# Esercitazione 3

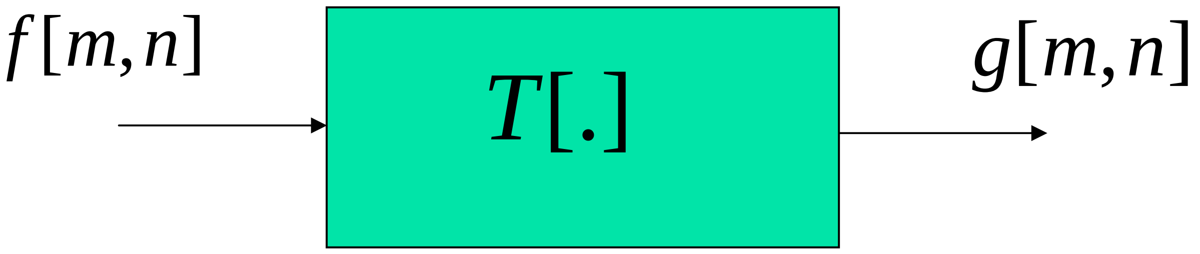
Corso di Elaborazione e
Trasmissione delle Immagini

Pisa, 12 ottobre 2005

# Argomenti proposti

- Prodotto di convoluzione
- Filtraggio nel dominio spaziale
- Trasformata di Fourier di segnali 2D spazio-discreti
- Filtraggio nel dominio delle frequenze spaziali

# Sistemi spazio discreti




Linearità

$$g[m,n] = T\left[\sum_{i=1}^{N}\sum_{i=1}^{N}\alpha_i f_i[m,n]\right] = \sum_{i=1}^{N}\sum_{i=1}^{N}\alpha_i T\big[f_i[m,n]\big] = \sum_{i=1}^{N}\sum_{i=1}^{N}\alpha_i g_i[m,n]$$

Invarianza alla traslazione

$$g'[m,n] = T\big[f[m-m_0, n-n_0]\big] = g[m-m_0, n-n_0]$$

# Prodotto di convoluzione

$$g[m,n] = \sum_{r=-\infty}^{\infty} \sum_{s=-\infty}^{\infty} f[r,s]h[m-r,n-s]$$

Estensione del prodotto di convoluzione

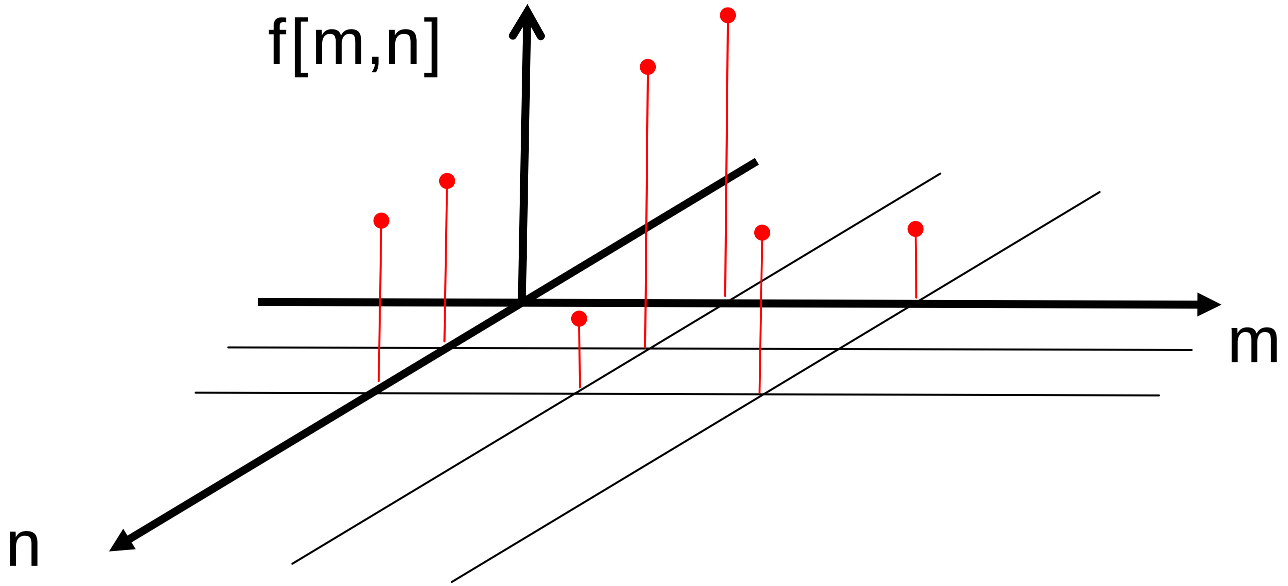
$f[m,n] \qquad (0, M_f - 1) X (0, N_f - 1)$

