EVALUATION OF THE ELECTRICAL FORCES ACTING ON A DETACHING BUBBLE

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ABSTRACT

The present study is focused to investigate the behavior of gas bubbles injected in two fluids, namely FC72, and HFE7100, of different physical properties, from an orifice drilled in flat plate, under the action of an external electrostatic field. In this way, the mechanical actions are separated from the thermal and mass transfer phenomena. The study is limited to the so-called “quasi-static” regime, i.e. when the process is occurring slowly and the role played by dynamical actions is negligible. A simple experimental apparatus was built and operated to study, with the aid of high speed cinematography, the growth and detachment of bubbles, and a dedicated code was developed to postprocess the images, obtaining the main geometrical parameters of the bubbles. The additional forces due to the application of electric field were derived starting from the Maxwell stress tensor, and their expressions, along with the bubble geometry derived from data processing, were implemented in Comsol Multiphysics code, that calculated the electric field configuration around the bubble and the value of the electric forces acting on it. The results show a fairly good agreement and allow for validating the electrical force calculations, that are strongly dependent on electric coupling condition at the interface and fluid properties, for future use in modeling boiling with electric field.

INTRODUCTION

The study of the physics of bubble detachment is precursory for the understanding of the boiling phenomena. In particular, when the attention is devoted to microgravity environment, the application of an electric force is particularly interesting, as it can act as a replacement for the lacking buoyancy.

The present study is focused to investigate the behavior of gas bubbles injected in two fluids, of different physical properties, from an orifice drilled in flat plate, under the action of an external electrostatic field. In this way, the mechanical actions are separated from the thermal phenomena. The study is limited to the so-called “quasi-static” regime, i.e. when the process is occurring slowly and the role played by dynamical actions is negligible.

EXPERIMENTS

Experimental apparatus and data processing

The experimental apparatus and the data conditioning procedures have been described elsewhere [4], [5], and only a brief outlook will be given here. The experimental cell (Fig.2) consisted of a polycarbonate box of about 2.5 dm³ volume, open to the atmosphere and monitored by a temperature sensor. A circular orifice (0.14±0.05 mm diameter) was drilled in a flat brass plate 30x30 mm², laid horizontally inside it, and gas (nitrogen) was injected through it into the fluid via a mass flow controller. Two different working fluids were adopted, as detailed in Tab.1. The level of the liquid above the plate was varied from a fluid to another in order to keep the hydraulic head constant and equal to 980 Pa (100 mm H₂O).

An axisymmetric electric field could be generated above the orifice by imposing a voltage up to 20 kV dc to a second brass plate, laid parallel to the first one and with a hole of 4 mm drilled in it and coaxial with the orifice (see Fig.2). In this way, a simple two-dimensional geometric configuration is obtained, useful for code implementation.

During the experiments reported herein, video images of the bubbles were taken with a high speed camera (Phantom V4.0 by Vision Research) at a frame rate up to 1500 fps and with a resolution of about 130 pixel/mm. The dedicated digital processing software is based on Matlab “Image Acquisition” toolbox, and is able to evaluate, among other parameters, the bubble volume, the coordinates of the bubble profile, the incidence angle, and the curvature radius at the bubble apex.

Fig. 1 – Schematization of bubble growth from an orifice (a) and on a flat surface (b).
The volumic electric force and the Maxwell stress tensor

The most generally accepted expression for the volumic electric force that acts on a fluid, to be included in the momentum equation, is [6]

\[
\mathbf{f}_e^v = \rho E - \frac{\varepsilon_0}{2} \text{grad} \varepsilon_x - \frac{1}{2} \varepsilon_0 \text{grad} E^2 - \rho \varepsilon_0 \frac{\partial \varepsilon_x}{\partial \rho} \text{grad} \rho \varepsilon_x
\]

(1)

Only the first term (Coulomb's force) depends on the sign of the electric field. It is present when free charge buildup occurs and in such cases it generally predominates over the other electrical forces. The other two terms depend on the gradient of the electric field and of the dielectric constant other electrical forces. The other two terms depend on the local intensity of electric field, the fluid density and its variation with temperature. A correct evaluation of the local value of electric field at the interface relies on the adoption of appropriate boundary conditions on it, which in turn depend on the assumption made on the electric properties of the fluids, and the problem is still debated [8], [9].

