Corpi bidimensionali: corpi in cui due dimensioni prevalgono sulla terza (spessore).

Superficie media: luogo dei punti dello spazio che si trovano a metà dello spessore.

Classificazione in dipendenza di:

- forma della superficie media
- direzione rispetto dei carichi rispetto alla superficie media

Corpi bidimensionali: corpi in cui due dimensioni prevalgono sulla terza (spessore).
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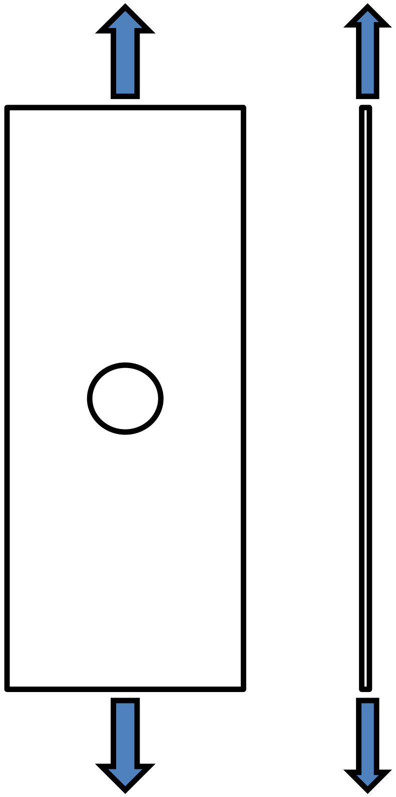
Classificazione in dipendenza di:

- forma della superficie media
- direzione rispetto dei carichi rispetto alla superficie media

Lastre:

- superficie media piana (***piano medio***)
- carichi applicati giacenti sul piano medio



Corpi bidimensionali: corpi in cui due dimensioni prevalgono sulla terza (spessore).

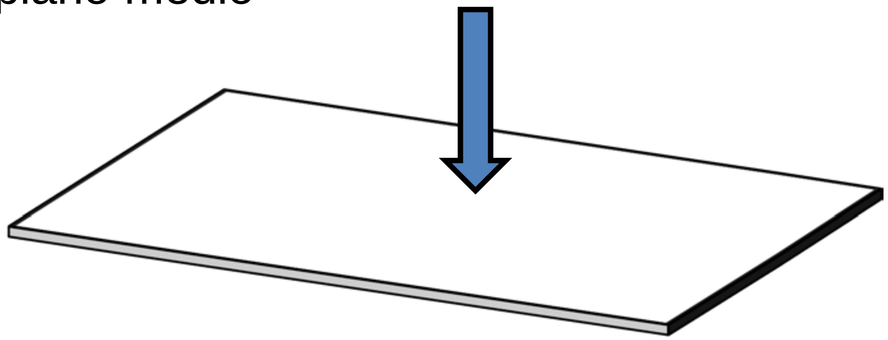
Superficie media: luogo dei punti dello spazio che si trovano a metà dello spessore.

Classificazione in dipendenza di:

- forma della superficie media
- direzione rispetto dei carichi rispetto alla superficie media

Piastre:

- superficie media piana (***piano medio***)
- carichi applicati ortogonalmente al piano medio



Corpi bidimensionali: corpi in cui due dimensioni prevalgono sulla terza (spessore).

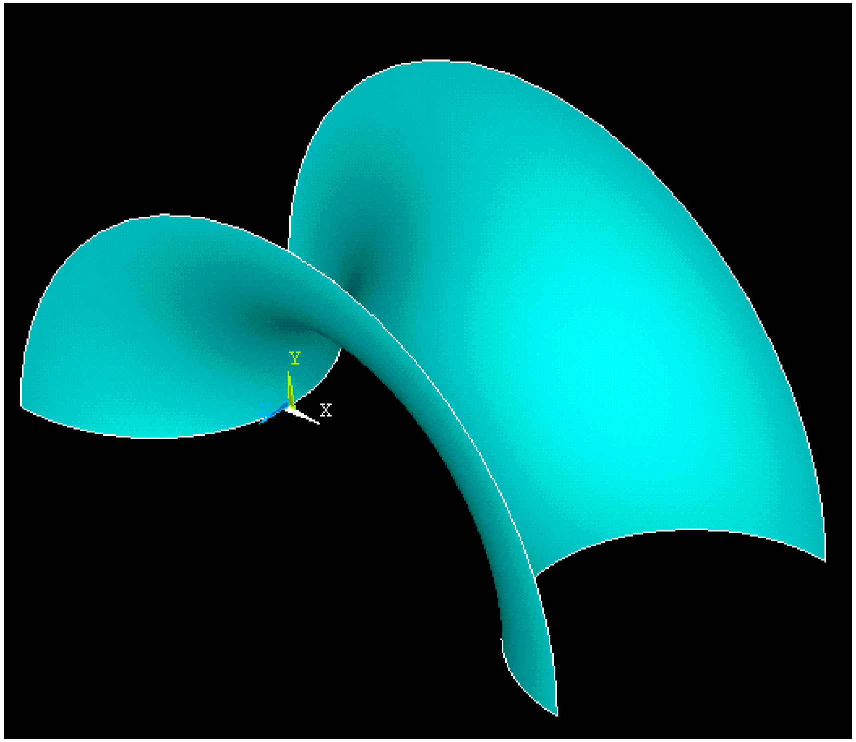
Superficie media: luogo dei punti dello spazio che si trovano a metà dello spessore.

