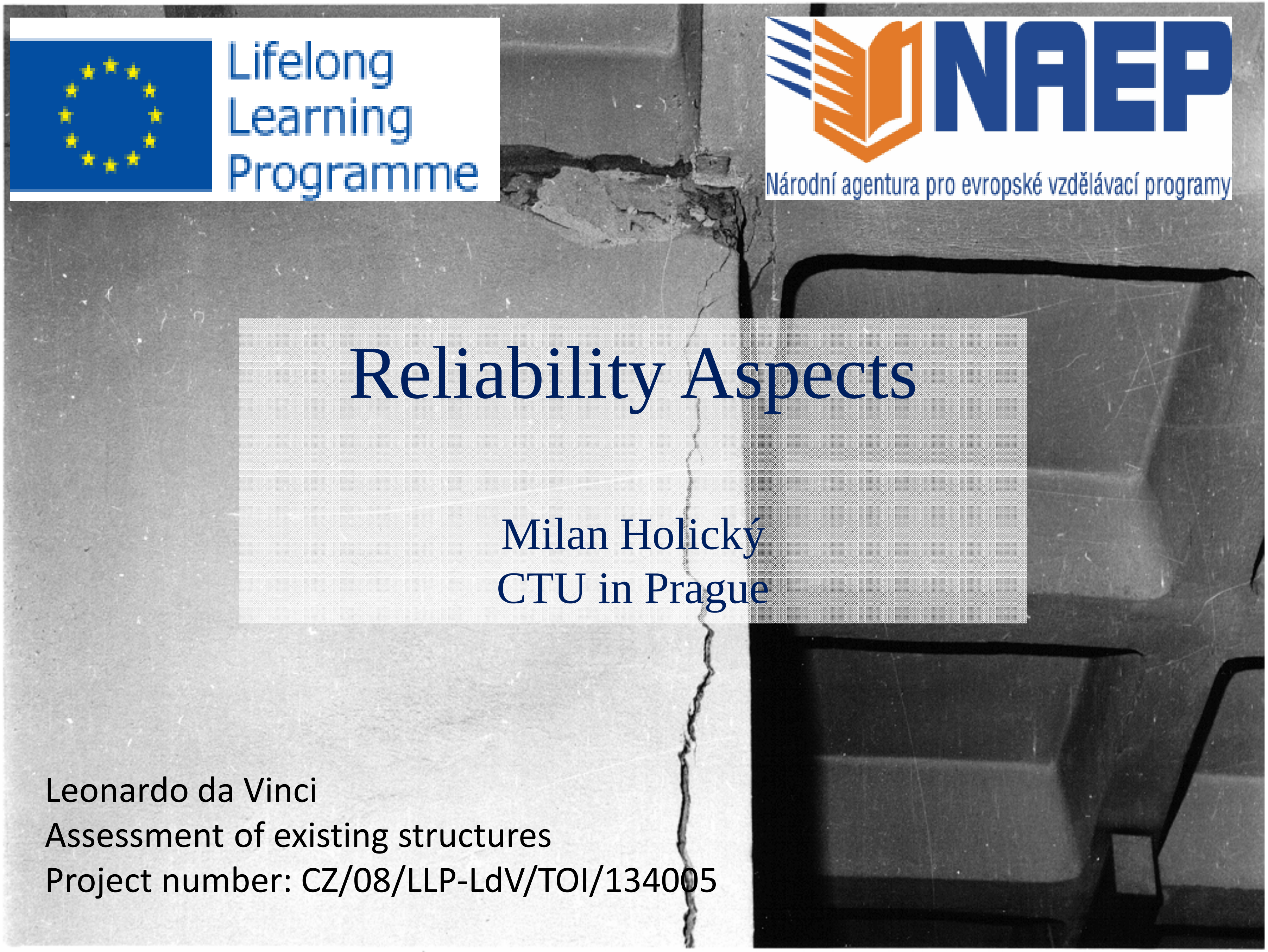


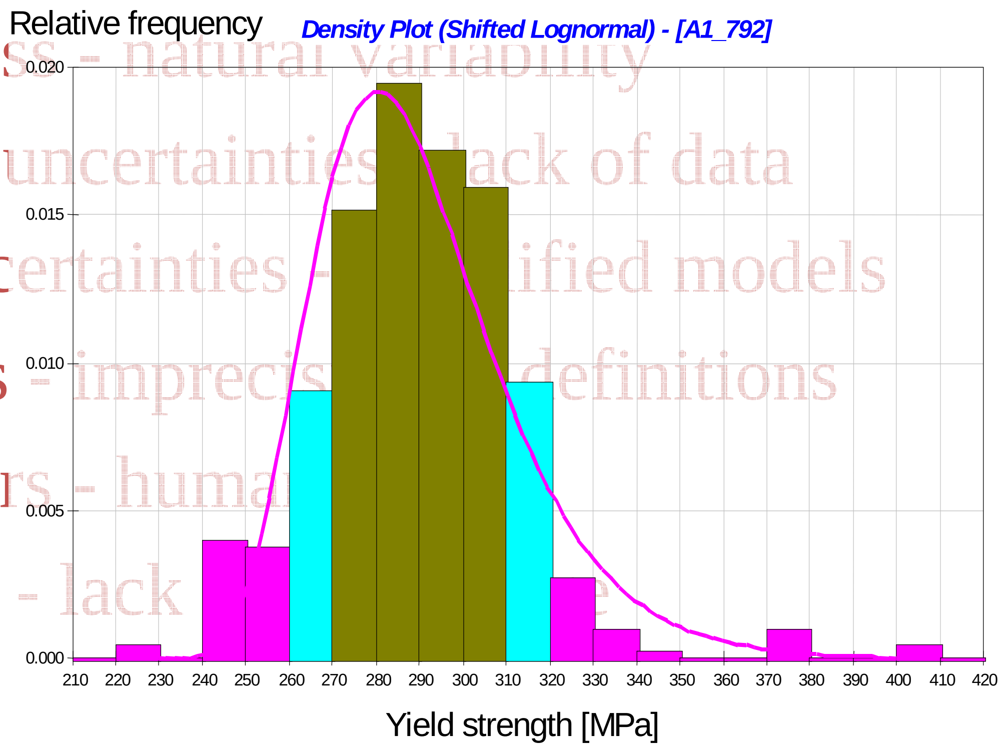
Reliability Aspects

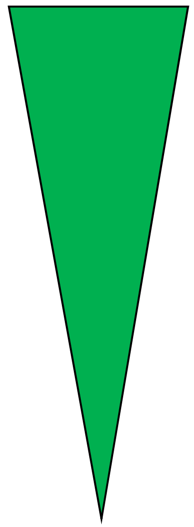
Milan Holický
CTU in Prague

Leonardo da Vinci
Assessment of existing structures
Project number: CZ/08/LLP-LdV/TOI/134005

Uncertainties are always present

- Uncertainties (aleatoric and epistemic) Decription
  - randomness - natural variability
  - statistical uncertainties - lack of data
  - model uncertainties - simplified models
  - vagueness - imprecise definitions
  - gross errors - human
  - ignorance - lack





- Tools
  - theory of probability and statistics, fuzzy logic
  - reliability theory and risk engineering

Some uncertainties are difficult to quantify

Eurocode EN 1990:

Reliability

- ability of a structure to fulfil all required functions during a specified period of time under given conditions