$h[m,n] \qquad (0, M_h - 1) X (0, N_h - 1)$

$g[m,n] \qquad (0, M_f + M_h - 2) X (0, N_f + N_h - 2)$

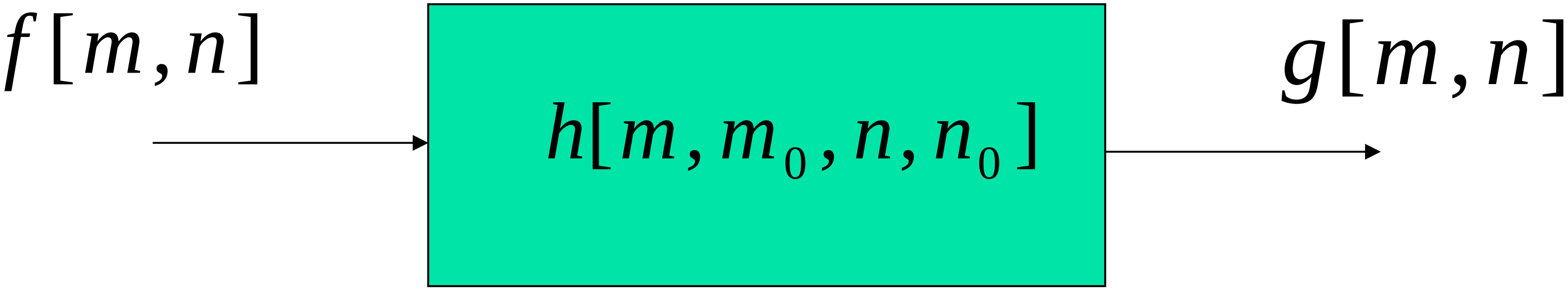
# Proprietà impulso discreto

$$f[m,n] = f[m,n] \otimes\otimes \delta[m,n] = \sum_{r=-\infty}^{\infty}\sum_{s=-\infty}^{\infty} f[r,s]\delta[m-r,n-s]$$




