AIMETA 2017
XXIII Conference
The Italian Association of Theoretical and Applied Mechanics
Luigi Ascione, Valentino Berardi, Luciano Feo, Fernando Fraternali and Antonio Michele Tralli (eds.)
Salerno, Italy, 4–7 September 2017

ENERGY HARVESTING FROM BRIDGE VIBRATIONS WITH PIEZOELECTRIC DEVICES – A FEASIBILITY STUDY

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Keywords: energy harvesting, piezoelectric materials, road bridges.

Abstract. *We present a feasibility study on the use of piezoelectric devices to harvest the energy connected to the vibrations induced on road bridges by travelling vehicles. We have selected an existing urban bridge as case study and collected the available documentation about its original design. Furthermore, the results of a past experimental campaign on the bridge have provided experimental evidence about the natural frequencies and mode shapes of the structure. Next, we have set up a three-dimensional finite element model of the bridge, which is currently being calibrated to match the results of the experimental dynamic analysis.*

Besides, we have developed a mechanical model of a laminated cantilever beam with a top piezoelectric layer and a concentrated mass on its free end. Our model applies to laminated beams with general (asymmetric) stacking sequences, thus representing an extension of similar models of the literature. The partial differential equation of motion has been determined and solved in the case of free vibrations under both open- and short-circuit electrical boundary conditions. As a numerical example, a piezoelectric cantilever beam has been designed with the same first natural frequency of the case study bridge.

1 INTRODUCTION

Methods for energy harvesting aim to transform the kinetic energy linked to structural vibrations or motion of fluids – such as wind and water currents – into electrical energy by using electromagnetic induction, piezoelectricity, or other useful material properties [1]. Piezoelectricity is the property of some materials to generate an electric voltage if subject to mechanical deformation [2]. Devices based on the piezoelectric effect have proven effective to harvest energy from environmental or machinery vibrations. A simple, yet effective configuration for such devices consists of cantilever beams – made of either metallic or composite materials – with layers of piezoelectric material, which oscillate together with the structure to which they are connected [3]. Recently, the use of piezoelectric cantilevers has been proposed to scavenge energy from the vibrations of bridges under moving loads. However, studies of the literature are until now based on simplified models of bridges, which consider only their in-plane behaviour as one-dimensional beams and not their real three-dimensional structure [4].

In this paper, we present a feasibility study on the use of piezoelectric devices to harvest the energy connected to the vibrations induced on road bridges by travelling vehicles. To this aim, an existing urban bridge has been chosen as case study with the aim of evaluating the amount of energy that can be harvested under real operational conditions. As a first step of the study, we have collected previous documentation about the original design and a later experimental campaign on the bridge. The latter has provided experimental evidence about the natural frequencies and mode shapes of the structure. Next, we have set up a three-dimensional finite element model of the bridge, which is currently being calibrated to match the results of the experimental dynamic analysis.

Besides, we have developed a mechanical model of a laminated cantilever beam with a top piezoelectric layer and a concentrated mass on its free end. Our model applies to laminated beams with general stacking sequences. In particular, the effects stemming from asymmetric stacking sequences – *i.e.* the elastic bending-extension coupling and the additional inertial terms due to the eccentricity between the centre of mass and laminate mid-plane – can be rigorously taken into account. In this respect, our model represents an extension of similar models of the literature [5]. The partial differential equation of motion has been determined and solved in the case of free vibrations under both open- and short-circuit electrical boundary conditions. As a numerical example, a piezoelectric cantilever beam has been designed with the same first natural frequency of the case study bridge.