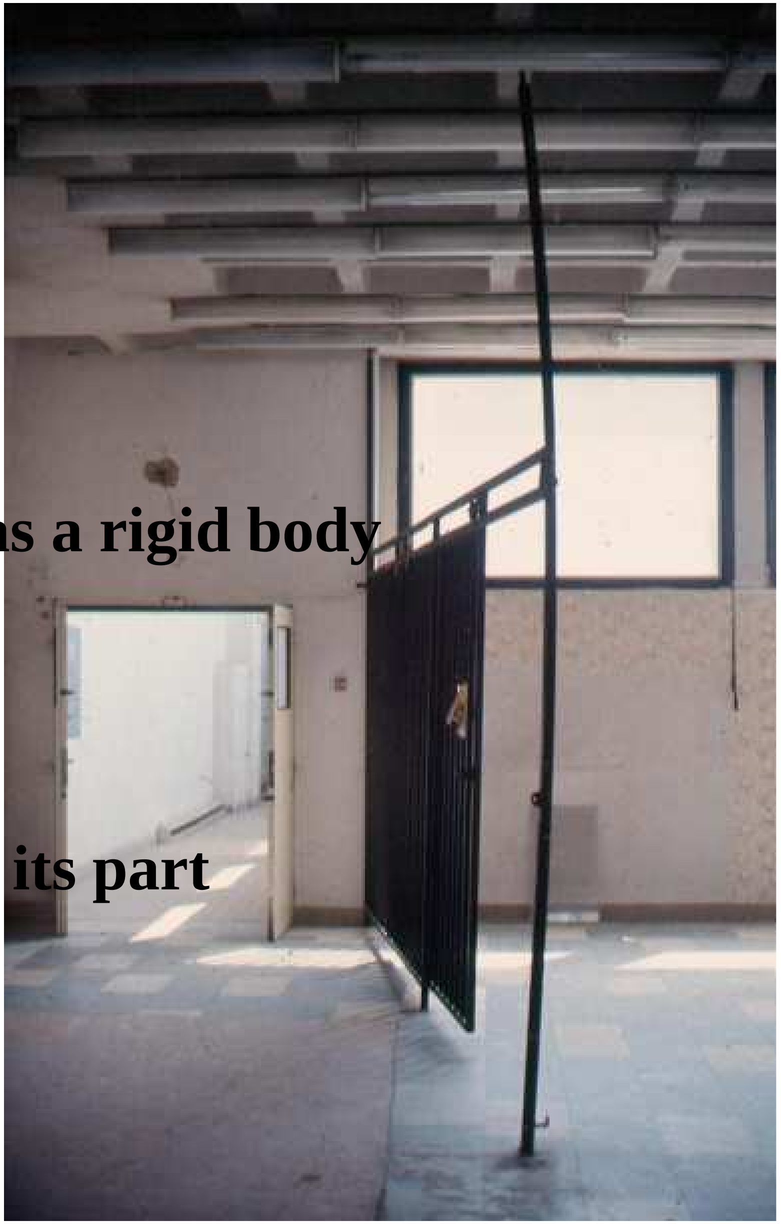
Failure probability P_f

- most important measure of structural reliability

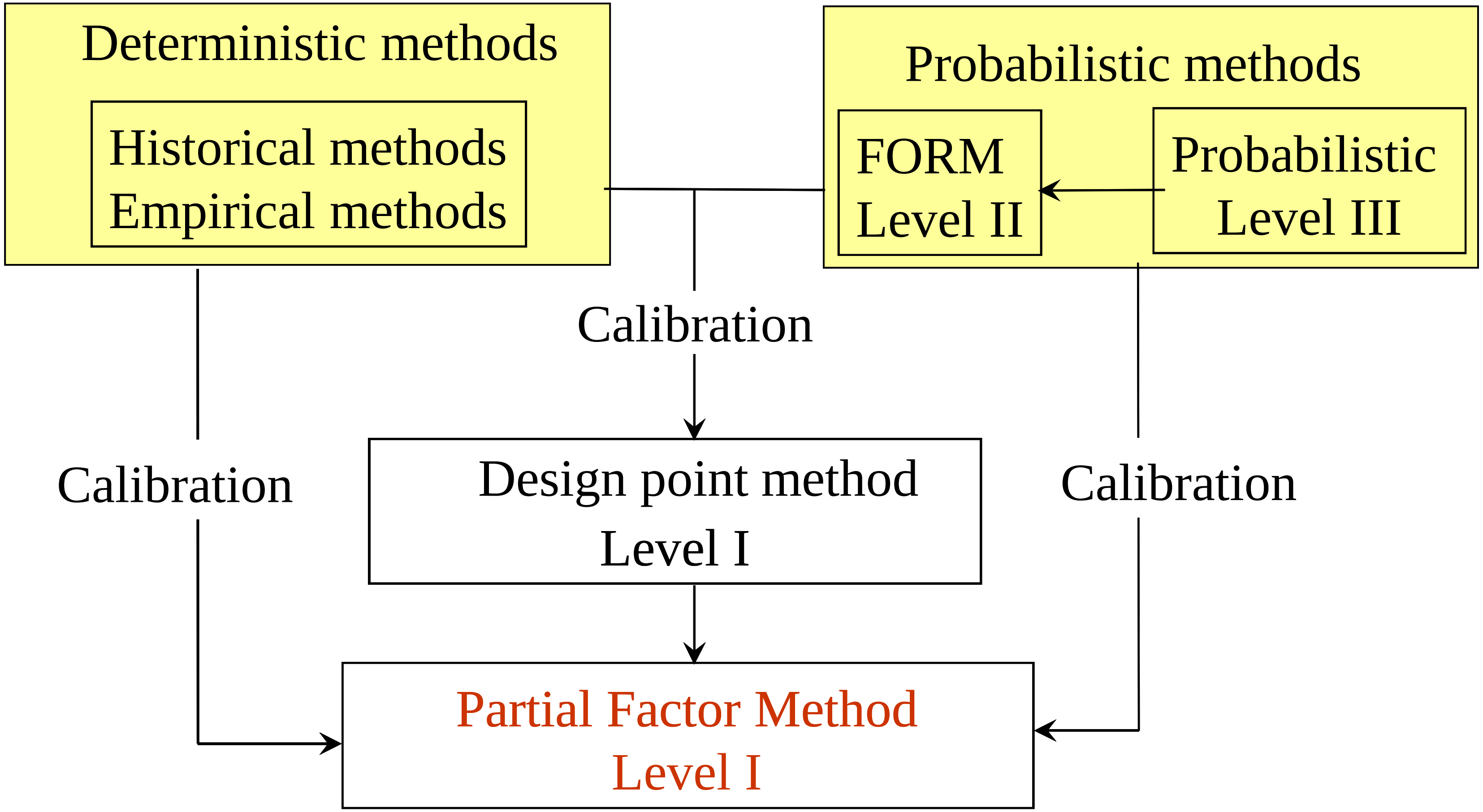
Limit State Approach

- **Limit states** - **states beyond which the structure no longer fulfils the relevant design criteria**

- **Ultimate limit states**
  - **loss of equilibrium of a structure as a rigid body**
  - **rupture, collapse, failure**
  - **fatigue failure**
- **Serviceability limit states**
  - **functional ability of a structure or its part**
  - **users comfort**
  - **appearance**



Reliability Methods



Reliability measures: failure probability p_f and reliability index β

p_f	10^{-1}	10^{-2}	10^{-3}	**10^{-4}**	10^{-5}	10^{-6}	10^{-7}
β	1,3	2,3	3,1	**3,7**	4,2	4,7	5,2

Fundamental case for normal distribution

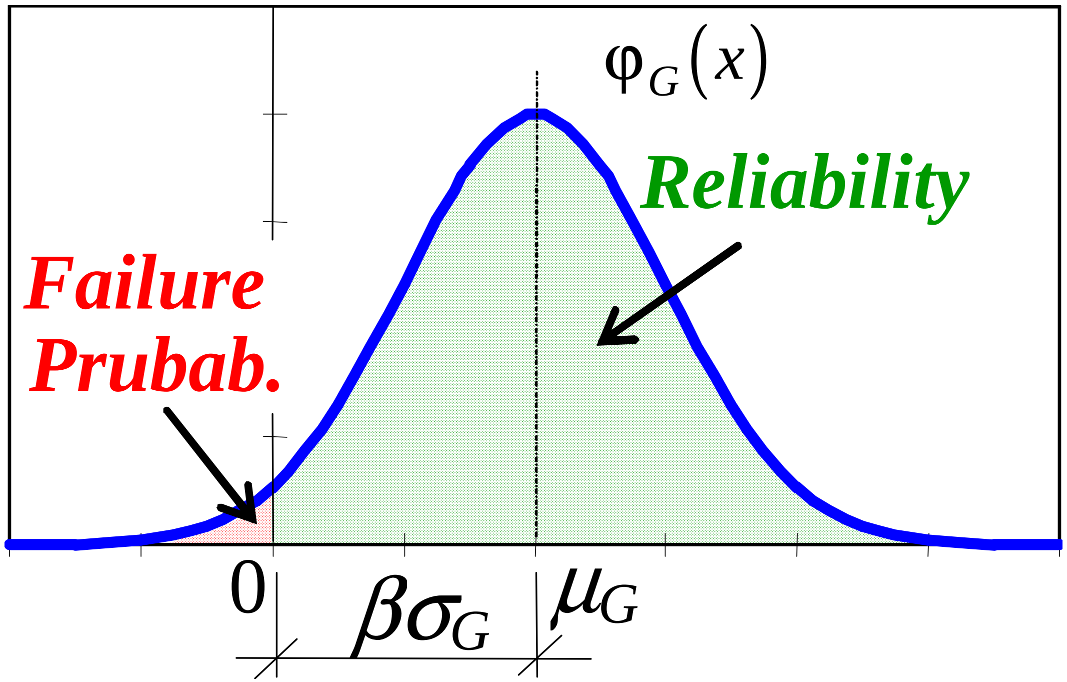
$E \leq R$

$G = R - E$ $\qquad \mu_G = \mu_R - \mu_E\,,\ \sigma_G^2 = \sigma_R^2 + \sigma_E^2$

