EXPLICIT EXPRESSIONS FOR THE CRACK LENGTH CORRECTION PARAMETERS FOR THE DCB, ENF, AND MMB TESTS ON MULTIDIRECTIONAL LAMINATES

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Keywords: Multidirectional laminates, Delamination toughness, DCB, ENF, and MMB tests, Crack length correction parameters, Beam-theory model

ABSTRACT

The *double cantilever beam* (DCB) and *end-notched flexure* (ENF) tests are the simplest and most commonly used testing methods to determine the delamination toughness of laminated specimens under fracture modes I and II, respectively. For I/II mixed-mode fracture, a widespread testing method is the *mixed-mode bending* (MMB) test, which can be regarded as the superposition of the DCB and ENF tests. For unidirectional (UD) laminated specimens, American, European and Japanese standards exist for such tests [1]. However, delamination toughness characterization of multidirectional (MD) composite laminates is still an open issue [2].

Several theoretical models are used in the literature to interpret the experimental results of the DCB, ENF, and MMB tests. The *simple beam-theory* (SBT) model considers the specimen as an assemblage of three rigidly connected Euler-Bernoulli beams [3]. The *corrected beam-theory* (CBT) model better accounts for the actual deformation of the specimens by considering the transverse shear deformability and the effects of deflections and rotations at the crack tip. This result is accomplished by replacing the actual delamination length, a, by an increased delamination length, $a + \chi h$ (where h is the specimen's half-thickness and χ is the so-called *crack length correction parameter*), in the SBT formulas for the compliance, C, and energy release rate, G [4, 5]. Actually, the current ASTM standard for the MMB test suggests formulas for the mode I and II crack length correction parameters, χ_{I} and χ_{II}, which can be used for UD laminated specimens [6]. For MD laminated specimens, de Morais and Pereira have proposed a *modified beam-theory* (MBT) model, where the crack length correction parameters are computed by considering the homogenised flexural and shear moduli [7].

The authors have developed an *enhanced beam-theory* (EBT) model of the MMB test, wherein the laminated specimen is considered as an assemblage of two identical sublaminates partly connected by a deformable interface. The sublaminates are modelled as extensible, flexible, and shear-deformable laminated beams. The interface is regarded as a continuous distribution of linearly elastic–brittle springs. An exact analytical solution for the internal forces, displacements, and interfacial stresses of the MMB test specimen has been deduced [8]. Furthermore, useful approximate expressions have been determined for the compliance and energy release rate of the DCB, ENF, and MMB test specimens. Such quantities can be expressed by introducing the following crack length correction parameters [9]:

$$\chi_{\mathrm{I}}^{\mathrm{EBT}} = \frac{1}{h}\sqrt{\frac{D_1}{C_1} + \sqrt{\frac{2D_1}{k_z}}} \quad \text{and} \quad \chi_{\mathrm{II}}^{\mathrm{EBT}} = \frac{1}{h}\frac{1}{\sqrt{2k_x\left(\frac{1}{A_1} + \frac{h^2}{4D_1}\right)}}, \tag{1}$$

where A_1, C_1, and D_1 are the sublaminates' extensional stiffness, shear stiffness, and bending stiffness, respectively, and k_x and k_z are the elastic constants (per unit area) of the distributed springs in the tangent and normal directions to the interface plane, respectively. Eqs. (1) define crack length

correction parameters having the same physical meaning that they have in the CBT model. The EBT expressions, however, have been deduced based on a rigorous analytical solution. Furthermore, they of course apply not only to UD, but also to MD laminated specimens. It is worth noting that comparisons between the CBT and EBT models for UD laminated specimens show very good agreement (see, for instance, Fig. 1).

In the present work, we demonstrate the application of the EBT model to MD laminated specimens with several stacking sequences and compare our theoretical predictions with experimental results and numerical analyses. The first obtained results look very promising.

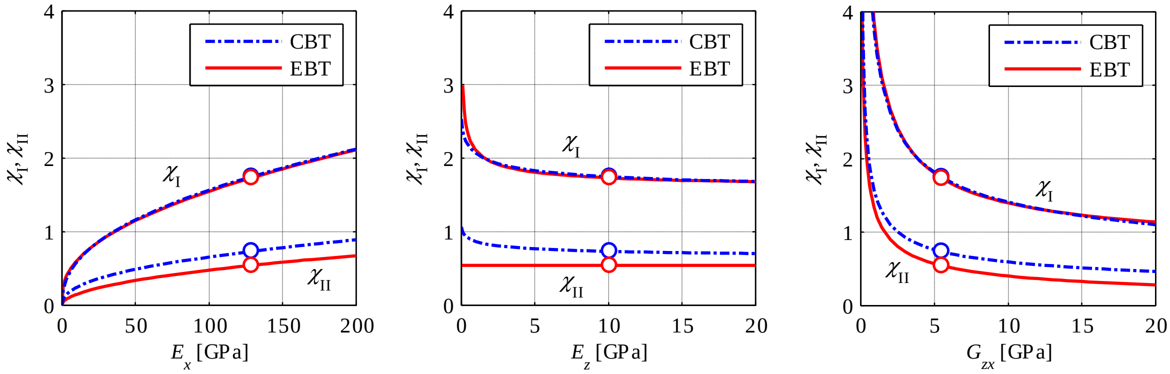

Figure 1: Crack length correction parameters as functions of the elastic moduli of a homogenous orthotropic specimen (from [9]).

REFERENCES

[1] D.F. Adams, L.A. Carlsson and R.B. Pipes, *Experimental Characterization of Advanced Composite Materials – 3rd edition*, CRC Press, Boca Raton, 2003.

[2] J. Andersons and M. König, Dependence of fracture toughness of composite laminates on interface ply orientations and delamination growth direction, *Composites Science and Technology*, **64**, 2004, pp. 2139-2152 (doi: 10.1016/j.compscitech.2004.03.007).

[3] K. Friedrich (editor), *Application of Fracture Mechanics to Composite Materials*, Elsevier, Amsterdam, 1989.

[4] J.G. Williams, End corrections for orthotropic DCB specimens, *Composites Science and Technology*, **35**, 1989, pp. 367-376 (doi: 10.1016/0266-3538(89)90058-4).

[5] Y. Wang and J.G. Williams, Corrections for mode II fracture toughness specimens of composites materials, *Composites Science and Technology*, **43**, 1992, pp. 251-256 (doi: 10.1016/0266-3538(92)90096-L).

[6] ASTM, *Standard Test Method for Mixed Mode I-Mode II Interlaminar Fracture Toughness of Unidirectional Fiber Reinforced Polymer Matrix Composites*, D6671/D6671M-06, American Society for Testing and Materials, West Conshohocken, PA, 2006 (doi: 10.1520/D6671_D6671M-06).

[7] A.B. de Morais and A.B. Pereira, Interlaminar fracture of multidirectional glass/epoxy laminates under mixed-mode I + II loading, *Mechanics of Composite Materials*, **43**, 2007, pp. 233-244 (doi: 10.1007/s11029-007-0023-1).

[8] S. Bennati, P. Fisicaro and P.S. Valvo, An enhanced beam-theory model of the mixed-mode bending (MMB) test – Part I: literature review and mechanical model, *Meccanica*, **48**, 2013, pp. 443–462 (doi: 10.1007/s11012-012-9686-3).

[9] S. Bennati, P. Fisicaro and P.S. Valvo, An enhanced beam-theory model of the mixed-mode bending (MMB) test – Part II: applications and results, *Meccanica*, **48**, 2013, pp. 465–484 (doi: 10.1007/s11012-012-9682-7).