As a work in progress, the following steps are foreseen in the present study:

(i) the mechanical model of the piezoelectric cantilever beam will be extended to obtain the dynamic response under support excitation; to this aim, a previously developed method for the analysis of laminated plates will be suitably extended [6][7];

(ii) general electrical boundary conditions will be considered to evaluate the obtainable electrical energy output;

(iii) dynamic transient analysis of the finite element model of the case study bridge will be carried out to determine its response under moving vehicles;

(iv) the outputs of previous dynamic analyses in terms of displacement vs. time histories at selected points of the bridge will be used as input for the piezoelectric cantilever model;

(v) lastly, an optimised design of the piezoelectric devices and their arrangement on the bridge will be pursued to maximise the efficiency of the energy harvesting system.

2 A CASE STUDY: THE CITTADELLA BRIDGE IN PISA

2.1 General description

The Cittadella bridge is an urban bridge located in Pisa, Italy, built in 1953. It crosses the Arno river connecting the northern and southern parts of the town. The current road section includes a one-way (North to South) vehicle lane, a bike lane, and two lateral walkways.

As for the structural scheme, the Cittadella bridge can be classified as a symmetrical, three-span cantilever bridge. The two outer spans (almost 29 m long) are covered by two girders, which are simply supported at the abutments and lean continuously over the riverbed piers. The inner span (about 48 m long) consists of two lateral cantilevers and a central girder resting on Gerber internal supports (Fig. 1).

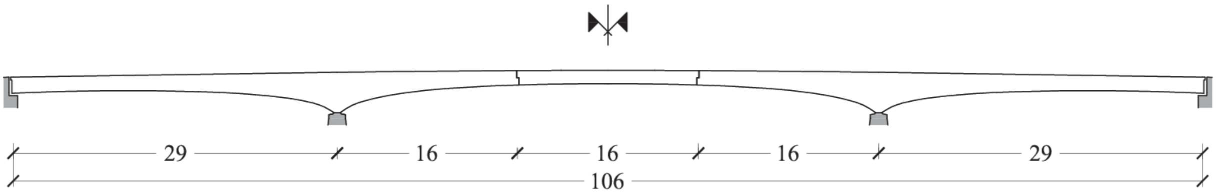

Figure 1: Side view of the Cittadella bridge.

The bridge is made of reinforced concrete with an internal steel structure. Actually, a peculiar construction technique was used to build the bridge. First, a so-called Melan beam, made of welded steel profiles, was built. This acted as a centring for the successive phase of concrete casting. After the curing of concrete, the Melan beam collaborates with concrete together with ordinary steel reinforcement bars. The cross section of the bridge presents a bi-cellular box geometry, with variable overall height (from 1.5 m on the abutments to 3.5 m on the piers) and thickness of the walls (from 0.15 to 0.40 m).

2.2 Experimental dynamic analysis

In 1993, the engineering company A.I.C.E. Consulting S.r.l. [8] conducted an experimental campaign on the bridge. Through the use of accelerometers placed at characteristic sections of the bridge, the first natural frequencies were determined under free environmental vibrations. From comparison of the acquisitions coming from couples of accelerometers placed at symmetric points with respect to both the transverse and longitudinal directions of the bridge, it was possible to associate each natural frequency to a corresponding mode shape (see Tab. 1).

Mode number	Frequency [Hz]	Mode shape
1	2.40	symmetric flexural
2	3.40	antisymmetric flexural
3	6.10	symmetric flexural

Table 1: Experimentally determined natural frequencies and mode shapes

2.3 Finite element model of the bridge

A finite element model of the bridge has been defined by using the commercial software Straus7 [9]. Beam elements are used to model the internal steel structure of the Melan beam. Plate elements are used to model the concrete slabs of the bi-cellular box section. Additional non-structural masses are introduced to model the effects of constructive elements, such as road pavement, railings, etc. (Fig. 2).

