Reliability Aspects

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Assessment of existing structures
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Uncertainties are always present

- randomnness
- natural variability
- statistical uncertainties
- lack of data
- model uncertainties
- simplified models
- vagueness
- imprecision in definitions
- gross errors
- ignorance
- lack of knowledge

Some uncertainties are difficult to quantify

- theory of probability and statistics, fuzzy logic
- reliability theory and risk engineering

Some uncertainties are difficult to quantify
Reliability

- ability of a structure to fulfil all required functions during a specified period of time under given conditions

Failure probability $P_f$

- most important measure of structural reliability
Limit State Approach

• **Limit states** - states beyond which the structure no longer fulfils the relevant design criteria

• **Ultimate limit states**
  – loss of equilibrium of a structure as a rigid body
  – rupture, collapse, failure
  – fatigue failure

• **Serviceability limit states**
  – functional ability of a structure or its part
  – users comfort
  – appearance
Reliability Methods

Reliability measures: failure probability $p_f$ and reliability index $\beta$

<table>
<thead>
<tr>
<th>$p_f$</th>
<th>$10^{-1}$</th>
<th>$10^{-2}$</th>
<th>$10^{-3}$</th>
<th>$10^{-4}$</th>
<th>$10^{-5}$</th>
<th>$10^{-6}$</th>
<th>$10^{-7}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>1,3</td>
<td>2,3</td>
<td>3,1</td>
<td><strong>3,7</strong></td>
<td>4,2</td>
<td>4,7</td>
<td>5,2</td>
</tr>
</tbody>
</table>
Fundamental case for normal distribution

\[ E \leq R \]

\[ G = R - E \]
\[ \mu_G = \mu_R - \mu_E, \sigma_G^2 = \sigma_R^2 + \sigma_E^2 \]

Transformation of \( G \) to standardized variable \( U = (G - \mu_G)/\sigma_G \)

For \( G = 0 \) the standardized variable \( u_0 = (0 - \mu_G)/\sigma_G \)

Reliability index :
\[ \beta = -u_0 = \frac{\mu_G}{\sigma_G} = \frac{\mu_R - \mu_E}{\sqrt{\sigma_R^2 + \sigma_E^2}}^{1/2} \]

\[ \Phi_G(x) \]

Failure Prubab.

Reliability

Failure probability
\[ p_f = \Phi(-\beta) \]
Fundamental case \( E < R \)

Limit state function: \( g(X) = G = R - E = 0 \)

Load \( E = e_0 \) known, resistance \( R \) random: \( \mu_R, \sigma_R, (\alpha_R) \)

Transformation of \( R \) to standardized variable \( U = (R - \mu_R)/\sigma_R \)

For \( R = e_0 \) the standardized variable \( u_0 = (e_0 - \mu_R)/\sigma_R \)

Reliability index for normal dist.: \( \beta = -u_0 = (\mu_R - e_0)/\sigma_R \)

Failure probability

\[ p_f = \Phi_R(e_0) \]

For normal distribution

\[ p_f = \Phi(-\beta) \]
An example of the fundamental case

\[ Z = R - E \]

\[ \mu_Z = \mu_R - \mu_E = 100 - 50 = 50 \]

\[ \sigma_Z^2 = \sigma_R^2 + \sigma_E^2 = 14^2 \]

\[ \beta = \frac{\mu_Z}{\sigma_Z} = 3.54 \]

\[ P_f = P(Z < 0) = \Phi_Z(0) = 0.0002 \]
Reliability margin and index $\beta$

$\beta = \mu_Z / \sigma_Z = 3.54$

$P_f = P(Z < 0) = \Phi_Z(0) = 0.0002,$

$\beta$ is the distance of the mean of reliability margin from the origin.
Probabilistic approach

\[ Z = R - E \]

\[ P_f = P(Z < 0) = \int \int \varphi_R(r) \varphi_E(e) drde \]

Techniques:

Numerical integration (NI)

Monte Carlo (MC)

First order Second moment method (FOSM)

Third moment method (accounting for skewness)