Classificazione in dipendenza di:

- forma della superficie media
- direzione rispetto dei carichi rispetto alla superficie media

Gusci:

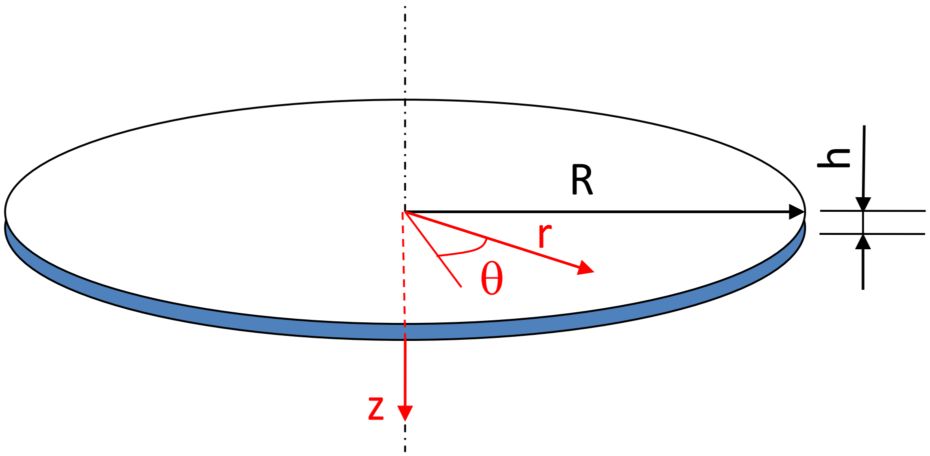
- superficie media curva
- carichi applicati sia sul piano medio che ortogonalmente ad esso



Piastre Circolari caricate in modo Assialsimmetrico.

Ipotesi semplificative generali per le piastre:

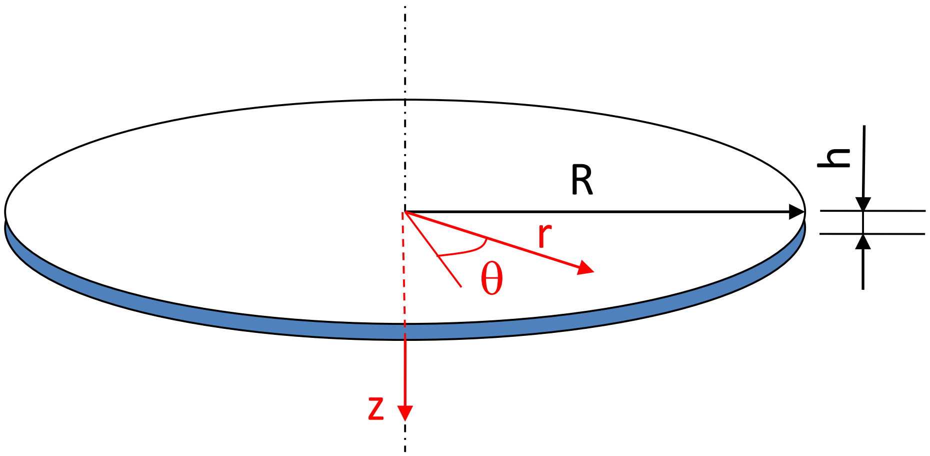
- spostamenti verticali del piano medio della piastra sotto carico molto minori dello spessore della piastra stessa
- punti dello spessore che, prima della deformazione, giacevano su di una retta ortogonale al piano medio, dopo la deformazion continuano a formare una retta ortogonale al piano medio deformato **(ipotesi di Kirchoff)**
- tensioni normali agenti ortogonalmente al piano medio della piastra trascurabili **(stato piano di tensione)**.