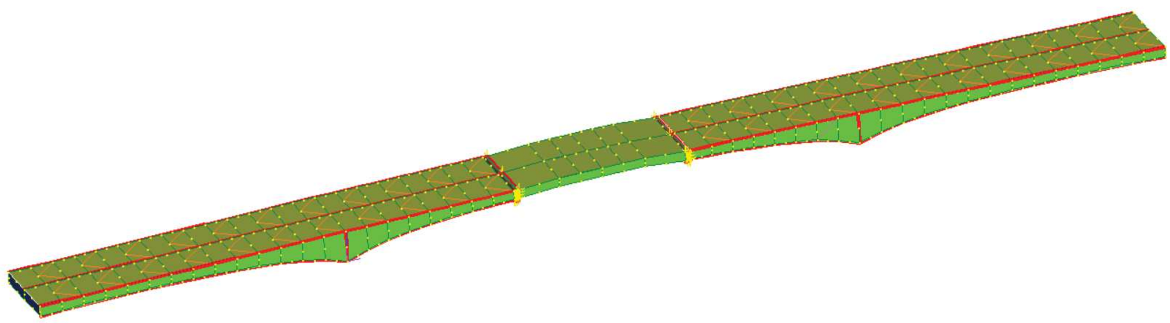

Figure 2: Finite element model of the Cittadella bridge.

The model is currently being calibrated to retrace the experimentally determined dynamic response in terms of natural frequencies and mode shapes under free vibrations. Subsequently, we will simulate the behaviour of the bridge under travelling vehicles. The dynamic response in terms of displacement vs. time histories at selected points of the bridge will be used as an input for the mechanical model of the piezoelectric cantilever beams. The final goal will be to evaluate quantitatively the amount of energy that can be harvested under normal operative conditions of the bridge.

3 MECHANICAL MODEL OF A PIEZOELECTRIC CANTILEVER BEAM

3.1 Model geometry

Let us consider a laminated cantilever beam of length L and width b, comprised of a support layer of elastic material and a top thin layer of piezoelectric material. We denote t_b and t_p the thicknesses of the support and the piezoelectric layer, respectively (Fig. 3). In turn, the support layer may be either homogeneous or made of several layers of different thicknesses and materials. Both cases are treated unitedly in what follows, as we use classical lamination theory to model the support layer as a homogenised laminate [10].

A global Cartesian reference system is fixed with the origin at the geometric centre of the clamped-end cross section, the x-axis aligned with the cantilever longitudinal direction, the z-axis normal to the lamination plane, and the y-axis aligned so as to complete a right-handed reference frame.

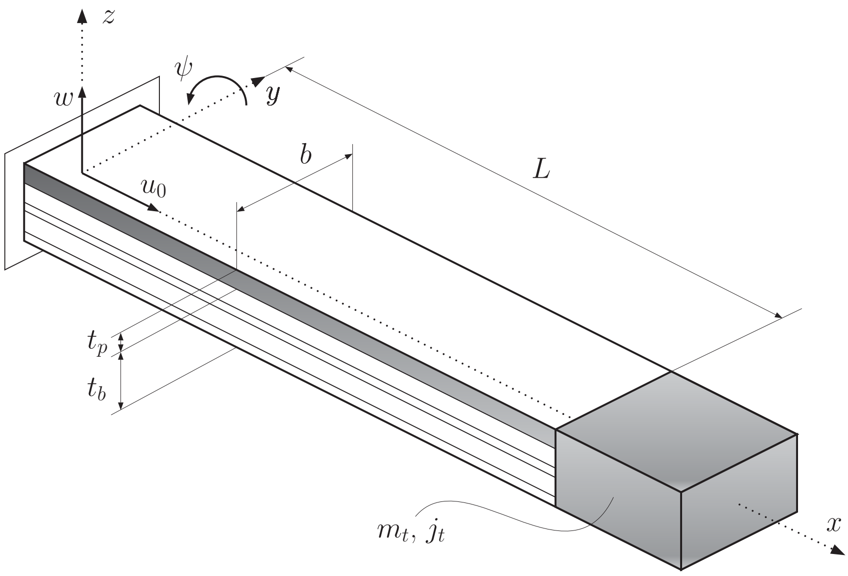

Figure 3: Piezoelectric cantilever beam.