First Order Reliability Methods (FORM)
Probabilistic models

<table>
<thead>
<tr>
<th></th>
<th>distribution</th>
<th>mean</th>
<th>sd</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>resistance</td>
<td>100</td>
<td>10</td>
</tr>
<tr>
<td>E</td>
<td>load effect</td>
<td>50</td>
<td>5</td>
</tr>
</tbody>
</table>

Load effect $E$, Gumbel distribution, $\mu_E = 50$, $\sigma_E = 5$

Resistance $R$, log-normal distribution, $\mu_R = 100$, $\sigma_R = 10$
Eurocode concepts of partial factors

Reliability index $\beta$

Action reliability index
$\beta_E = \alpha_E \beta$

- Main action $\beta_E = -0.7 \beta$
- Accompanying $\beta_E = -0.28 \beta$

Resistance reliability index
$\beta_R = \alpha_R \beta$

- Resistance $\beta_R = 0.8 \beta$
Partial factor

- Design value
  
  for normal and lognormal distribution
  \[ x_d = \mu (1 - \alpha \beta V) \]
  
  for lognormal distribution: \( x_d \)
  \[ = \mu \exp(-\alpha \beta \sigma - 0.5 \sigma^2) \]

- Characteristic value
  
  for normal \( x_k = \mu (1 - kV) \)
  
  for lognormal \( x_k = \mu \exp(-k \sigma - 0.5 \sigma^2) \)

- Partial factor \( \gamma_m = \frac{x_k}{x_d} \)
## Partial factor approach

<table>
<thead>
<tr>
<th></th>
<th>distribution</th>
<th>mean</th>
<th>sd</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>resistance</td>
<td>Lognormal</td>
<td>100</td>
</tr>
<tr>
<td>E</td>
<td>load effect</td>
<td>Gumbel</td>
<td>50</td>
</tr>
</tbody>
</table>

Probability density $\phi_E(x), \phi_R(x)$

Load effect $E_k$, Gumbel distribution, $\mu_E = 50, \sigma_E = 5$

Resistance $R_k$, log-normal distribution, $\mu_R = 100, \sigma_R = 10$

$R_k/\gamma_m > E_k \gamma_Q$
## Indicative target reliabilities in ISO 13822

<table>
<thead>
<tr>
<th>Limit states</th>
<th>Target reliability index, $\beta$</th>
<th>Reference period</th>
</tr>
</thead>
<tbody>
<tr>
<td>Serviceability</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reversible</td>
<td>0.0</td>
<td>Intended remaining working life</td>
</tr>
<tr>
<td>Irreversible</td>
<td>1.5</td>
<td>Intended remaining working life</td>
</tr>
<tr>
<td>Fatigue</td>
<td></td>
<td></td>
</tr>
<tr>
<td>inspectable</td>
<td>2.3</td>
<td>Intended remaining working life</td>
</tr>
<tr>
<td>not inspectable</td>
<td>3.1</td>
<td>Intended remaining working life</td>
</tr>
<tr>
<td>Ultimate</td>
<td></td>
<td></td>
</tr>
<tr>
<td>very low consequences of failure</td>
<td>2.3</td>
<td>$L_S$ years*</td>
</tr>
<tr>
<td>low consequence of failure</td>
<td>3.1</td>
<td>$L_S$ years*</td>
</tr>
<tr>
<td>medium consequence of failure</td>
<td>3.8</td>
<td>$L_S$ years*</td>
</tr>
<tr>
<td>high consequence of failure</td>
<td>4.3</td>
<td>$L_S$ years*</td>
</tr>
</tbody>
</table>

* $L_S$ is a minimum standard period for safety (e.g. 50 years)
Updating distributions

$$P(x|I) = \frac{P(x) P(I|x)}{P(I)}$$

$$f_X(x|I) = C f_X(x) P(I|x)$$

updated prior likelihood

prior distribution $f_X(x)$

updated distribution $f_X(x|I)$

$f_X(x)$, $f_X(x|I)$

prior $x_d$, updated $x_d$
Formal Updating formulas

\[ f_Q''(q \mid \hat{x}) = C f_Q'(q) L(\mid \hat{x} \mid q) \]

\[ f_X^U(x) = \int_{-\infty}^{\infty} f_X(x \mid q) f_Q''(q \mid \hat{x}) dq \]