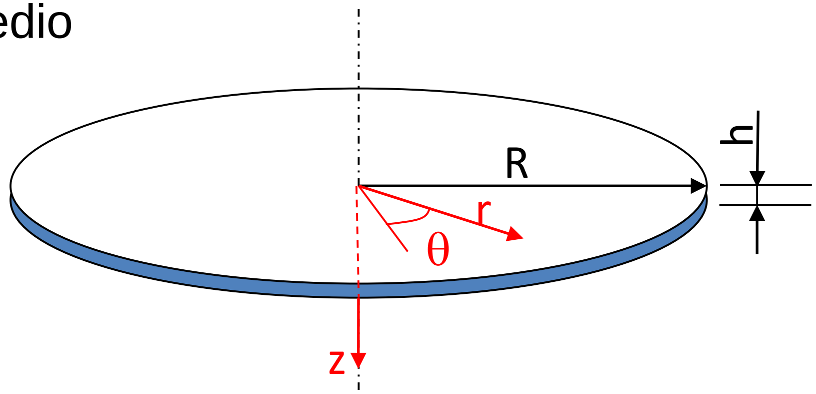
Piastre Circolari caricate in modo Assialsimmetrico.

Assunzioni semplificative specifiche:

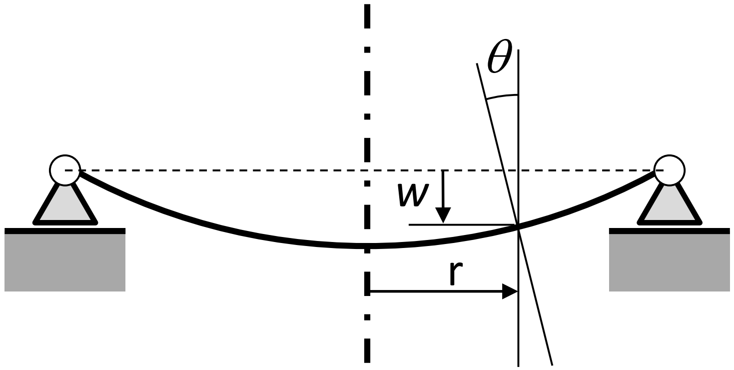
- la componente di spostamento in direzione circonferenziale è nulla per simmetria
- la componente di spostamento in senso radiale si assume trascurabile in base all'ipotesi di piccole deflessioni della piastra
- la componente di spostamento verticale w, la sola significativa, risulta, per simmetria, funzione della sola coordinata radiale, r.
- il piano medio non varia le sue dimensioni con la deformazione (esattamente come la fibra media di una trave inflessa)



Sistema di riferimento cilindrico con asse «z» coincidente con asse di simmetria ed origine sul piano medio

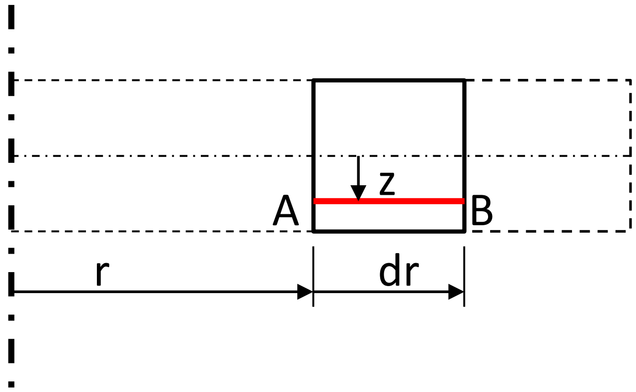


Deformata della piastra definita dalla sola funzione w(r).



$$\theta = -\frac{dw}{dr}$$

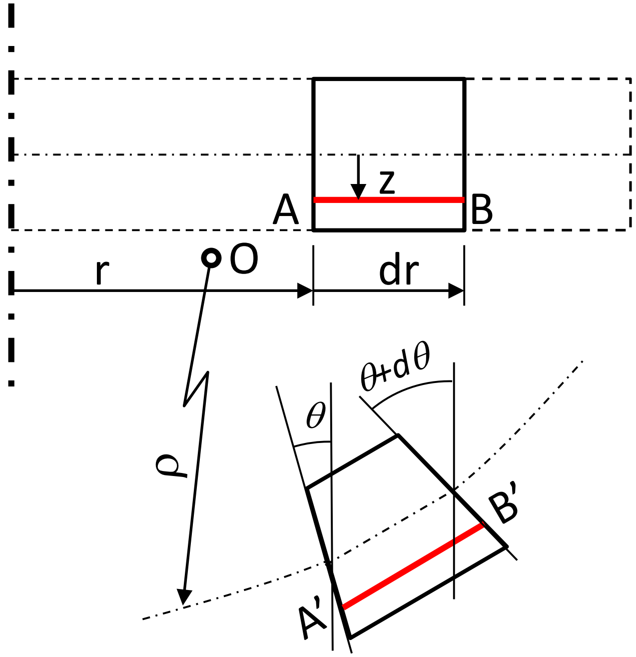
Componenti di deformazione/1



Elemento di piastra

Segmento AB alla quota «z»