In the absence of free charge, the components of \( \mathbf{T} \) can be expressed as

\[
t_a = \varepsilon_0 \varepsilon_x E_x E_y - \frac{\varepsilon_0}{2} E^2 \left[ \varepsilon_0 - \rho \frac{\partial \varepsilon_x}{\partial \rho} \right] \delta_{x_a}
\]

(4)

The electrostrictive term \( \rho \left( \varepsilon_0 \frac{\partial \varepsilon_x}{\partial \rho} \right) \) is very close to 0 for gases [6]; for non-polar fluids, it is given by the Clausius-Mossotti law

\[
\rho \left( \varepsilon_0 \frac{\partial \varepsilon_x}{\partial \rho} \right) = \frac{\varepsilon_x - 1}{3} \varepsilon_x + 2 \varepsilon_x + 4n^2 \varepsilon_x + n^4
\]

(5)

while for polar fluids, to the authors' best knowledge, the only available expression in literature is [11]

\[
\rho \left( \varepsilon_0 \frac{\partial \varepsilon_x}{\partial \rho} \right) = \frac{\varepsilon_x - 1}{3} \varepsilon_x + 2 \varepsilon_x + n^4
\]

(6)

where \( n \) is the refraction index of the fluid.

***Momentum balance across an interface***

Let us consider a gas-liquid interface, of curvature \( K \), and velocity \( w \), and a control volume of vanishing thickness surrounding it, as sketched in Fig. 3. \( n, t \) and \( b \) are the normal, tangent and bi-normal unit vectors, respectively. The mass and momentum balance on the volume (Panton, [12], ch.23) are, respectively

\[
\int_S [\rho \mathbf{n}_d \cdot (\mathbf{v}_f - \mathbf{w}) - \rho \mathbf{n}_s \cdot (\mathbf{v}_g - \mathbf{w})]dA = 0
\]

(7)

\[
\int_S [\rho \mathbf{n}_d \cdot (\mathbf{v}_f - \mathbf{w}) \mathbf{v}_f + \rho \mathbf{n}_s \cdot (\mathbf{v}_g - \mathbf{w}) \mathbf{v}_g]dA =
\]

\[
\int_S \{ [\mathbf{n}_d \cdot (\mathbf{T}_{w,f} - \mathbf{T}_{w,g})] - \mathbf{n}_d \cdot \mathbf{p}_f \} +
\]

\[
- [\mathbf{n}_s \cdot (\mathbf{T}_{w,f} - \mathbf{T}_{w,g})] - \mathbf{n}_s \cdot \mathbf{p}_g \}dA
\]

(8)

Being the integration domain arbitrary, after little manipulation the local equation for interface equilibrium is obtained as

\[
m''(\mathbf{v}_f - \mathbf{v}_g) = \mathbf{n}(\mathbf{p}_f - \mathbf{p}_g) + \mathbf{n} (\mathbf{T}_{w,f} - \mathbf{T}_{w,g}) + \mathbf{n} \cdot (\mathbf{T}_{w,f} - \mathbf{T}_{w,g}) - \mathbf{v}_f \nabla \sigma - 2\sigma k \mathbf{n} K
\]

(9)

where the terms represent (from left to right) the recoil force, the pressure difference, the viscous stress, the electric stress, the Marangoni force and the capillary force and \( m'' \) is the...
mass velocity. It is interesting to consider the components of Eq.(9) in the absence of mass flow and in the direction normal and tangent to the interface, \( \mathbf{n} \) and \( \mathbf{t} \), respectively

\[
0 = (\tau_{n,f} - \tau_{n,g}) + (\tau_{t,f} - \tau_{t,g}) - \frac{d\sigma}{dm}
\]

(10)

\[
0 = -(\rho_f - \rho_g) - (\tau_{n,f} - \tau_{n,g}) - (\tau_{t,f} - \tau_{t,g}) - 2\sigma K
\]

(11)

It is therefore clear that the tangential and normal electric stresses contribute to determine the shape of the interface. In particular, the role of the electrostatic pressure has to be included even if the fluid is considered incompressible. It can be shown that in the absence of free charge, the tangential electric stress vanishes [9]. The additional pressure difference due to the electric field application is given by

\[
\Delta p_{el} = \rho_f - \rho_g = n \left( \mathbf{T}_{el} - \mathbf{T}_{el} \right) \cdot \mathbf{n}
\]

(12)

and it is worth remarking that the electric field plays a role in modifying this term too.