Transformation of G to standardized variable $U = (G - \mu_G)/\sigma_G$

For $G = 0$ the standardized variable $u_0 = (0 - \mu_G)/\sigma_G$

Reliability index : $\qquad \beta = -u_0 = \dfrac{\mu_G}{\sigma_G} = \dfrac{\mu_R - \mu_E}{(\sigma_R^2 + \sigma_E^2)^{1/2}}$



Failure probability

$p_f = \Phi(-\beta)$

Fundamental case $E < R$

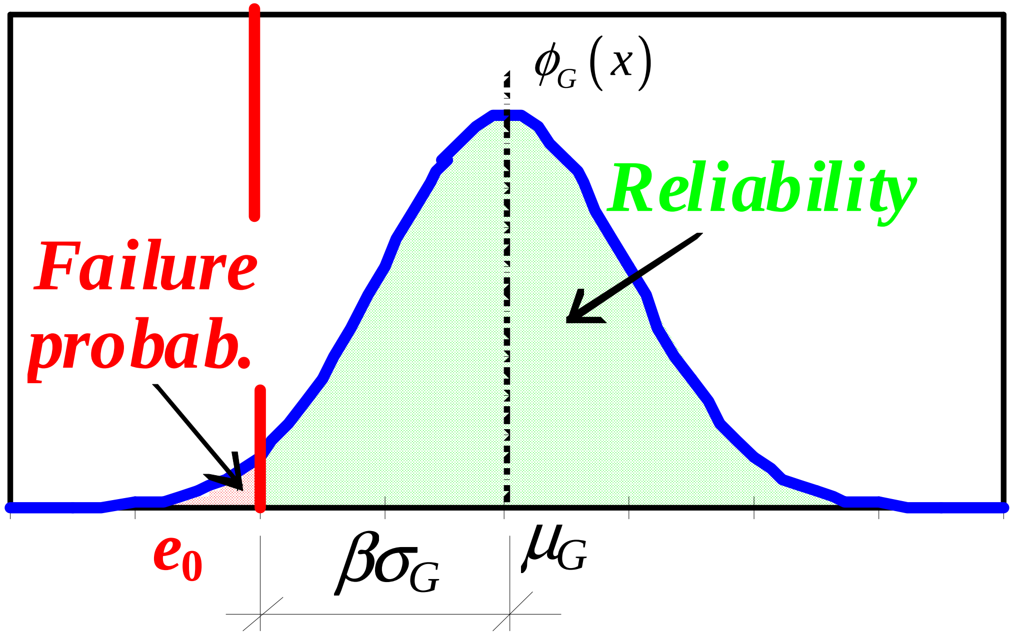
Limit state function: $g(X) = G = R - E = 0$

Load $E=e_0$ known, resistance R random: μ_R, σ_R, (α_R)