Componenti di deformazione/2

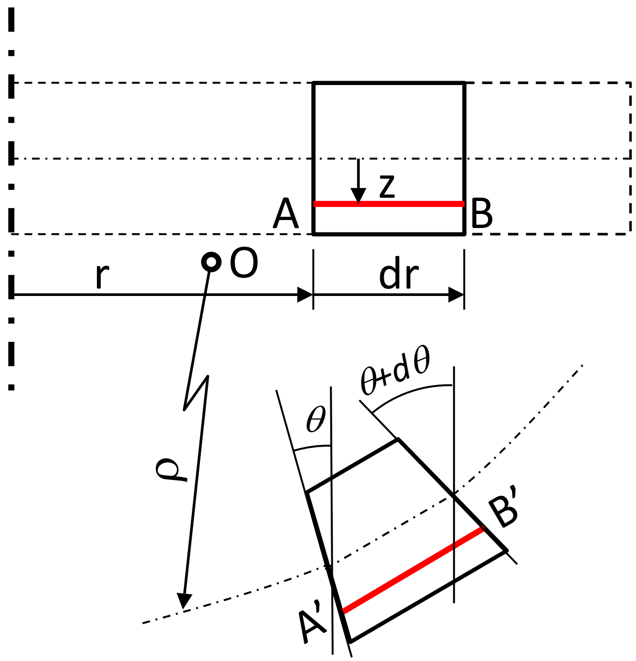


dopo l'inflessione:

- il segmento AB assume la posizione A'B'
- Il segmento dr appartenente al piano medio (lunghezza immutata) può essere approssimato con un arco di circonferenza di centro O e raggio:

$$\rho := \frac{dr}{d\theta}$$

Componenti di deformazione/3



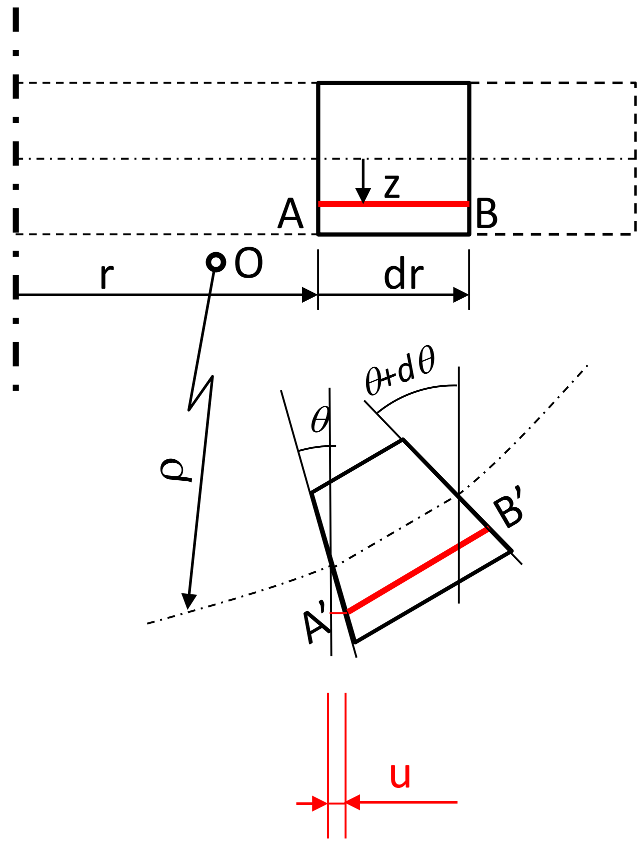
Lunghezza di AB:

$$\overline{A'}\,\overline{B'} = (\rho + z)d\theta = dr + z \cdot d\theta$$

Deformazione radiale:

$$\varepsilon_{rr} = \frac{(dr + z \cdot d\theta) - dr}{dr} = z \cdot \frac{d\theta}{dr}$$

Componenti di deformazione/4



Il punto A subisce uno spostamento radiale :

$$u \approx z \cdot \theta$$

La circonferenza per A, di lunghezza iniziale:

$$2\pi \cdot r$$

dopo la deformazione assume lunghezza:

$$2\pi \cdot (r+u)$$

Producendo una deformazione circonferenziale:

$$\varepsilon_{\theta\theta} = \frac{2\pi(r+z\cdot\theta) - 2\pi\cdot r}{2\pi\cdot r} = \frac{z\theta}{r}$$

Componenti di deformazione/4

Stato piano di tensione:

$$\varepsilon_{rr} = \frac{\sigma_{rr}}{E} - \nu \frac{\sigma_{\theta\theta}}{E}$$

$$\varepsilon_{\theta\theta} = \frac{\sigma_{\theta\theta}}{E} - \nu \frac{\sigma_{rr}}{E}$$