$$f[m,n] = f[m,n]\delta[m-m_0,n-n_0] = f[m_0,n_0]\delta[m-m_0,n-n_0]$$

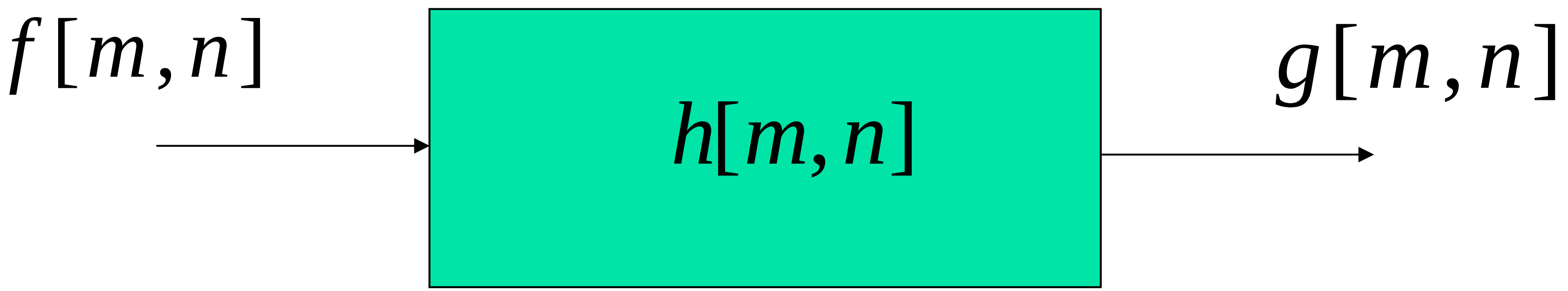
# Sistemi Lineari




$$h[m,m_0,n,n_0] = T\left[\delta[m-m_0,n-n_0]\right]$$

$$g[m,n] = \sum_{r=-\infty}^{\infty}\sum_{s=-\infty}^{\infty} f[r,s]h[m,r,n,s]$$

# Sistemi Lineari e Invarianti alla traslazione




$$h[m-m_0, n-n_0] = T\left[\delta[m-m_0, n-n_0]\right]$$

$$g[m,n] = \sum_{r=-\infty}^{\infty}\sum_{s=-\infty}^{\infty} f[r,s]h[m-r, n-s]$$