Transformation of R to standardized variable $U=(R - \mu_R)/\sigma_R$

For $R = e_0$ the standardized variable $u_0 =(e_0 - \mu_R)/\sigma_R$

Reliability index for normal dist.: $\beta = -u_0 = (\mu_R - e_0)/\sigma_R$



Failure probability

$$p_f = \Phi_R(e_0)$$

For normal distribution

$$p_f = \Phi(-\beta)$$

An example of the fundamental case

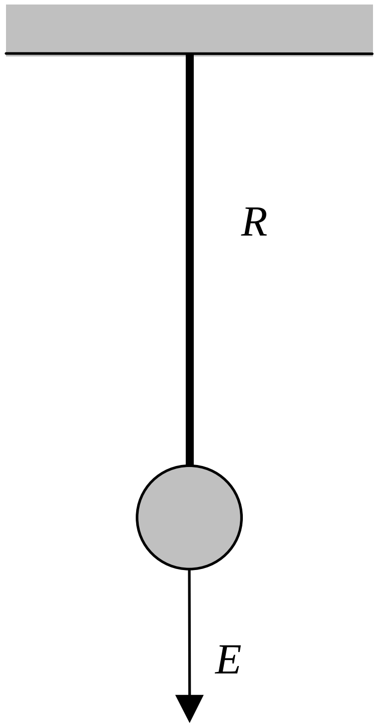
$$Z = R - E$$

$$\mu_Z = \mu_R - \mu_E = \quad 100 – 50 = 50$$

$$\sigma_Z^2 = \sigma_R^2 + \sigma_E^2 = 14^2$$

$$\beta = \mu_Z / \sigma_Z = 3.54$$

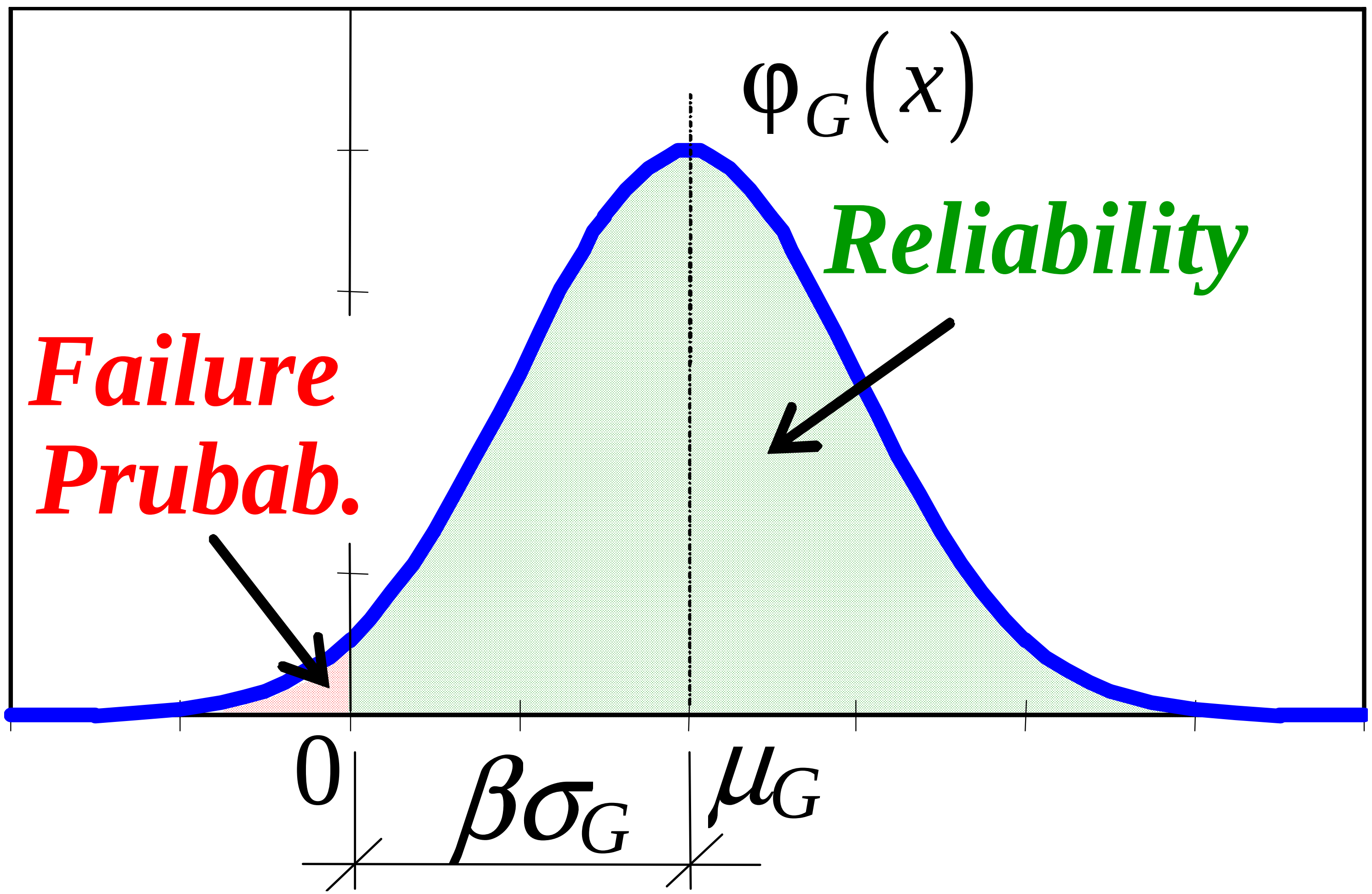
$$P_f = P(Z < 0) = \Phi_Z(0) = 0.0002$$



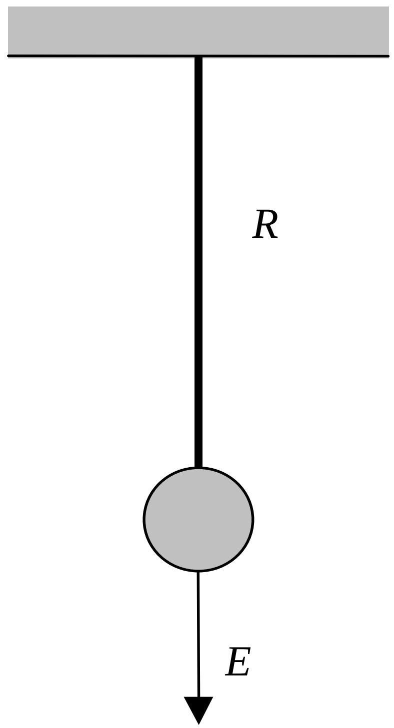
Relaibility margin and index β

$\beta = \mu_Z / \sigma_Z = 3.54$ $\qquad P_f = P(Z < 0) = \Phi_Z(0) = 0.0002,$

β Is the distance of the mean of reliability margin from the origin



Probabilistic approach



$Z = R - E$

$$P_f = P(Z < 0) = \iint_{Z(X)<0} \varphi_R(r)\varphi_E(e)\,dr\,de$$

Techniques:

Numerical integration (NI)

Monte Carlo (MC)

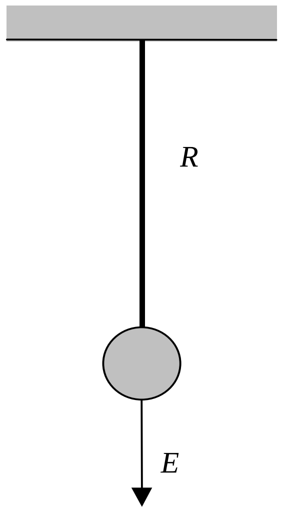
First order Second moment method (FOSM)

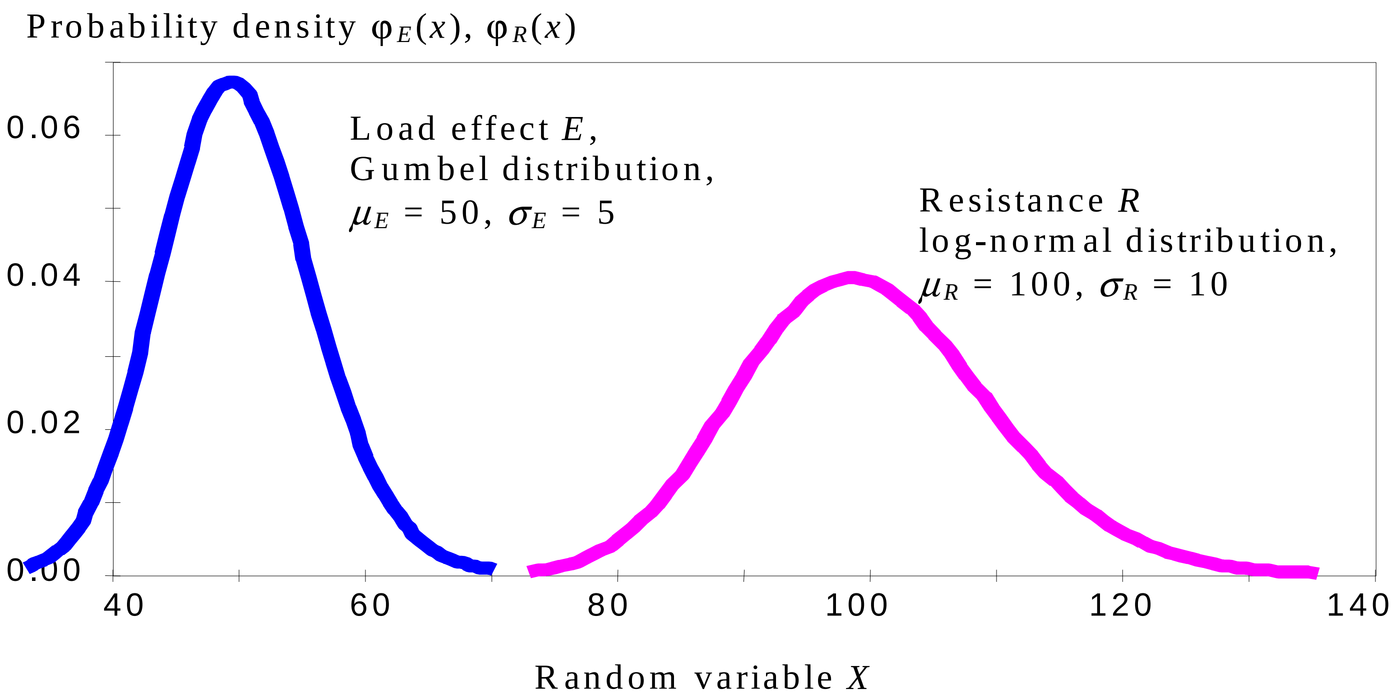
Third moment method (accounting for skewness)

First Order Reliability Methods (FORM)

Probabilistic models

		distribution	mean	sd
R	resistance	Lognormal	100	10
E	load effect	Gumbel	50	5





Eurocode concepts of partial factors

Reliability index β

Action reliability index

$\beta_E = \alpha_E \beta$

Resistance reliability index

$\beta_R = \alpha_R \beta$

Main action

$\beta_E = -0{,}7\,\beta$

Accompanying

$\beta_E = -0{,}28\,\beta$

Resistance

$\beta_R = 0{,}8\,\beta$

Partial factor

- Design value

for normal and lognormal distribution

$$x_{\mathrm{d}} = \mu(1 - \alpha\beta V)$$

for lognormal distribution: x_{d}

$$= \mu\, exp\left(-\alpha\beta\,\sigma - 0.5\,\sigma^2\right)$$

- Characteristic value

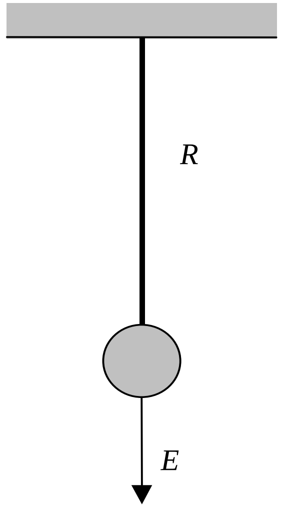
for normal $x_{\mathrm{k}} = \mu(1 - kV)$

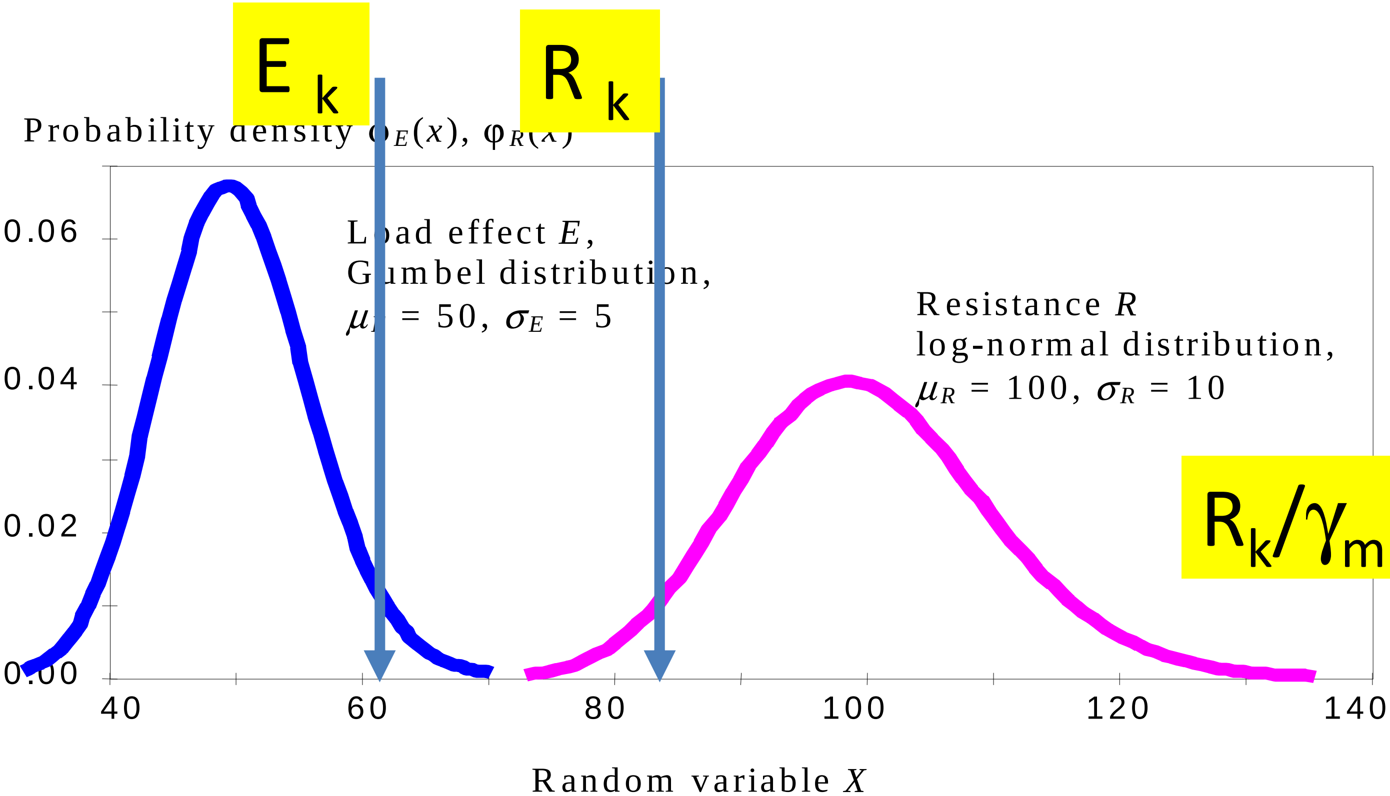
for lognormal $x_{\mathrm{k}} = \mu\ exp\left(-k\,\sigma - 0.5\,\sigma^2\right)$