Ask the expert!
Concluding remarks on reliability aspects

- Uncertainties are always present
- Probability Theory may be helpful
- Reliability targets depend on consequences of failure
- Reliability targets depend on costs of improving
- Existing structures may have a lower target reliability
- Reliability may be updated using inspection results
- There is a relation partial factor – reliability index
Fatigue steel structures
Find

\[ P(a(t+\Delta t) > d \mid a(t) = \ldots \text{ of } a(t) < \ldots) \]

no cracks found, but?

measured 1 mm, but?
**Example:** Resistance with unknown mean $m_R$ and known stand. Dev. $s_R = 17,5$

Assume we have 3 observations with mean $m_m = 350$
Then $m_R$ has $s_m = 17,5/\sqrt{3} = 10$.
If the load is to 304 then:

\[
\begin{align*}
  m_Z &= 350 - 304 = 46 \\
  s_Z &= \sqrt{(17,5^2 + 10^2)} = 20,2 \\
  \beta &= 2,27 \\
  P_f &= 0,0116
\end{align*}
\]

Now we have one extra observation equal to 350.
In that case the estimate of the mean $m_m$ does not change.
The standard deviation of the mean changes to $17,5/\sqrt{(4)} = 8,8$

\[
\begin{align*}
  m_Z &= 350 - 304 = 46, \\
  s_Z &= \sqrt{(17,5^2 + 8,8^2)} = 19,6, \\
  \beta &= 2,35 \\
  P_f &= 0,0095
\end{align*}
\]
Reliability level Beta (one year periods)
given a crack found at t=10 a
Existing Structures (NEN 8700)

Reliability index in case of assessment

Minimum $\beta < \beta_{new} - 1.0$

Human safety: $\beta > 3.6 - 0.8 \log T$
EXAMPLE

Resistance: \( R = \pi d^2 f_y / 4 \)

Load effect: \( E = V \rho \)

Failure if \( E > R \) or: \( V \rho > \pi d^2 f_y / 4 \)

Limit state: \( V \rho = \pi d^2 f_y / 4 \)

Limit state function: \( Z = R - E = \pi d^2 f_y / 4 - V \rho \)
Reliability index

Probability of Failure = $\Phi(-\beta) \approx 10^{-\beta}$

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>1.3</th>
<th>2.3</th>
<th>3.1</th>
<th>3.7</th>
<th>4.2</th>
<th>4.7</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(F) = \Phi(-\beta)$</td>
<td>$10^{-1}$</td>
<td>$10^{-2}$</td>
<td>$10^{-3}$</td>
<td>$10^{-4}$</td>
<td>$10^{-5}$</td>
<td>$10^{-6}$</td>
</tr>
</tbody>
</table>
Relation Partial factors and beta-level:

\[ \gamma = \exp\{\alpha \beta V - kV\} \approx 1 + \alpha \beta V \]

\( \alpha = 0.7-0.8 \)
\( \beta = 3.3 - 3.8 - 4.3 \) (life time, Annex B)
\( k = 1.64 \) (resistance)
\( k = 0.0 \) (loads)
\( V = \text{coefficient of variation} \)
Extensions

• load fluctuations
• systems
• degradation
• inspection
• risk analysis
• target reliabilities
## JCSS TARGET RELIABILITIES $\beta$