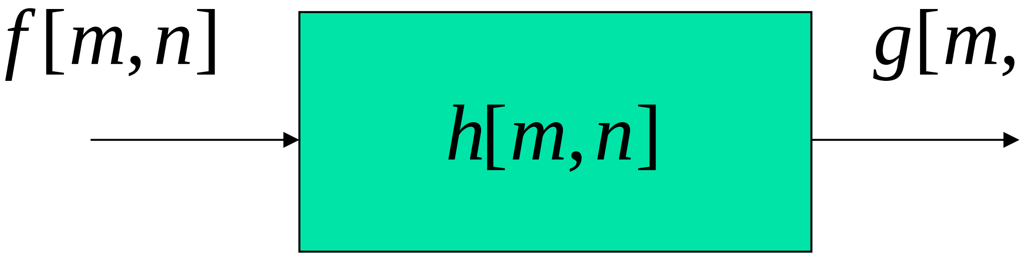
# Risposta in frequenza

$$\tilde{H}(X,Y) = \sum_{n}\sum_{m} h[m,n]\exp[-j2\pi(mX+nY)]$$

$$h[m,n] = \int_{[1]}\int_{[1]} \tilde{H}(X,Y)\exp[j2\pi(mX+nY)]dXdY$$

$$\tilde{G}(X,Y) = \tilde{F}(X,Y)\tilde{H}(X,Y)$$

# Filtraggio nel dominio spaziale (1)




$$g[m,n] = f[m,n] \otimes \otimes h[m,n]$$

$$h[m,n] = \frac{1}{16}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

# Filtraggio nel dominio spaziale (2)

Filtro ideale passa-basso

$$g_{PB}[m,n] = f[m,n] \otimes\otimes h_{PB}[m,n]$$

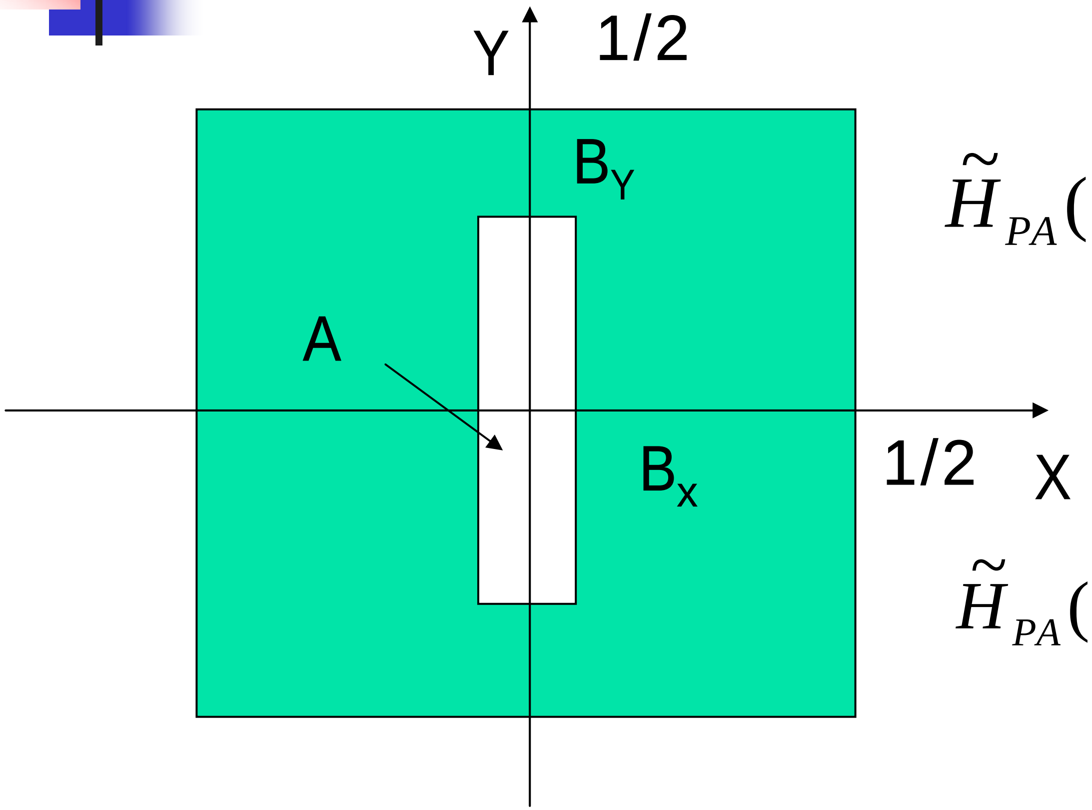
Filtro ideale passa-alto

$$g_{PA}[m,n] = f[m,n] \otimes\otimes h_{PA}[m,n]$$

$$= f[m,n] \otimes\otimes \left[\delta[m,n] - h_{PB}[m,n]\right]$$

$$= f[m,n] - g_{PB}[m,n]$$

# Filtri ideali passa-alto




$$\tilde{H}_{PA}(X,Y)=\begin{cases}0 & (X,Y)\in A\\ 1 & altrove\end{cases}$$

$$\tilde{H}_{PA}(X,Y)=1-\tilde{H}_{PB}(X,Y)$$

$$h_{PA}[m,n]=\delta[m,n]-h_{PB}[m,n]$$

# Trasformata di Fourier di segnali spazio-discreti

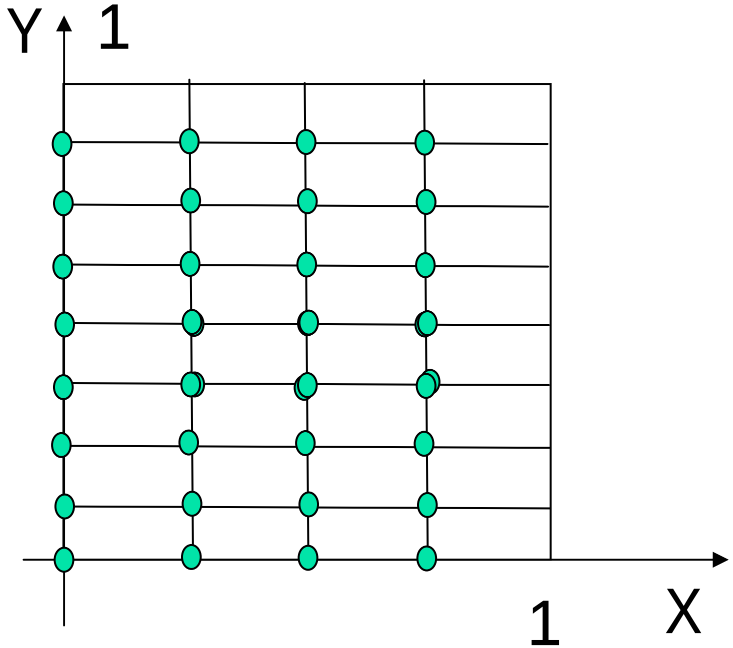
Trasformata

$$\tilde{F}(X,Y) = \sum_{m=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} f[m,n] \exp[-j2\pi(mX+nY)]$$

Antitrasformata

$$f[m,n] = \int_{[1]} \int_{[1]} \tilde{F}(X,Y) \exp[j2\pi(mX+nY)dXdY$$

# Campioni della Trasformata Discreta di Fourier




$$\tilde{F}(r\Delta X, s\Delta Y) = \tilde{F}\left(\tfrac{r}{M}, \tfrac{s}{N}\right)$$

$$\Delta X = \tfrac{1}{M}$$

$$\Delta Y = \tfrac{1}{N}$$

$$r = 0, 1, \ldots, M-1$$

$$s = 0, 1, \ldots, N-1$$

$$\tilde{F}\left(\tfrac{r}{M}, \tfrac{s}{N}\right) = \sum_{m=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} f[m,n] \exp[-j2\pi\left(\frac{mr}{M} + \frac{ns}{N}\right)]$$