Invertendo:

$$\sigma_{rr} = \frac{E}{1-\nu^2}\left(\varepsilon_{rr} + \nu\varepsilon_{\theta\theta}\right)$$

$$\sigma_{\theta\theta} = \frac{E}{1-\nu^2}\left(\varepsilon_{\theta\theta} + \nu\varepsilon_{rr}\right)$$

Relazioni costitutive

Stato piano di tensione:

$$\varepsilon_{rr} = \frac{\sigma_{rr}}{E} - \nu \frac{\sigma_{\theta\theta}}{E}$$

$$\varepsilon_{\theta\theta} = \frac{\sigma_{\theta\theta}}{E} - \nu \frac{\sigma_{rr}}{E}$$

Invertendo:

$$\sigma_{rr} = \frac{E}{1-\nu^2}\left(\varepsilon_{rr} + \nu\varepsilon_{\theta\theta}\right)$$

$$\sigma_{\theta\theta} = \frac{E}{1-\nu^2}\left(\varepsilon_{\theta\theta} + \nu\varepsilon_{rr}\right)$$

$$\varepsilon_{rr} = z \cdot \frac{d\theta}{dr}$$

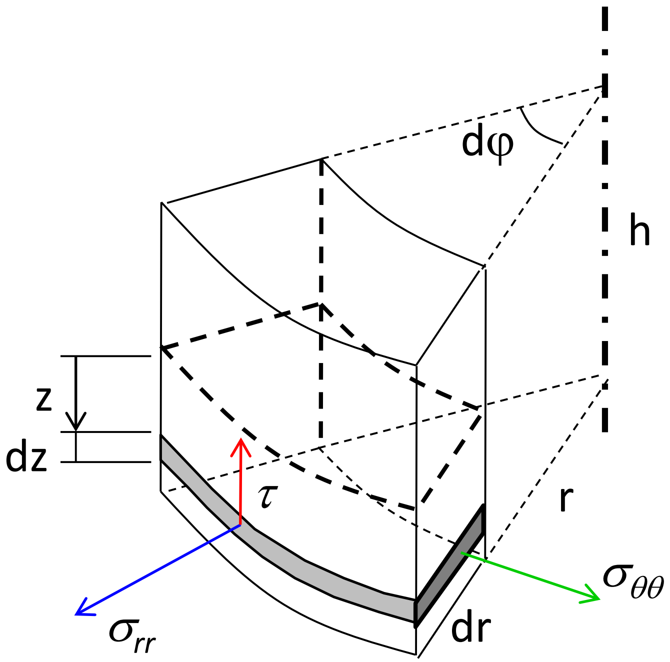
$$\varepsilon_{\theta\theta} = \frac{z\theta}{r}$$

$$\sigma_{rr} = \frac{E \cdot z}{1-\nu^2}\left(\frac{d\theta}{dr} + \nu \frac{\theta}{r}\right)$$

$$\sigma_{\theta\theta} = \frac{E \cdot z}{1-\nu^2}\left(\frac{\theta}{r} + \nu \frac{d\theta}{dr}\right)$$

Caratteristiche di sollecitazione generalizzate/1

Si considera un elemento di volume della piastra, ottenuto con tre incrementi delle coordinate.
Tensioni agenti (simmetria ed ipotesi di «plane stress»):

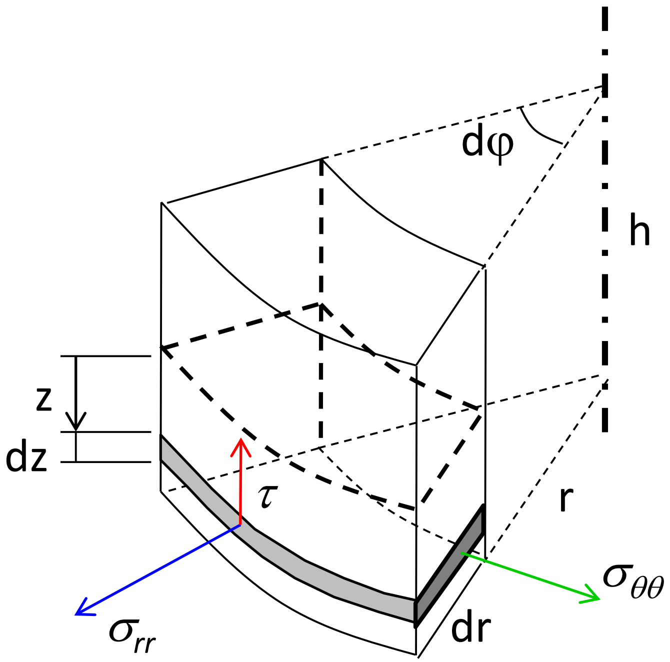


Caratteristiche di sollecitazione generalizzate/2

Dato che le tensioni normali dipendono linearmente da «z», per le relative risultanti si ha:

$$N_{rr} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_{rr} \cdot dz = 0$$

$$N_{\theta\theta} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_{\theta\theta} \cdot dz = 0$$