- Partial factor $\gamma_{\mathrm{m}} = \frac{x_k}{x_d}$

Partial factor approach

		distribution	mean	sd
R	resistance	Lognormal	100	10
E	load effect	Gumbel	50	5





$$R_k/\gamma_m > E_k \gamma_Q$$

Indicative target reliabilities in ISO 13822

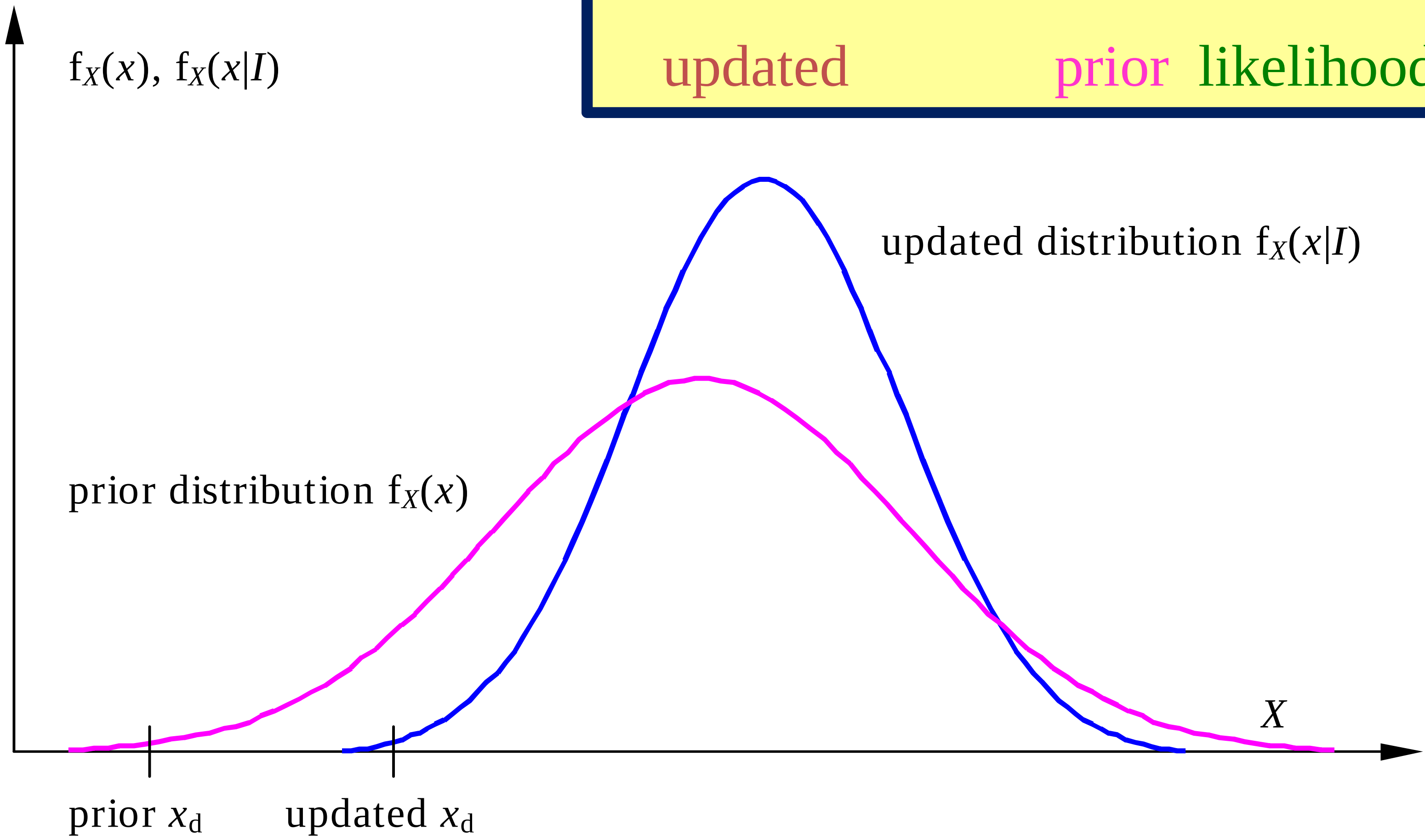
Limit states	Target reliability index, β	Reference period
Serviceability		
Reversible	0,0	Intended remaining working life
Irreversible	1,5	Intended remaining working life
Fatigue		
inspectable	2,3	Intended remaining working life
not inspectable	3,1	Intended remaining working life
Ultimate		
very low consequences of failure	2,3	L_S years*
low consequence of failure	3,1	L_S years*
medium consequence of failure	3,8	L_S years*
high consequence of failure	4,3	L_S years*
* L_S is a minimum standard period for safety (e.g. 50 years)		

Updating distributions

$$P(x|I) = P(x)\, P(I\,|\,x) \,/\, P(I)$$

$$f_X(x|I) = C\;\; f_X(x)\, P(\,I\,|\,x)$$

updated — prior — likelihood



Formal Updating formulas

$$f_Q''(q \mid \hat{x}) = C\, f_Q'(q)\, L(\mid \hat{x} \mid q)$$

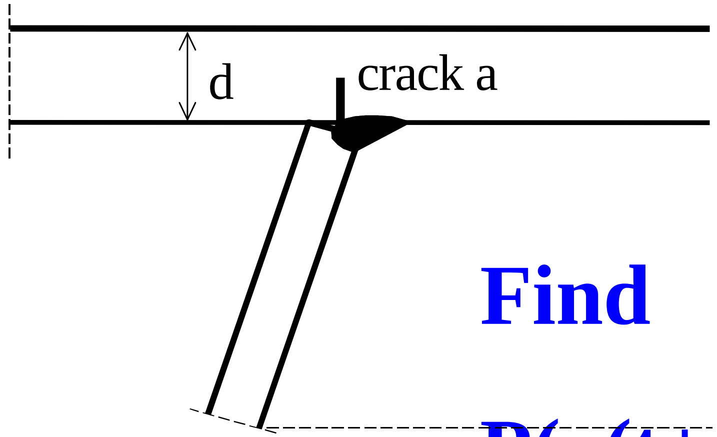
$$f_X^U(x) = \int_{-\infty}^{\infty} f_X(x|q)\, f_Q''(q \mid \hat{x})\, dq$$

Ask the expert !