# Campioni della Trasformata Discreta di Fourier

Antitrasformata

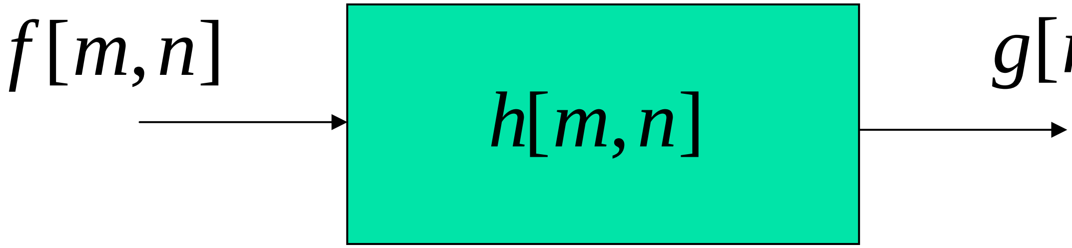
$$f[m,n] = \int_{[1]}\int_{[1]} \tilde{F}(X,Y)\exp[j2\pi(mX + nY)dXdY$$

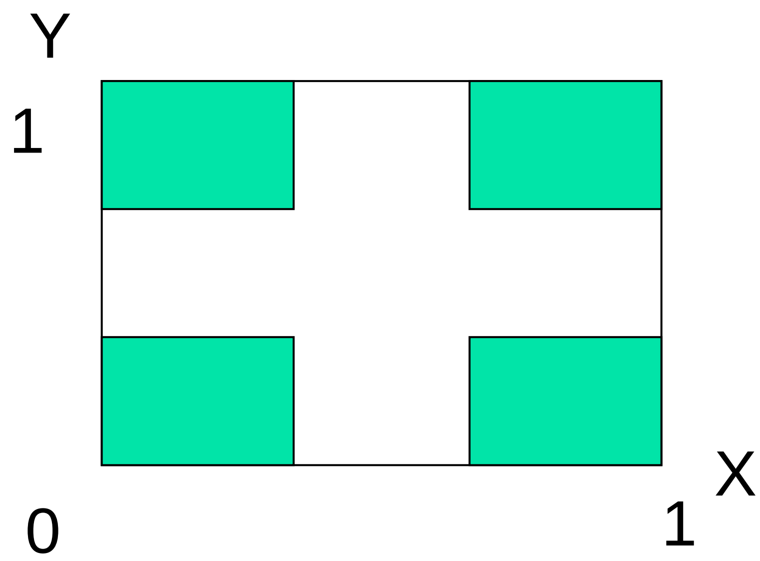
$$\cong \sum_{r=0}^{M-1}\sum_{s=0}^{N-1} \tilde{F}(\tfrac{r}{M},\tfrac{s}{N})\exp[j2\pi\left(\frac{mr}{M} + \frac{ns}{N}\right)]\frac{1}{M}\frac{1}{N}$$

$$f[m,n] = \sum_{r=0}^{M-1}\sum_{s=0}^{N-1} \tilde{F}(\tfrac{r}{M},\tfrac{s}{N})\exp[j2\pi\left(\frac{mr}{M} + \frac{ns}{N}\right)]\frac{1}{M}\frac{1}{N}$$

