which represent a rigid displacement of the whole system.

Through the stated solution strategy, the internal forces in beams are determined as follows:

$$N_1(s) = \frac{W}{d_1 h_1 - d_2 h_2}\left\{\sum_{i=1}^{6} F_i \left[\frac{\lambda_i^2}{k_z} - c_1 - c_2 + \frac{d_1 + d_2}{\lambda_i^2}\right] \exp(\lambda_i\, s) + F_7 s + F_8\right\},$$
$$N_2(s) = -\frac{W}{d_1 h_1 - d_2 h_2}\left\{\sum_{i=1}^{6} F_i \left[\frac{\lambda_i^2}{k_z} - c_1 - c_2 + \frac{d_1 + d_2}{\lambda_i^2}\right] \exp(\lambda_i\, s) + F_7 s + F_9\right\};$$
$$Q_1(s) = -W \sum_{i=1}^{6} \frac{F_i}{\lambda_i} \exp(\lambda_i\, s) - \frac{W}{d_1 h_1 - d_2 h_2}\left(\frac{a_1 + a_2}{d_1 h} + h_1\right) F_7,$$
$$Q_2(s) = W \sum_{i=1}^{6} \frac{F_i}{\lambda_i} \exp(\lambda_i\, s) - \frac{W}{d_1 h_1 - d_2 h_2}\left(\frac{a_1 + a_2}{d_2 h} + h_2\right) F_7; \quad (16)$$
$$M_1(s) = \frac{W}{d_1 h_1 - d_2 h_2}\left\{\sum_{i=1}^{6} F_i \left[\left(\frac{\lambda_i^2}{k_z} - c_1 - c_2\right) h_1 + \frac{d_2 h}{\lambda_i^2}\right] \exp(\lambda_i\, s) - \frac{(a_1 + a_2)\, F_7 s + F_8 a_1 + F_9 a_2}{d_1 h}\right\},$$
$$M_2(s) = \frac{W}{d_1 h_1 - d_2 h_2}\left\{\sum_{i=1}^{6} F_i \left[\left(\frac{\lambda_i^2}{k_z} - c_1 - c_2\right) h_2 + \frac{d_1 h}{\lambda_i^2}\right] \exp(\lambda_i\, s) - \frac{(a_1 + a_2)\, F_7 s + F_8 a_1 + F_9 a_2}{d_2 h}\right\}.$$

The values of the integration constants, F_1, F_2, …, F_9, are determined for each specific problem by substituting equations (16) into the boundary conditions (9). In simpler cases, the resulting set of 9 linear equations can be solved analytically. In more complex cases, it is more convenient to solve that equation set numerically.

INTERFACIAL CRACK PROPAGATION

Interfacial crack propagation occurs when the energy release rate, G, attains a critical value, $G_c(\psi)$, computed according to a suitable mixed-mode crack-growth criterion [1], where

$$\psi = \arctan\sqrt{\frac{G_{\mathrm{II}}}{G_{\mathrm{I}}}} \tag{17}$$

is the mode-mixity angle.

In the considered two-layer system, crack propagation may initiate at either of the two end sections, O or B, which, depending on the particular problem being analysed, correspond to existing crack tips or free edges. Here, the energy release rates are

$$G_O = G_{O,\mathrm{I}} + G_{O,\mathrm{II}} \qquad \text{and} \qquad G_B = G_{B,\mathrm{I}} + G_{B,\mathrm{II}}, \tag{18}$$

where

$$G_{O,\mathrm{I}} = \frac{\sigma_O^2}{2k_z}, \qquad G_{O,\mathrm{II}} = \frac{\tau_O^2}{2k_x} \qquad \text{and} \qquad G_{B,\mathrm{I}} = \frac{\sigma_B^2}{2k_z}, \qquad G_{B,\mathrm{II}} = \frac{\tau_B^2}{2k_x} \tag{19}$$

are the respective contributions related to fracture modes I and II, and

$$\begin{aligned}
\sigma_O &= \sigma|_{s=0} = \sum_{i=1}^{6} F_i, & \tau_O &= \tau|_{s=0} = -\frac{\sum_{i=1}^{6} F_i \left[\frac{\lambda_i^3}{k_z} - (c_1+c_2)\,\lambda_i + \frac{d_1+d_2}{\lambda_i}\right] + F_7}{d_1h_1 - d_2h_2}, \\
\sigma_B &= \sigma|_{s=b} = \sum_{i=1}^{6} F_i \exp(\lambda_i b), & \tau_B &= \tau|_{s=b} = -\frac{\sum_{i=1}^{6} F_i \left[\frac{\lambda_i^3}{k_z} - (c_1+c_2)\,\lambda_i + \frac{d_1+d_2}{\lambda_i}\right] \exp(\lambda_i b) + F_7}{d_1h_1 - d_2h_2}.
\end{aligned} \tag{20}$$

Finally, the mode-mixity angles at the two end sections can be computed,

$$\psi_O = \arctan\sqrt{\frac{G_{O,\mathrm{II}}}{G_{O,\mathrm{I}}}} \qquad \text{and} \qquad \psi_B = \arctan\sqrt{\frac{G_{B,\mathrm{II}}}{G_{B,\mathrm{I}}}}. \tag{21}$$

APPLICATION: THE SINGLE-LAP JOINT (SLJ) TEST

As an example of application, the single-lap joint (SLJ) test (Fig. 3) is considered [9].