3.2 Constitutive relationships

3.2.1 Piezoelectric material

Under small electric field conditions, the constitutive relationships for a piezoelectric material are [11]:

$$\begin{aligned} \epsilon_k &= s^E_{km}\sigma_m + d^c_{jk}E_j \\ D_i &= d^d_{im}\sigma_m + e^\sigma_{ij}E_j \end{aligned} \tag{1}$$

where ϵ_k is the mechanical strain, σ_m is the mechanical stress, D_i is the electrical displacement, and E_j is the electric field; furthermore, s_{km} is the compliance under zero or constant electrical field (indicated by the superscript E) and e^σ_{ij} is the dielectric permittivity under zero or constant stress (indicated by the superscript σ); lastly, d^c_{jk} and d^d_{im} are the piezoelectric coefficients which defines respectively strain per unit field at constant stress and electric displacement per unit stress at constant electric field; superscripts c and d have been added to differentiate between the converse and the direct piezoelectric effect, though in practice they have the same exact numerical value [12].

By following the de Saint Venant's assumption on the stress field, i.e. $\sigma_y = \sigma_z = \tau_{yz} = 0$, and ignoring the Poisson's effect [7], the former three-dimensional equations can be reduced to the one-dimensional case and solved for σ_x and D_z, giving:

$$\begin{aligned} \sigma_x &= \overline{Q}_{11}\epsilon_x - P_cV/t_p \\ D_z &= P_c\epsilon_x + e_pV/t_p \end{aligned} \tag{2}$$

where $\overline{Q}_{11}$ denotes the reduced stiffness [10], $P_c = \overline{Q}^{(p)}_{11}d_{31}$ is a piezoelectric coupling factor [5], in which $\overline{Q}^{(p)}_{11}$ is the first element of the stiffness matrix of the piezoelectric material and $e_p = (e_z - P_cd_{31})$ is an equivalent dielectric permittivity constant. The electric field, E_z, supposed constant over the small layer thickness, has been replaced with the generated voltage, V.

3.2.2 Piezoelectric laminated cantilever beam

In line with Euler-Bernoulli beam theory, the stress resultants of interest acting on the cantilever cross section are the axial force, N_x, and the bending moment, M_y (Fig. 4), which can be expressed in terms of the normal stress, σ_x, as follows:

$$N_x = \sum_{k=1}^{n_t} \int_{z_{k-1}}^{z_k} \sigma_x^{(k)}\, bdz + \int_{z_p - t_p}^{z_p} \sigma_x^{(p)}\, bdz \tag{3}$$
$$M_y = \sum_{k=1}^{n_t} \int_{z_{k-1}}^{z_k} \sigma_x^{(k)} \cdot z\, bdz + \int_{z_p - t_p}^{z_p} \sigma_x^{(p)} \cdot z\, bdz$$

where z_i is the ordinate of the bottom surface of the i-th layer, n_t is the number of ordinary layers, and z_p is the ordinate of the top surface of the piezoelectric layer. Furthermore, we denote with $h = (t_b + t_p)/2$ the half-thickness of the cross section (Fig. 5).

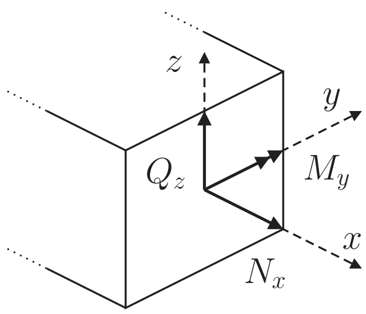

Figure 4: Positive resultant forces and bending moment.