Se e solo se $f[m,n] \neq 0$ $m = 0,1,\ldots, M-1; n = 0,1,\ldots, N-1$

# Filtraggio nel dominio delle frequenze spaziali

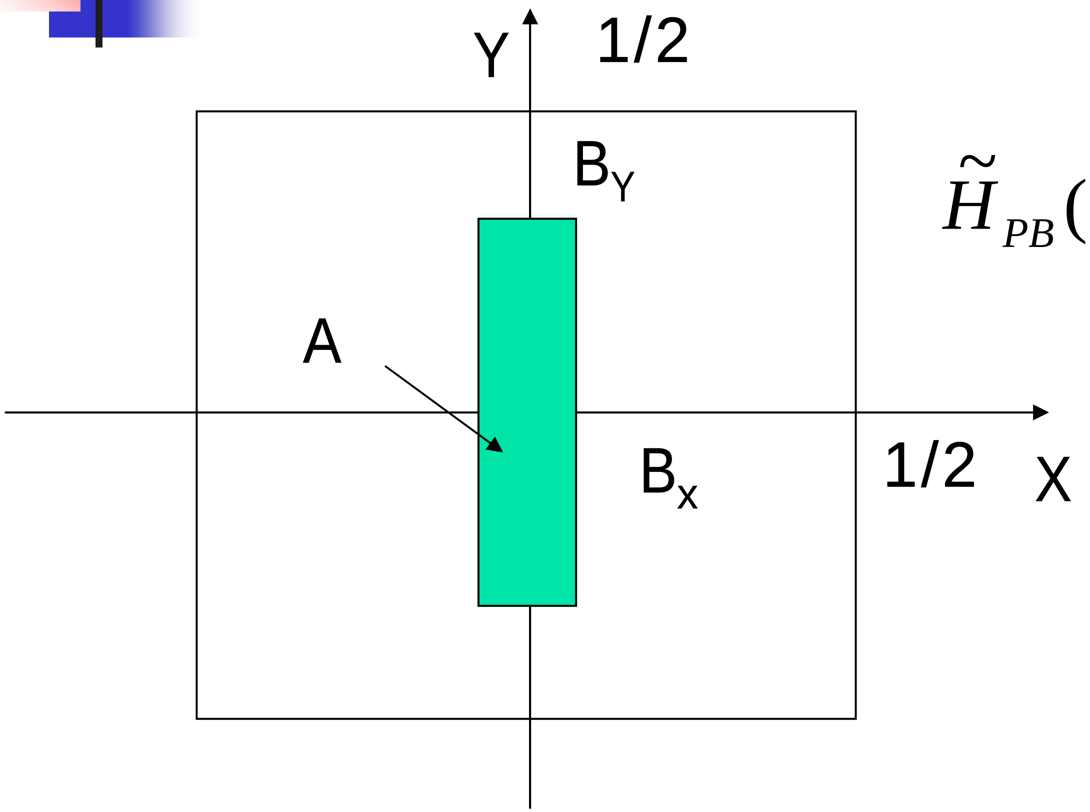



$$\tilde{G}\left(\frac{r}{M},\frac{s}{N}\right)=\tilde{F}\left(\frac{r}{M},\frac{s}{N}\right)\,\tilde{H}\left(\frac{r}{M},\frac{s}{N}\right)$$

Filtro passa-basso
in termini di trasformata discreta

# Filtri ideali passa-basso



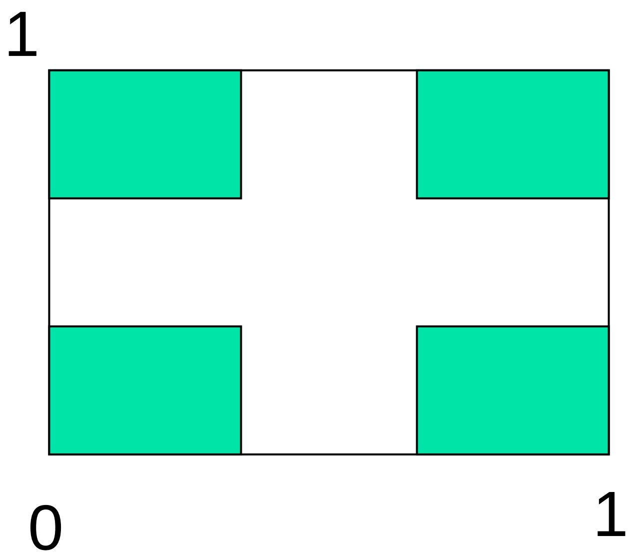

$$\tilde{H}_{PB}(X,Y) = \begin{cases} 1 & (X,Y) \in A \\ 0 & altrove \end{cases}$$

$$h_{PB}[m,n] = 2B_X\, 2B_Y \sin c(2B_X m) \sin c(2B_Y n)$$

# Filtri ideali

$$\tilde{G}[r,s] = \tilde{F}[r,s]\,\tilde{H}[r,s]$$

Filtro passa-basso




Filtro passa-alto