Caratteristiche di sollecitazione generalizzate/3

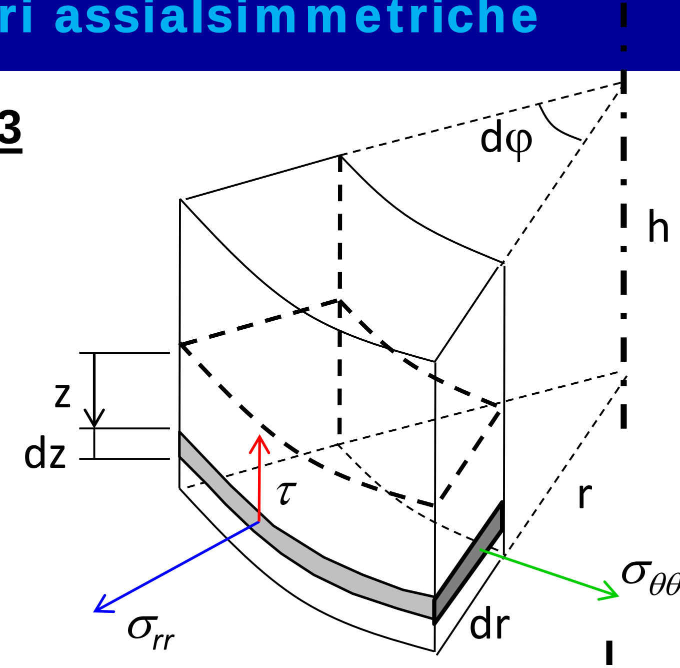
Per i momenti risultanti, invece si ottiene:

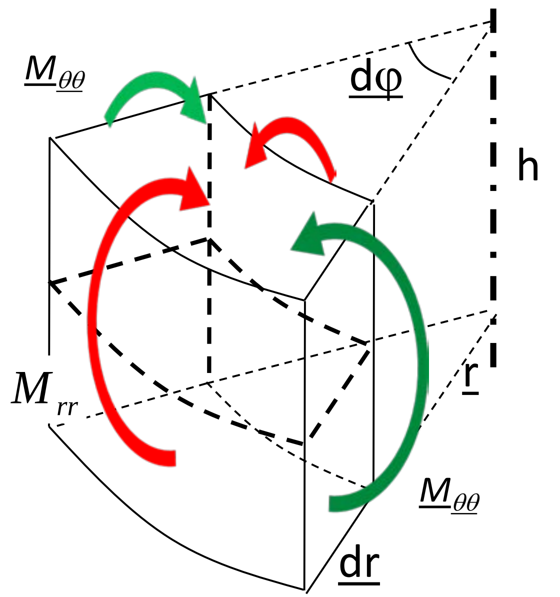
$$M_{rr} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_{rr} \cdot z \cdot dz = \frac{E \cdot z}{1-\nu^2}\left(\frac{d\theta}{dr} + \nu \frac{\theta}{r}\right)\int_{-\frac{h}{2}}^{\frac{h}{2}} z^2 \cdot dz =$$

$$= \frac{E}{1-\nu^2}\left(\frac{d\theta}{dr} + \nu \frac{\theta}{r}\right)\frac{h^3}{12} = D\left(\frac{d\theta}{dr} + \nu \frac{\theta}{r}\right)$$