Concluding remarks on reliability aspects

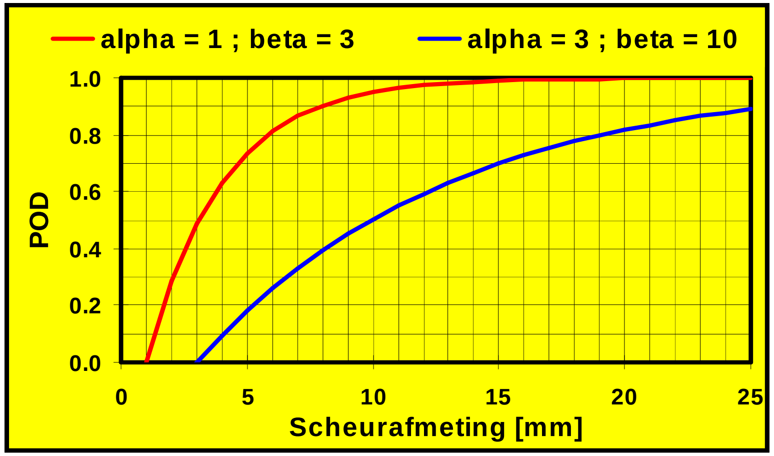
- Uncertainties are always present
- Probability Theory may be helpful
- Reliability targets depends on consequences of failure
- Reliability targets depend on costs of improving
- Existing structures may have a lower target reliability
- Reliability may be updated using inspection results
- There is a relation partial factor – reliability index

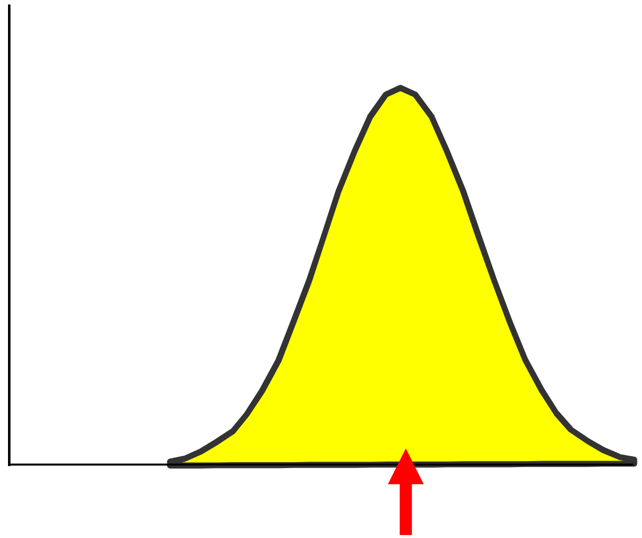
Fatigue steel structures



Find

P(a(t+Δt) > d | a(t) = .. of a(t)<..)





no cracks found, but?

measured 1 mm, but?

Example: Resistance with unknown mean m_R and known stand. Dev. s_R =17,5

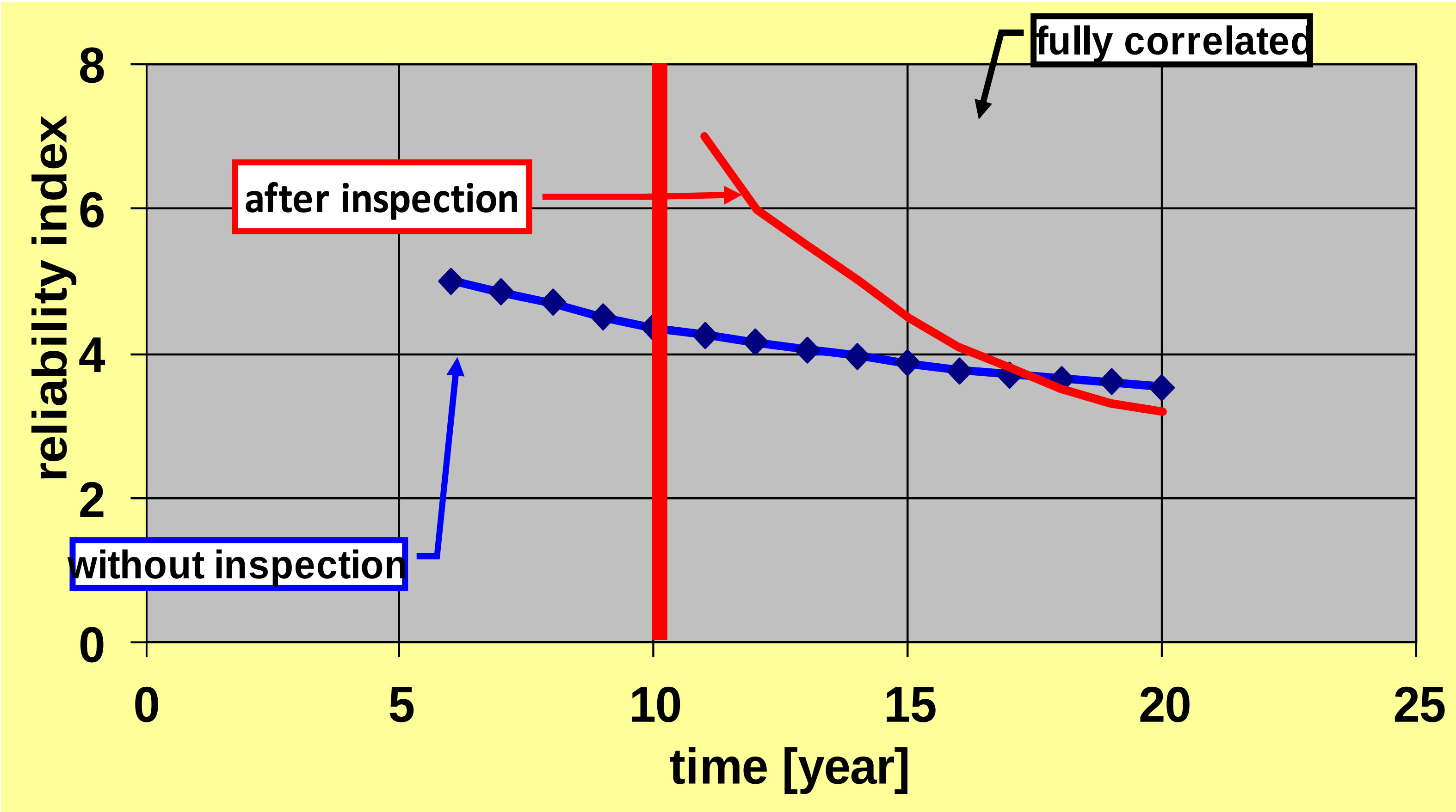
Assume we have 3 observations with mean m_m = 350
Then m_R has s_m = 17,5/√3 = 10.
If the load is to 304 then:

m_Z= 350-304=46
s_Z =$\sqrt{(17,5^2 + 10^2)}$ = 20,2
β=2,27
P_f=0,0116

Now we have one extra observation equal to 350.
In that case the estimate of the mean m_m does not change.
The standard deviation of the mean changes to 17,5/√(4) = 8,8

m_Z= 350-304=46,
s_Z =$\sqrt{(17,5^2 + 8,8^2)}$ = 19,6,
β=2,35
P_f=0,0095

Reliabilty level Beta (one year periods)
given a crack found at t=10 a



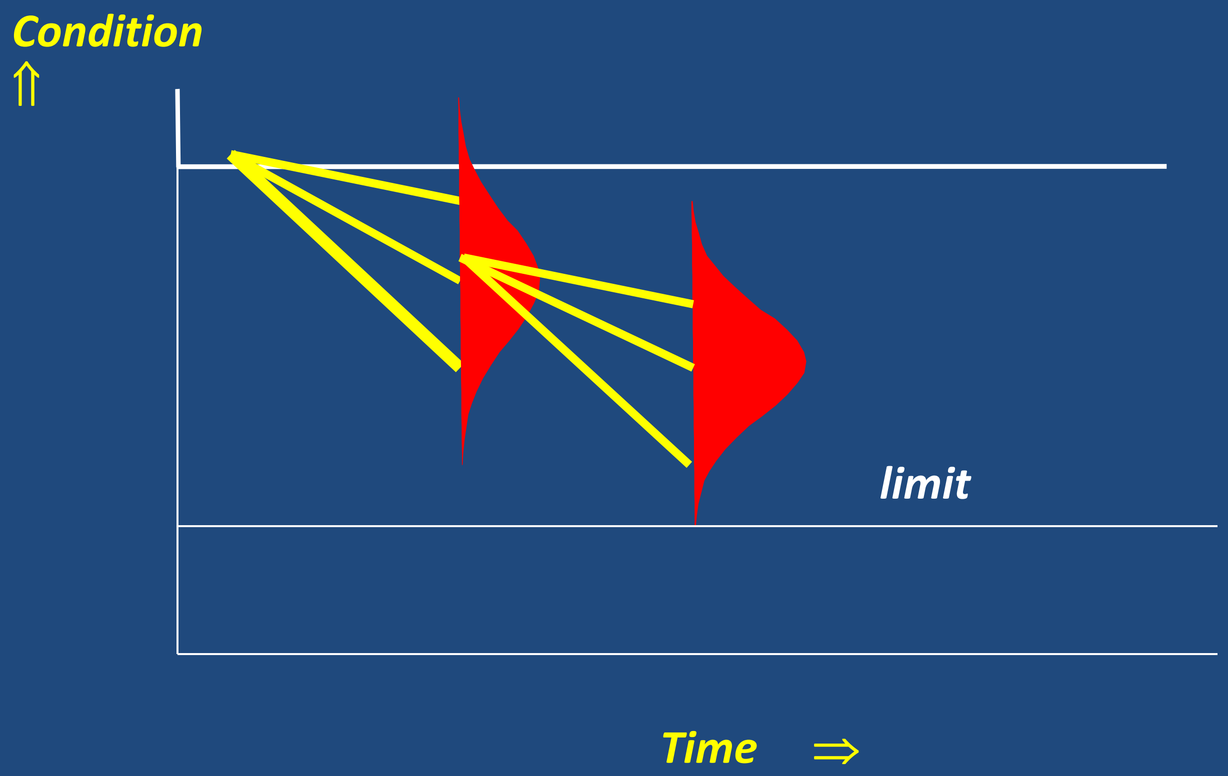
Existing Structures (NEN 8700)