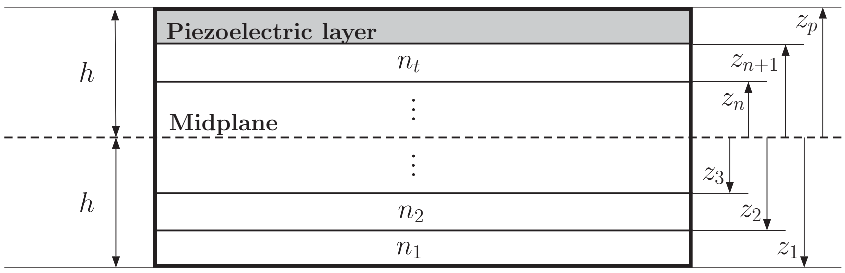

Figure 5: Geometry of laminated composite beam section.

By substituting the first of Eq.s 2 in 3, we can rewrite the former expressions in the more compact form:

$$N_x = \mathbb{A} \cdot \epsilon_0 + \mathbb{B} \cdot k - P_c bV \tag{4}$$
$$M_y = \mathbb{B} \cdot \epsilon_0 + \mathbb{D} \cdot k - \frac{P_c bV}{2}(2h - t_p)$$

where $\mathbb{A}$, $\mathbb{B}$ and $\mathbb{D}$ respectively are the homogenised extensional, coupling, and bending stiffness constants of a laminate [10]. Eq.s 4 represent the constitutive laws for a laminated beam, inclusive for the top layer of piezoelectric material. Moreover, by imposing that the overall axial force is null, we can solve Eq.s 4 for ϵ_0, finding:

$$\epsilon_0 = -\frac{\mathbb{B}}{\mathbb{A}} \cdot k + \frac{P_c b V}{\mathbb{A}} \tag{5}$$

By substituting the obtained expression into Eq.s 4, and expressing ϵ_0 and k via the kinematic field, we obtain:

$$\begin{aligned} u_{0,x} &= \frac{\mathbb{B}}{\mathbb{A}} w_{,xx} + \frac{P_c b}{\mathbb{A}} V \\ M_y &= -\mathbb{D}^* w_{,xx} - \Gamma_p V \end{aligned} \tag{6}$$

where $\mathbb{D}^* = \mathbb{D} - \mathbb{B}^2/\mathbb{A}$ and $\Gamma_p = P_c b(\mathbb{B}_p/\mathbb{A}_p - \mathbb{B}/\mathbb{A})$. Eq.s 6 are the modified constitutive relationships which relate the overall bending moment M_y, the section curvature k, and the voltage V in a laminated cantilever beam with a layer of piezoelectric material.

3.3 Partial differential equation of motion

The equation of motion can be derived by considering the equilibrium of all the forces acting along the x and z-directions and of the bending moments acting around the y-axis on an elementary beam segment (Fig. 6). Developing calculations leads to:

$$M(x,t)_{,xx} + I_2^* w(x,t)_{,xxtt} - I_0 w(x,t)_{,tt} = 0 \tag{7}$$

where I_0 and I_2^* are both inertial constants, in detail $I_2^* = (I_2 - I_1^2/I_0)$ and their expressions can be found in [6]. By substituting the second of Eq.s 6 into 7 and accounting that voltage is independent from the x-coordinate, we obtain the governing differential equation of the problem:

$$\mathbb{D}^* w(x,t)_{,xxxx} - I_2^* w(x,t)_{,xxtt} + I_0 w(x,t)_{,tt} = 0 \tag{8}$$

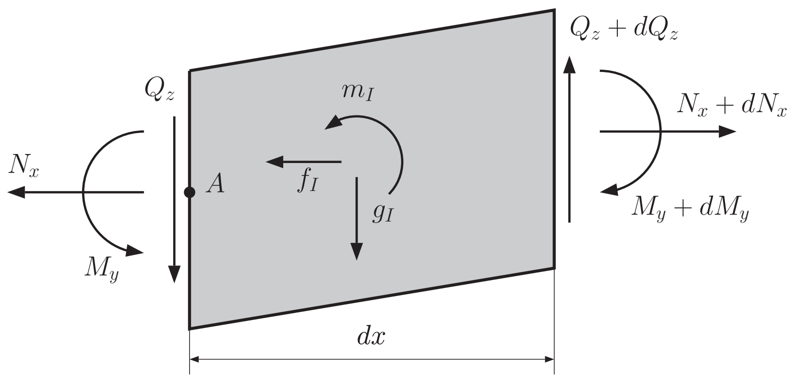

Figure 6: Equilibrium of a differential element of the beam.