for a one year reference period

<table>
<thead>
<tr>
<th>Cost to increase safety</th>
<th>Minor</th>
<th>Moderate</th>
<th>Large</th>
</tr>
</thead>
<tbody>
<tr>
<td>Large</td>
<td>$\beta=3.1$ ($p_F \approx 10^{-3}$)</td>
<td>$\beta=3.3$ ($p_F \approx 5 \times 10^{-4}$)</td>
<td>$\beta=3.7$ ($p_F \approx 10^{-4}$)</td>
</tr>
<tr>
<td>Normal</td>
<td>$\beta=3.7$ ($p_F \approx 10^{-4}$)</td>
<td>$\beta=4.2$ ($p_F \approx 10^{-5}$)</td>
<td>$\beta=4.4$ ($p_F \approx 5 \times 10^{-6}$)</td>
</tr>
<tr>
<td>Small</td>
<td>$\beta=4.2$ ($p_F \approx 10^{-5}$)</td>
<td>$\beta=4.4$ ($p_F \approx 5 \times 10^{-5}$)</td>
<td>$\beta=4.7$ ($p_F \approx 10^{-6}$)</td>
</tr>
</tbody>
</table>
Human life safety

• Include value for human life in $D$
• Still reasons for IR and SR
• Example: $p < 10^{-4}$ / year
### Example NEN 8700 (Netherlands)

**Minimum values for the reliability index β with a minimum reference period**

<table>
<thead>
<tr>
<th>Consequence class</th>
<th>Minimum reference period for existing building</th>
<th>β-NEW wn</th>
<th>β-NEW wd</th>
<th>β-EXISTING wn</th>
<th>β-EXISTING wd</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1 year</td>
<td>3.3</td>
<td>2.3</td>
<td>1.8</td>
<td>0.8</td>
</tr>
<tr>
<td>1</td>
<td>15 years</td>
<td>3.3</td>
<td>2.3</td>
<td>1.8&lt;sup&gt;a&lt;/sup&gt;</td>
<td>1.1&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td>2</td>
<td>15 years</td>
<td>3.8</td>
<td>2.8</td>
<td>2.5&lt;sup&gt;a&lt;/sup&gt;</td>
<td>2.5&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td>3</td>
<td>15 years</td>
<td>4.3</td>
<td>3.3</td>
<td>3.3&lt;sup&gt;a&lt;/sup&gt;</td>
<td>3.3&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
</tbody>
</table>

Class 0: As class 1, but no human safety involved

wn = wind not dominant
wd = wind dominant
(a) = in this case is the minimum limit for personal safety normative
1) Updating distributions (eg concrete strength)

2) Updating failure probability $P\{F \mid I\}$

Example: $I = \{\text{crack} = 0.6 \text{ mm}\}$

see JCSS document on Existing Structures en ISO13822
\[ P(A \cap B) = P(A | B)P(B) \]
\[ P(F \cap I) = P(F | I)P(I) \]
\[ P(F | I) = \frac{P(F \cap I)}{P(I)} \]

Two types of information I:

- equality type: \( h(x) = 0 \)
- inequality type: \( h(x) < 0; \ h(x) > 0 \)

\( x = \) vector of basic variables

\[ P(F | I) = \frac{P(Z(t_2) < 0 \cap h(t_1) > 0)}{P(h(t_1) > 0)} \]
## Target Reliabilities in EN 1990, Annex B

<table>
<thead>
<tr>
<th>Reliability classes</th>
<th>Consequences for loss of human life, economical, social and environmental consequences</th>
<th>Reliability index $\beta$</th>
<th>Examples of buildings and civil engineering works</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\beta_a$ for $T_a= 1$ yr</td>
<td>$\beta_d$ for $T_d= 50$ yr</td>
</tr>
<tr>
<td>RC3 – high</td>
<td>High</td>
<td>5.2</td>
<td>4.3</td>
</tr>
<tr>
<td>RC2 – normal</td>
<td>Medium</td>
<td>4.7</td>
<td>3.8</td>
</tr>
<tr>
<td>RC1 – low</td>
<td>Low</td>
<td>4.2</td>
<td>3.3</td>
</tr>
</tbody>
</table>
Formal Updating formulas

\[ f_Q''(q \mid \hat{x}) = C f_Q'(q) L(\mid \hat{x} \mid q) \]

\[ f^U_X(x) = \int_{-\infty}^{\infty} f_X(x \mid q) f_Q''(q \mid \hat{x}) dq \]
Cost optimisation / design versus assessment

\[ P_F = 10^{-\beta} \]