Reliability index in case of assessment

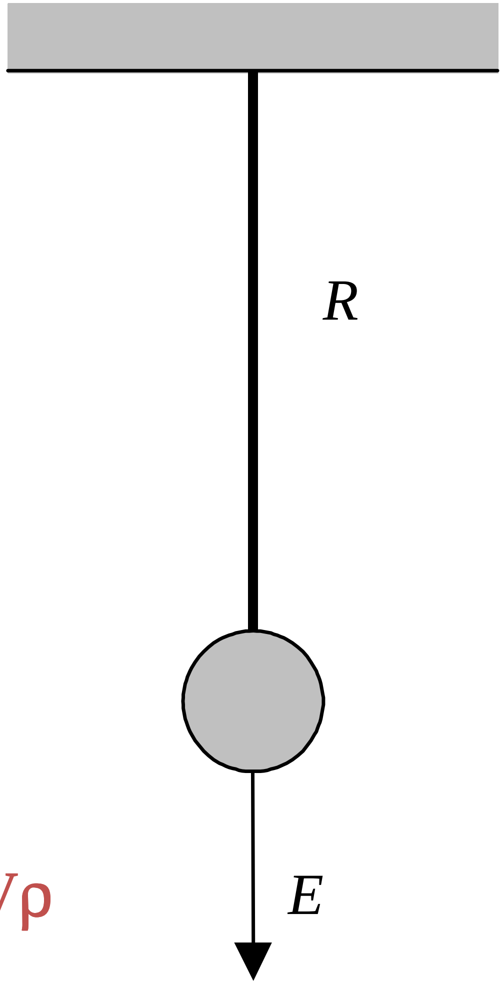
Minimum $\beta < \beta_{new} - 1.0$

Human safety: $\beta > 3.6 - 0.8 \log T$

Inspection en monitoring



EXAMPLE



Resistance: $R = \pi d^2 f_y / 4$

Load effect: $E = V\rho$

Failure if E>R or: $V\rho > \pi d^2 f_y / 4$

Limit state: $V\rho = \pi d^2 f_y / 4$

Limit state function: $Z = R - E = \pi d^2 f_y / 4 - V\rho$

Reliability index

Probability of Failure = $\Phi(-\beta) \approx 10^{-\beta}$

β	1.3	2.3	3.1	3.7	4.2	4.7
$P(F)=\Phi(-\beta)$	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}	10^{-6}

Relation Partial factors and beta-level:

$$\gamma = \exp\{\alpha \beta V - kV\} \approx 1 + \alpha \beta V$$

α = 0.7-0.8

β = 3.3 - 3.8 - 4,3 (life time, Annex B)

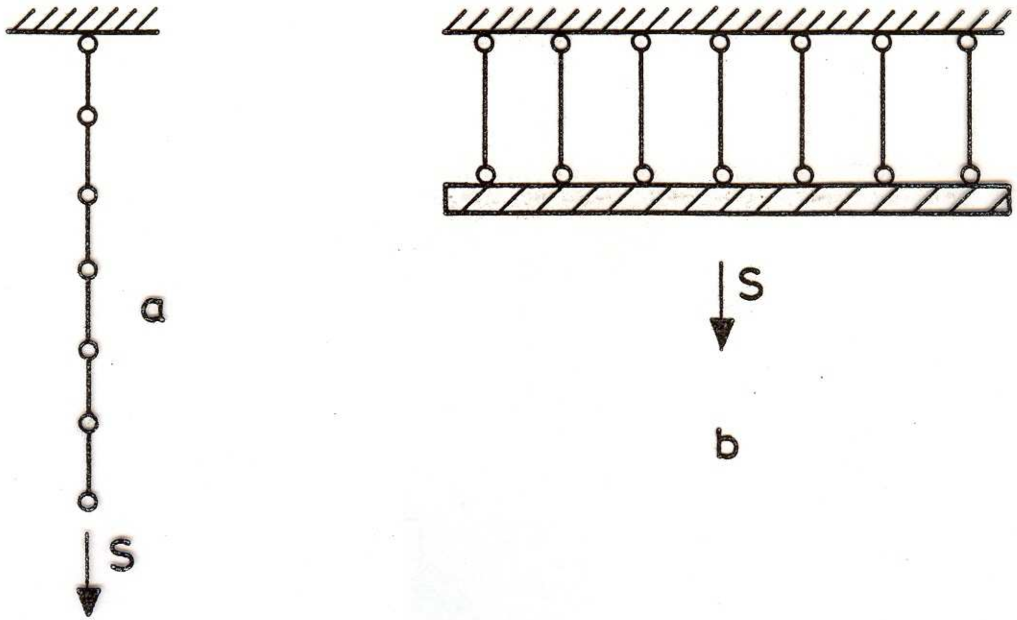
k = 1.64 (resistance)

k = 0.0 (loads)

V = coefficient of variation

Extensions

- load fluctuations
- systems
- degradation
- inspection
- risk analysis
- target reliabilities

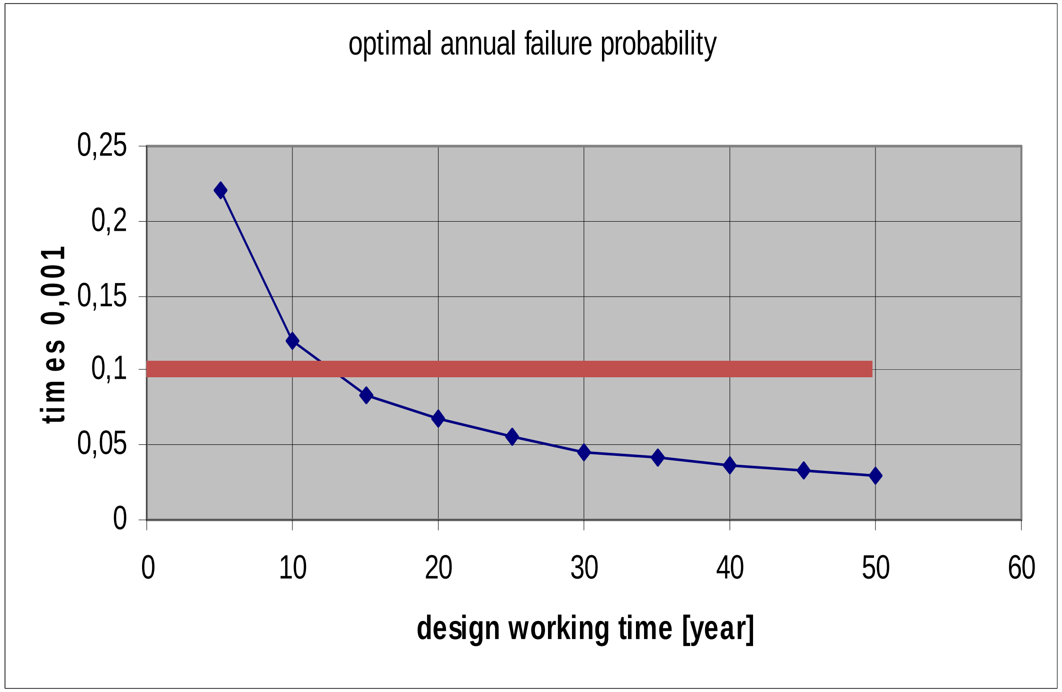


JCSS TARGET RELIABILITIES β
for a one year reference period

	Consequences of failure ⇒		
Cost to increase safety ⇓	Minor	Moderate	Large
Large	β=3.1 (p_F≈10^{-3})	β=3.3 (p_F ≈ 5 10^{-4})	β=3.7 (p_F ≈ 10^{-4})
Normal	β=3.7 (p_F≈10^{-4})	β=4.2 (p_F ≈ 10^{-5})	β=4.4 (p_F ≈ 5 10^{-6})
Small	β=4.2 (p_F≈10^{-5})	β=4.4 (p_F ≈ 5 10^{-5})	β=4.7 (p_F ≈ 10^{-6})