3.4 Analysis of free vibrations

For the free vibration analysis, a solution for Eq. 8 is sought in the form:

$$w(x,t) = \Phi(x) \cdot y(t) \tag{9}$$

Following this approach, by substituting Eq. 9 into 8, we are able to obtain two uncoupled equations of motion, one in the time domain and one in the space, respectively [13]:

$$y(t)_{,tt} + \omega^2 y(t) = 0 \tag{10}$$
$$\Phi(x)_{,xxxx} + \omega^2 \frac{I_2^*}{\mathbb{D}^*}\Phi(x)_{,xx} - \omega^2 \frac{I_0}{\mathbb{D}^*}\Phi(x) = 0$$

The first equation is the classical solution for the free vibrations of an undamped SDOF; the second one can be solved by looking for the roots of the characteristic polynomial, yielding:

$$\Phi(x) = \sum_{i=1}^{4} C_i \cdot e^{\lambda_i x} \tag{11}$$

where:

$$\lambda_{1,2} = \pm\sqrt{\frac{I_2^*}{2\mathbb{D}^*} \cdot \left(1 + \sqrt{1 + \frac{4I_0\mathbb{D}^*}{(I_2^*\omega)^2}}\right)} \cdot j\omega \;= \pm\mu_1(\omega) \cdot j \quad \in \mathbb{C} \tag{12}$$
$$\lambda_{3,4} = \pm\sqrt{\frac{I_2^*}{2\mathbb{D}^*} \cdot \left(-1 + \sqrt{1 + \frac{4I_0\mathbb{D}^*}{(I_2^*\omega)^2}}\right)} \cdot \omega \;= \pm\mu_2(\omega) \quad \in \mathbb{R}$$

Setting equal to zero the imaginary part of the right-hand side of the former equation leads to the final expression for the solution:

$$\Phi(x) = A_1 cos[\mu_1(\omega)x] + A_2 sin[\mu_1(\omega)x] + A_3 e^{-\mu_2(\omega)x} + A_4 e^{\mu_2(\omega)x} \tag{13}$$

where A_i are four real constants that must be evaluated so as to satisfy the boundary conditions. These are:

$$\Phi(0) = 0 \qquad \Phi_{,x}(0) = 0 \tag{14}$$
$$\Phi_{,xx}(L) + k_1\Phi_{,x}(L) = 0 \qquad \Phi_{,xxx}(L) + k_2\Phi(L) = 0$$

In these equations, $\Phi(L)_{,x}$ stands for the first derivative of $\Phi(x)$ calculated in $x = L$, and so on. The first two conditions stand respectively for the initial deflection and slope to be null, the second two describe the values of bending moment and shear force at the free end in the presence of a tip mass m_t, with moment of inertia j_t. The values of the constants are $k_1 = [\omega^2 j_t + \Gamma_p^2/(\alpha C0)]/\mathbb{D}^*$, and $k_2 = \omega^2 m_t^*/\mathbb{D}^*$. The system derived from the imposition of the boundary conditions, which is linear in A_i, can then be written in its final matrix form. Since the system of equations is homogeneous in the variables A_i, the only possible non-trivial solutions are the ones which nullify the determinant of the associated coefficient matrix. Setting to zero this determinant and looking for its roots finally yields to the eigenfrequencies of the beam.

3.5 Electrical boundary conditions

Two limit electrical boundary conditions are investigated in this paper:

- short circuit (SC);
- open circuit (OC).

3.5.1 Short-circuit condition

The short-circuit condition entails that the voltage across the piezoelectric layer is null. This condition is trivial, since it leads to the same solution as for an ordinary Euler-Bernoulli beam.