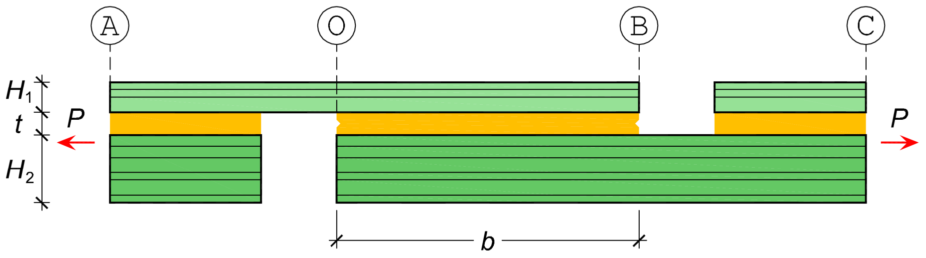

Fig. 3: The single-lap joint (SLJ) test

The forces acting at the end sections of the layered element (Fig. 4) in this case are:

$$\begin{aligned} &N_1^O = P, \quad Q_1^O = 0, \quad M_1^O = Ph_2; \quad N_1^B = 0, \quad Q_1^B = 0, \quad M_1^B = 0; \\ &N_2^O = 0, \quad Q_2^O = 0, \quad M_2^O = 0; \quad N_2^B = P, \quad Q_2^B = 0, \quad M_2^B = -Ph_1. \end{aligned} \tag{22}$$

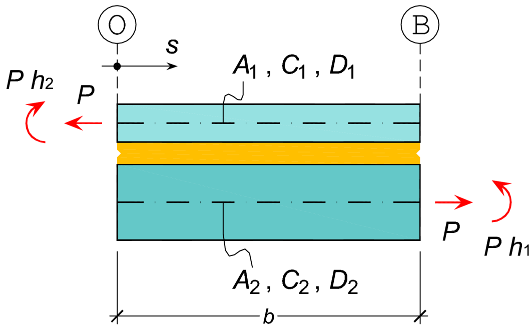

Fig. 4: Two-layer system corresponding to the single-lap joint (SLJ) test specimen

A specimen with the following sizes is considered [10]: $W = 25$ mm, $H = 4$ mm, $b = 50$ mm, $H_1 = 1$ mm, $H_2 = 3$ mm. The elastic moduli are $E_x = 106.3$ GPa and $G_{zx} = 40.0$ GPa. Hence, the beams' stiffnesses are $A_1 = E_x H_1 = 106300$ N/mm, $C_1 = 5\, G_{zx} H_1 / 6 = 33333$ N/mm, $D_1 = E_x H_1^3 / 12 = 8858$ N mm, $A_2 = E_x H_2 = 318900$ N/mm, $C_2 = 5\, G_{zx} H_2 / 6 = 100000$ N/mm, $D_2 = E_x H_2^3 / 12 = 239170$ N mm. The assumed interface constants are $k_x = 1500$ N/mm^3 and $k_z = 4100$ N/mm^3. The applied load is $P = 5000$ N.

Figure 5 shows the interfacial normal and tangential interfacial stresses as functions of the abscissa s. The peak values at section O are $\sigma_O = 180$ MPa and $\tau_O = 61$ MPa. Correspondingly, the energy release rate is $G_O = 5221$ J/m^2 and the mode-mixity angle is $\psi_O = 29.2°$. The values at section B are $\sigma_B = 0.1$ MPa and $\tau_B = 9.6$ MPa. Correspondingly, the energy release rate is $G_B = 36.6$ J/m^2 and the mode-mixity angle is $\psi_B = 67.0°$. Although, in general, the mixed-mode crack-growth criterion should be specified in order to predict crack growth, for the case under examination it is more likely that crack propagation would initiate from section O, where considerably higher values of interfacial stresses and energy release rate are detected.

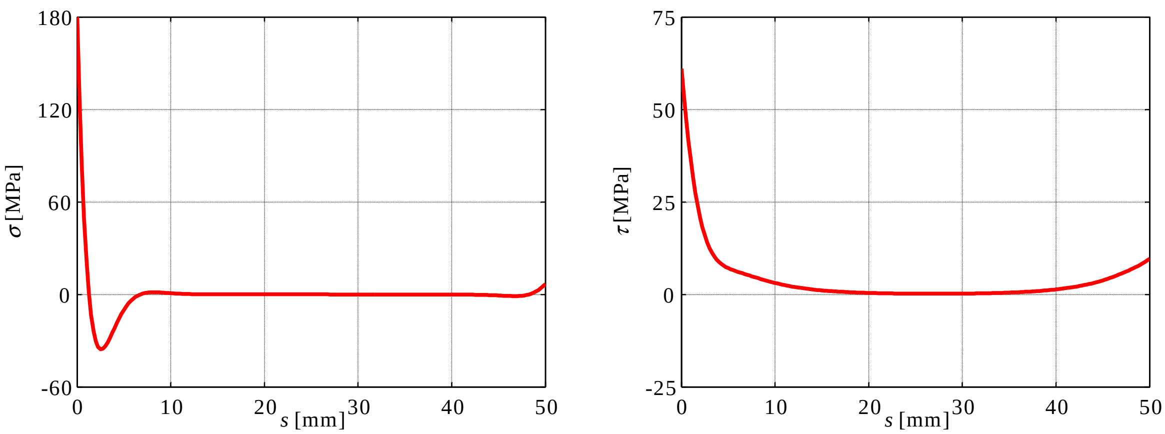

Fig. 5: SLJ specimen: interfacial stresses

CONCLUSIONS

The problem of an interfacial crack propagating between two different elastic layers has been analysed. The layers have been modelled as shear-deformable laminated beams of finite length, connected one another by an interface of negligible thickness, and subjected to general loads at their ends. A complete explicit solution has been obtained for the internal forces and interfacial stresses. Hence, explicit expressions for the energy release rate and mode mixity angle have been deduced.

As an example, the model has been applied to the case of an unbalanced single-lap joint test. However, the obtained solution can be effectively used to deal with a wide gamut of interfacial fracture problems, ranging from test specimens to real structures.

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