For non-conducting fluids with no interface charge, having considered Eq.(4) and boundary conditions, Eq.(12) can be reformulated as [7], [9]

\[
\Delta p_{el} = \frac{\varepsilon_0}{2} \left[ \varepsilon_{R,f} (1 - \varepsilon_{R,f}) E_{R,f}^2 + \varepsilon_{R,f} (1 - \varepsilon_{R,f}) E_{R,f}^2 + \rho_f \left( \frac{\varepsilon_{R,f}}{\varepsilon_{R,f}} \right) \right]
\]

(13)

**Determination of the shape of a bubble**

Equation (11) represents a modified version of Laplace-Young equation, extended to include the electric stress

\[
(p_g - p_f) + \Delta p_{el} = 2\sigma K
\]

(14)

On the other hand, in static conditions, both the internal and external pressure are related to the vertical coordinate by the Stevin equation

\[
p_g = p_{0,g} + \rho_g g y, \quad p_f = p_{0,f} + \rho_f g y
\]

(15)

where \( p_{0,g} \) is the pressure at the bubble apex in the two phases. Of course \( p_{0,g} \) and \( p_{0,f} \) are again linked by the extended Laplace-Young equation

\[
p_{0,g} - p_{0,f} = \frac{2\sigma}{R_0} - \Delta p_{el,0}
\]

(16)

where the curvature radii at the apex are identical due to the axial symmetry. Combining Eqs. (14), (15) and (16) and eliminating the pressures, the capillary equation is finally obtained

\[
\sigma \left( \frac{1}{R_1} + \frac{1}{R_2} \right) = \frac{2\sigma}{R_0} + (\Delta p_{el} - \Delta p_{el,0}) - (\rho_f - \rho_g) g y
\]

(17)

In Eq.(17), the gravitational head and the electric stress are responsible of deviation of the bubble from spherical shape: in fact, in the absence of electric field, bubbles grow as perfect spheres in microgravity [13].

To model the quasi-static bubble growth, solutions of the capillary equation, complying with a boundary condition at the three phase line (as outlined in the introduction), are sought by varying the parameter \( R_0 \) in order to have an increase in volume: when the volume reaches a maximum and cannot be further increased, detachment is assumed to occur, according to [14].

**Overall momentum balance on a slowly growing bubble**

![Fig. 4 – Overall force balance on a bubble in quasi-static conditions.](image)

The overall momentum balance on a control volume surrounding the bubble, on the liquid side, for a growth with negligible inertial effects, is written as (see Fig.4)

\[
\int_B \rho g V dV + \int_C \left( \mathbf{T}_{el} - \mathbf{T}_{el} \right) \cdot \mathbf{n} dS + \int_C \sigma dL = 0
\]

(18)

Where, in particular, \( \mathbf{T}_{el} \) is the Maxwell stress tensor on the liquid side, and \( \mathbf{n} \) and \( \mathbf{t} \) are the normal outward and tangent unit vectors, respectively.

Integration of the \( \mathbf{z} \)-component, neglecting viscous terms, gives

\[
F_b + F_p = F_a + F_e
\]

(19)

where the buoyancy, internal overpressure, surface tension and electric forces are given by

\[
F_b = V (\rho_f - \rho_g) g
\]

\[
F_p = \frac{\pi D^2}{4} (\rho_f - \rho_g) g H
\]

\[
F_a = \pi D_a \sigma \sin \alpha_0
\]

\[
F_e = \int j \cdot \mathbf{T}_{el} \cdot \mathbf{n} dS
\]

Eq.(19) shows that, during quasi-static growth, bubble equilibrium is determined by buoyancy, excess of internal pressure, surface tension at the bubble neck, and electric force. This balance must hold at any stage of bubble growth, by means of an adjustment of the incidence angle \( \alpha_0 \) at the orifice rim, and the bubble detachment occurs when it is no longer possible to fulfill it further to a volume increase. Since \( F_a \) must be maximum in this condition, the incidence angle at detachment is 90°.

**Balance of forces in the absence of electric field**

To demonstrate the force equilibrium, integration of capillary equation has been performed in the absence of electric field, following the numerical scheme originally proposed by Pitt [15] and already adopted in a former paper [4]. The process of bubble growth is illustrated in Fig.5.

In Figs.6-8 the curvature radius at the bubble top, the incidence angle and the value of the forces are plotted versus the bubble volume, normalized to the Tate’s value.

\[
V_T = \frac{\pi D_a \sigma}{(\rho_f - \rho_g) g}
\]

(21)

It can be easily checked that \( V_T \) is the result of the balance of the buoyancy forces versus the surface tension one, with an incidence angle of 90°.
The analysis of Figs. 6 and 7 shows that the curvature radius initially decreases down to the value corresponding to a semisphere with the same diameter of the orifice; at the same time, the incidence angle reaches 90° for the first time. Later on, the curvature radius decreases, in correspondence with bubble expansion; the incidence angle first decreases and then increases again, when the bubble neck starts to form. When the incidence angle attains 90° again, no more equilibrium solution is possible for the attached bubble, and detachment occurs.