3.5.2 Open-circuit condition

In open-circuit condition, the electrical constitutive relation can be re-written by integrating both sides of the second of Eq.s 2 over the piezoelectric surface, parallel to the xy-plane. Since the total free charge in the piezoelectric layer is zero, after some calculations, we obtain:

$$V(t) = \frac{\Gamma_p \Phi(L)_{,x}}{\alpha C_0} y(t) \tag{15}$$

where $C_0 = e_p(bL/t_p)$ is the piezoelectric layer equivalent capacitance and $\alpha = 1+(P_c^2 bt_p/e_p\mathbb{A})$ is a collector factor.

4 NUMERICAL EXAMPLE

To illustrate the model, we have carried out the preliminary design of a piezoelectric cantilever beam to be used on the case study bridge. Accordingly, the cantilever beam has been designed in such a way that its first natural frequency matches as close as possible the first natural frequencies of the bridge, $f_1 = 2.40$ Hz. For the sake of simplicity, we have considered a metallic (steel) support layer. The properties of the piezoelectric layers are the same used by Karimi *et al.* [4]. The design values of the geometrical, electrical, and mechanical parameters are shown in Tab. 2. The calculated natural frequencies for the first five vibration modes are shown in Tab. 3 for both SC and OC conditions.

t_p [mm]	t_b [mm]	b [mm]	L [mm]	m_t [kg]	j_t [mm^2 kg]	$\overline{Q}_{11}^{(p)}$ [GPa]	$\overline{Q}_{11}^{(b)}$ [GPa]	P_c [C/m^2]	e_z [C^2/Nm^2]
0.1	1.0	30.0	350.0	0.161	55.556	64	210	-12.16	$1451e_0$

Table 2: Geometrical values of the cantilever beam

Mode number	SC [Hz]	OC [Hz]
1	2.382	2.396
2	33.931	33.960
3	113.475	113.479
4	232.916	232.917
5	389.397	389.398

Table 3: Numerical values of the frequency output

5 CONCLUSIONS

We have presented a feasibility study on the use of piezoelectric devices to harvest energy from the vibrations of road bridges. Present developments and results include the following:

- an existing urban bridge, namely the Cittadella bridge in Pisa, Italy, has been selected as case study; the original design documents and the results of a past experimental campaign on the bridge have been collected; such information has been used to define a three-dimensional finite element model of the bridge;
- a mechanical model of a laminated piezoelectric cantilever beam has been developed; an analytic solution has been determined for the free vibrations under both open- and short-circuit electrical conditions;
- as a numerical example, a piezoelectric cantilever beam has been designed with the same first natural frequency of the case study bridge.

Future developments of the present study include the following actions:

- the model of the cantilever beam will be extended to obtain the dynamic response under support excitation and general electrical boundary conditions;
- dynamic transient analyses will be carried out on the finite element model of the bridge to determine its response under moving vehicles;
- the outputs of previous dynamic analyses in terms of displacement vs. time histories at selected points of the bridge will be used as input for the piezoelectric cantilever model;
- lastly, an optimised design of the piezoelectric devices and their arrangement on the bridge will be pursued to maximise the efficiency of the energy harvesting system.

6 ACKNOWLEDGEMENTS

This paper was prepared partly during one of the authors (J. Bonari) stay at the School of Mechanical Engineering, Babol Noshirvani University of Technology, Iran, in the period June – August 2017. The Italian authors wish to thank their Iranian counterparts, in particular Dr. R.-A. Jafari and Dr. M. Abedi, for hospitality, support, and scientific advice. Also, financial support from the University of Pisa is gratefully acknowledged for a scholarship dedicated to students who need to spend a period abroad for carrying out their Master's Thesis.

The engineering company A.I.C.E. Consulting S.r.l. has provided information about the original design documents and later experimental dynamic analysis of the Cittadella bridge. Its technical support is gratefully acknowledged.

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