Human life safety

- Include value for human life in *D*
- Still reasons for IR and SR
- Example: $p < 10^{-4}$ / year



Example NEN 8700 (Netherlands)

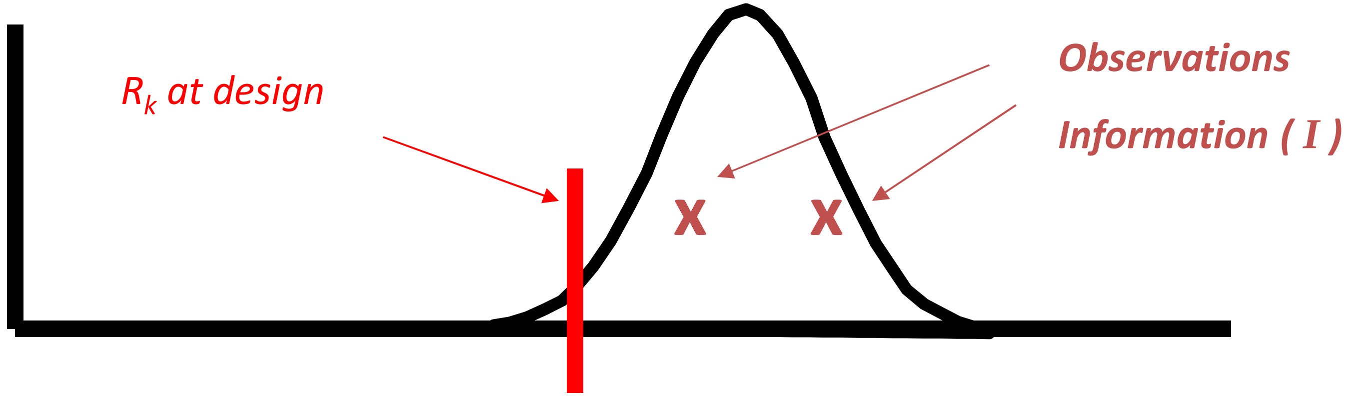
Minimum values for the reliability index β with a minimum reference period

Consequence class	Minimum reference period for existing building	β-NEW		β-EXISTING	
		wn	wd	wn	wd
0	1 year	3.3	2,3	1.8	0.8
1	15 years	**3.3**	2,3	1.8[a]	1.1[a]
2	15 years	**3.8**	2.8	2.5[a]	2.5[a]
3	15 years	**4.3**	3.3	3.3[a]	3.3[a]

Class 0: As class 1, but no human safety involved
wn = wind not dominant
wd = wind dominant
(a) = in this case is the minimum limit for personal safety normative

Updating

1) **Updating distributions (eg concrete strength)**



2) **Updating failure probability P{F | I }**

Example: I = {crack = 0.6 mm}

see JCSS document on Existing Structures en ISO13822

$$P(A \cap B) = P(A|B)P(B)$$

$$P(F \cap I) = P(F|I)P(I)$$

$$P(F|I) = \frac{P(F \cap I)}{P(I)}$$

Two types of information I:

equality type: $h(\mathbf{x}) = 0$

inequality type: $h(\mathbf{x}) < 0$; $h(\mathbf{x}) > 0$

$\mathbf{x}$ = vector of basic variables

$$P(F|I) = \frac{P(Z(t_2) < 0 \cap h(t_1) > 0)}{P(h(t_1) > 0)}$$

Target Reliabilities in EN 1990, Annex B

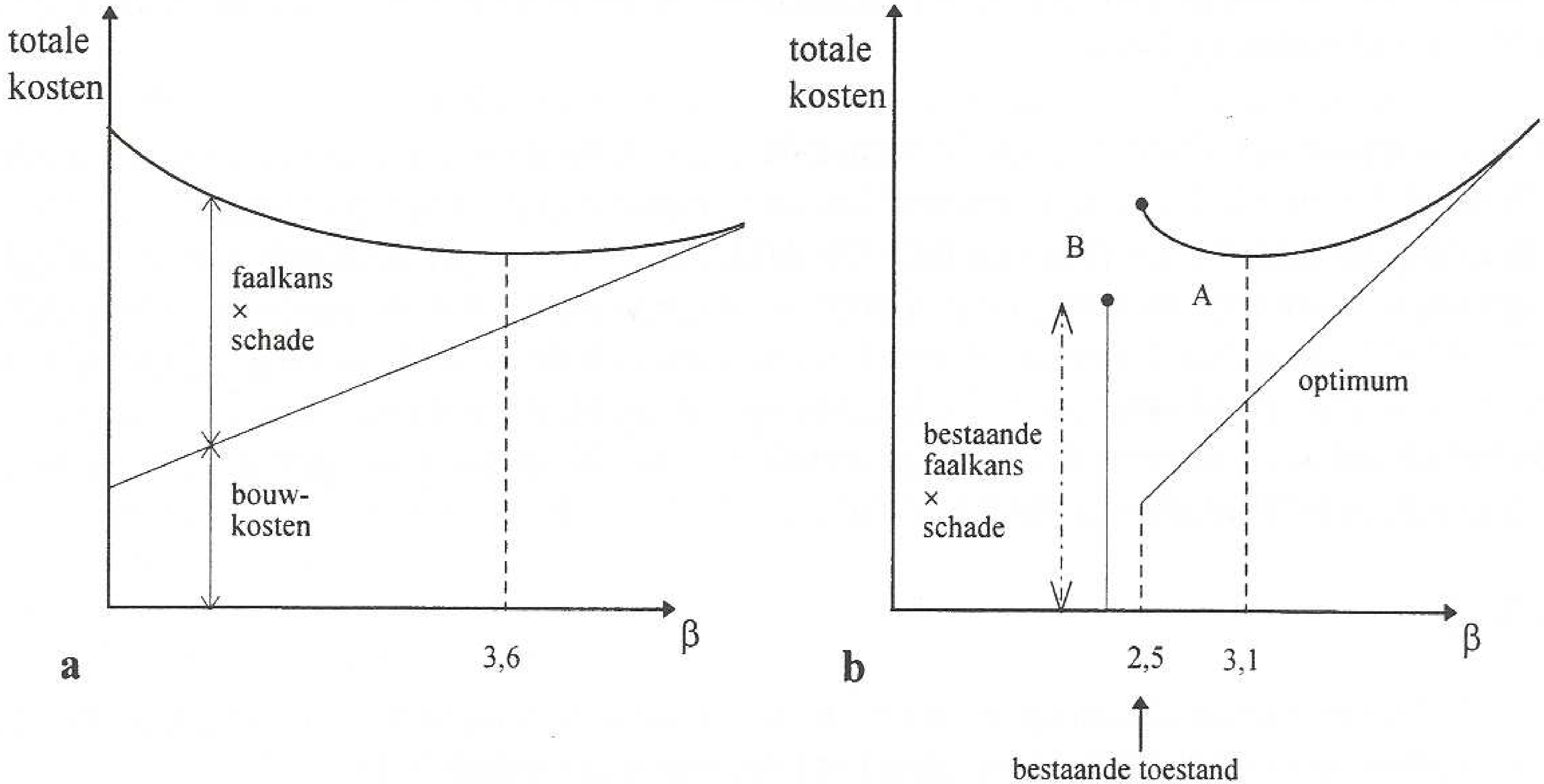
Reliability classes	Consequences for loss of human life, economical, social and environmental consequences	Reliability index β		Examples of buildings and civil engineering works
		β_a for T_a= 1 yr	β_d for T_d= 50 yr	
RC3 – high	High	5,2	4,3	Important bridges, public buildings
RC2 – normal	Medium	4,7	3,8	Residential and office buildings
RC1 – low	Low	4,2	3,3	Agricultural buildings, greenhouses

Formal Updating formulas

$$f_Q''(q \mid \hat{x}) = C\, f_Q'(q)\, L(\mid \hat{x} \mid q)$$

$$f_X^U(x) = \int_{-\infty}^{\infty} f_X(x|q)\, f_Q''(q \mid \hat{x})\, dq$$

Cost optimisation / design versus assessment



$P_F = 10^{-\beta}$