The force balance (Fig 8) shows that in a first stage the growth is mainly controlled by the balance between internal overpressure and surface tension; later on, as the curvature radius decreases, the overpressure term loses importance and the force balance is ruled by the buoyancy force (increasing monotonically with volume) and surface tension. In fact, detachment occurs for a value of the volume very close to Tate’s one.

### Evaluation of the Electric Force on a Bubble

The evaluation of the electric force is not trivial due to the fact that the electric field configuration is continuously modified by the bubble growth. This makes also very difficult to perform a direct integration of Eq.(17) in the presence of the electric force.

The electric force has thus been evaluated a posteriori, using the experimental bubble profile to evaluate the electric field configuration, and hence the Maxwell stress tensor, with the aid of Comsol Multiphysics code. This software has the useful characteristic of performing direct integration of a pre-defined, tailor-made, expression along a boundary (the
liquid-gas interface, in the present case). In particular, for non-conducting fluid with no interface charge, the resulting electric force on the bubble can be expressed as [7]

\[ F_{e,y} = \int_S \varepsilon_0 \varepsilon_{f} \left( n \cdot \mathbf{E}_f \right) - \frac{E^2}{2} \left[ \frac{\partial \rho_f}{\partial r} \right]_T \ dv \]  (22)

Equations (13) and (22) have been directly implemented in Comsol code. In a first instance, bubbles of ellipsoidal shape were studied. The analysis showed that, for a given bubble volume, the resulting force \( F_{e,y} \) is strongly dependent on the aspect ratio, as shown in Fig.10.

In a second phase, the electric forces were evaluated using the actual bubble profile. To this aim, the bubble perimeter was digitized with the aid of the dedicated Matlab image processing routine and directly implemented in Comsol code. The code gives the electric field configuration (Fig.12) and the values of \( \Delta \rho_{el} \) and resulting vertical electric force, \( F_{e,y} \), on the bubble.

Fig.13 shows the value of the local electric overpressure, as given by Eq.(13), on a bubble in FC72 at 20 kV: it can be seen that \( \Delta \rho_{el} \) is positive at bubble equator and negative at poles: this contributes to bubble elongation in the direction of the electric field.

These results of force calculations are reported in Fig.14 for a bubble in FC72: in this case too, the forces add to a value very close to zero.

**Fig. 10 - Values of electric force \( F_{e,y} \) for ellipsoidal bubbles of the same volume and different aspect ratio (HFE7100).**

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These results of force calculations are reported in Fig.14 for a bubble in FC72: in this case too, the forces add to a value very close to zero.

**Fig. 11 - Image processing and implementation in Comsol: a) bubble image b) digitized profile (via dedicated software), c) creation of Comsol geometrical object.**

**Fig. 12 – Electric field intensity and isopotential lines around the bubble, as calculated by Comsol code.**

**Fig. 13 – Values of electric overpressure \( \Delta \rho_{el} \) for a bubble in FC72 at 20 kV.**

**Fig. 14 – Force balance on a bubble in FC72, in the presence of electric field (20 kV).**

**CONCLUSIONS**

The understanding of the evolution of bubbles in “adiabatic” (i.e. absence of mass transfer) conditions, is useful to open the way to the study of the more complex boiling phenomena. In this paper, an evaluation is carried out of the forces induced by an externally applied electric field on a gas bubble originating from a circular orifice drilled on a flat plate, and a comparison is made with experimental results. A simple experimental apparatus was built and operated to study quasi-static bubble detachment in fluids of different physical and electrical properties with the aid of high speed cinematography. An axisymmetric electric field could be generated above the orifice by imposing a voltage up to 20 kV.
dc to a second plate, laid parallel to the first one and with a hole of 4 mm drilled in it and coaxial with the orifice. The resulting geometry is simple and suitable for fast implementation in numerical codes. All the experimental data were obtained by postprocessing of high speed video images taken during the experiment, via a dedicated software implemented in Matlab.

The additional forces due to the application of electric field were derived starting from the Maxwell stress tensor, and their expressions, along with the bubble geometry derived from data processing, were implemented in Comsol Multiphysics code, that calculated the electric field configuration around the bubble and the value of the electric forces acting on it.

The results show a fairly good agreement, and allow for validating the electrical force calculations, that are strongly dependent on electric coupling condition at the interface and fluid properties, for future use in modeling boiling in the presence of electric field.

ACKNOWLEDGEMENTS

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REFERENCES


NOMENCLATURE

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