$$D = \frac{E \cdot h^3}{12\left(1-\nu^2\right)}$$

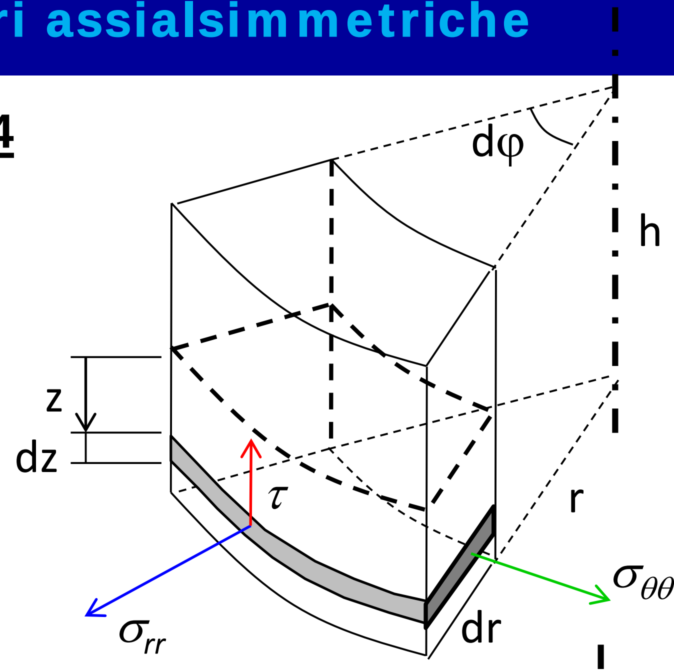
Rigidezza flessionale della piastra





Caratteristiche di sollecitazione generalizzate/4

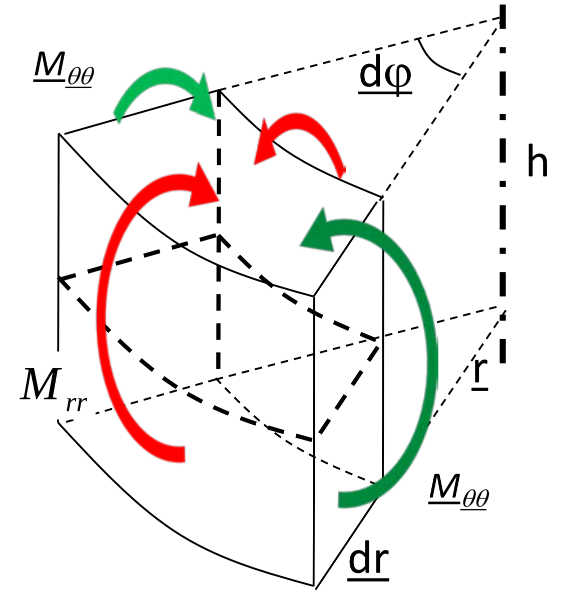
Per i momenti risultanti, invece si ottiene:



$$M_{rr} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_{rr} \cdot z \cdot dz = \frac{E \cdot z}{1-\nu^2}\left(\frac{d\theta}{dr} + \nu \frac{\theta}{r}\right)\int_{-\frac{h}{2}}^{\frac{h}{2}} z^2 \cdot dz =$$

$$= \frac{E}{1-\nu^2}\left(\frac{d\theta}{dr} + \nu \frac{\theta}{r}\right)\frac{h^3}{12} = D\left(\frac{d\theta}{dr} + \nu \frac{\theta}{r}\right)$$

$$D = \frac{E \cdot h^3}{12\left(1-\nu^2\right)}$$ **Rigidezza flessionale della piastra**

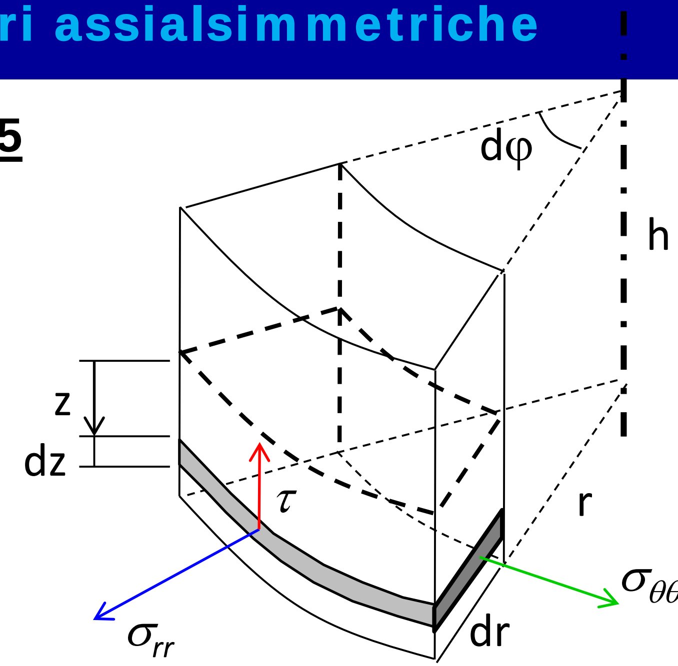


$$M_{\theta\theta} = D\left(\frac{\theta}{r} + \nu \frac{d\theta}{dr}\right)$$

Caratteristiche di sollecitazione generalizzate/5

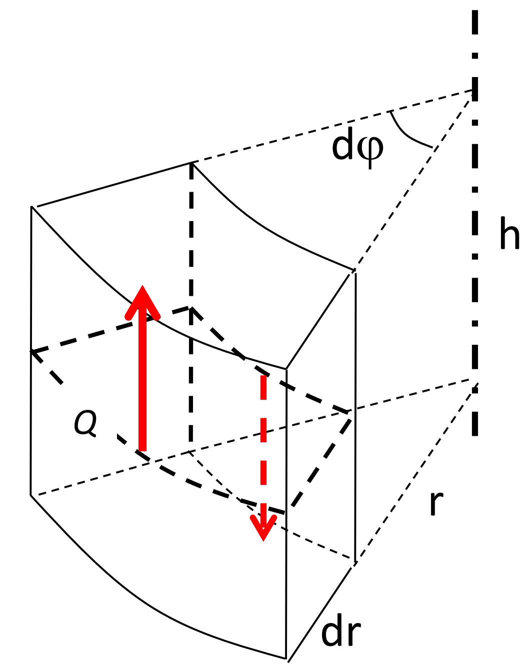
Si pone infine:

$$Q = \int_{-\frac{h}{2}}^{\frac{h}{2}} \tau \cdot dz$$



Osservazioni

- i momenti sono denominati in base alle tensioni che producono, invece che all'asse cui si riferiscono
- i momenti M_{rr} ed $M_{\theta\theta}$ e la forza di taglio Q sono calcolati per unità di lunghezza in direzione circonferenziale; essi si misurano rispettivamente in N*m/m ed in N/m e sono detti ***Caratteristiche di sollecitazione generalizzate.***



Relazione tra tensioni normali e momenti/1

$$M_{rr} = D\left(\frac{d\theta}{dr} + \nu\frac{\theta}{r}\right)$$

$$M_{\theta\theta} = D\left(\frac{\theta}{r} + \nu\frac{d\theta}{dr}\right)$$

$$\sigma_{rr} = \frac{E \cdot z}{1-\nu^2}\left(\frac{d\theta}{dr} + \nu\frac{\theta}{r}\right)$$

$$\sigma_{\theta\theta} = \frac{E \cdot z}{1-\nu^2}\left(\frac{\theta}{r} + \nu\frac{d\theta}{dr}\right)$$

Relazione tra tensioni normali e momenti/2

$$M_{rr} = D\left(\frac{d\theta}{dr} + \nu\frac{\theta}{r}\right)$$

$$M_{\theta\theta} = D\left(\frac{\theta}{r} + \nu\frac{d\theta}{dr}\right)$$

$$\sigma_{rr} = \frac{E\cdot z}{1-\nu^2}\left(\frac{d\theta}{dr} + \nu\frac{\theta}{r}\right)$$

$$\sigma_{\theta\theta} = \frac{E\cdot z}{1-\nu^2}\left(\frac{\theta}{r} + \nu\frac{d\theta}{dr}\right)$$

$$\sigma_{rr} = \frac{E\cdot z}{1-\nu^2}\frac{M_{rr}}{D}$$

$$\sigma_{\theta\theta} = \frac{E\cdot z}{1-\nu^2}\frac{M_{\theta\theta}}{D}$$

Relazione tra tensioni normali e momenti/3

$$M_{rr} = D\left(\frac{d\theta}{dr} + \nu\frac{\theta}{r}\right)$$

$$M_{\theta\theta} = D\left(\frac{\theta}{r} + \nu\frac{d\theta}{dr}\right)$$

$$\sigma_{rr} = \frac{E\cdot z}{1-\nu^2}\left(\frac{d\theta}{dr} + \nu\frac{\theta}{r}\right)$$

$$\sigma_{\theta\theta} = \frac{E\cdot z}{1-\nu^2}\left(\frac{\theta}{r} + \nu\frac{d\theta}{dr}\right)$$

$$\sigma_{rr} = \frac{E\cdot z}{1-\nu^2}\frac{M_{rr}}{D}$$

$$\sigma_{\theta\theta} = \frac{E\cdot z}{1-\nu^2}\frac{M_{\theta\theta}}{D}$$

$$D = \frac{E\cdot h^3}{12\left(1-\nu^2\right)}$$

Relazione tra tensioni normali e momenti/4

$$M_{rr} = D\left(\frac{d\theta}{dr} + \nu\frac{\theta}{r}\right)$$

$$M_{\theta\theta} = D\left(\frac{\theta}{r} + \nu\frac{d\theta}{dr}\right)$$

$$\sigma_{rr} = \frac{E \cdot z}{1-\nu^2}\left(\frac{d\theta}{dr} + \nu\frac{\theta}{r}\right)$$

$$\sigma_{\theta\theta} = \frac{E \cdot z}{1-\nu^2}\left(\frac{\theta}{r} + \nu\frac{d\theta}{dr}\right)$$

$$\sigma_{rr} = \frac{E \cdot z}{1-\nu^2}\frac{M_{rr}}{D}$$

$$\sigma_{\theta\theta} = \frac{E \cdot z}{1-\nu^2}\frac{M_{\theta\theta}}{D}$$

$$D = \frac{E \cdot h^3}{12\left(1-\nu^2\right)}$$

$$\sigma_{rr} = \frac{12M_{rr}}{h^3}z$$

$$\sigma_{\theta\theta} = \frac{12M_{\theta\theta}}{h^3}z$$

Relazione tra tensioni normali e momenti/5

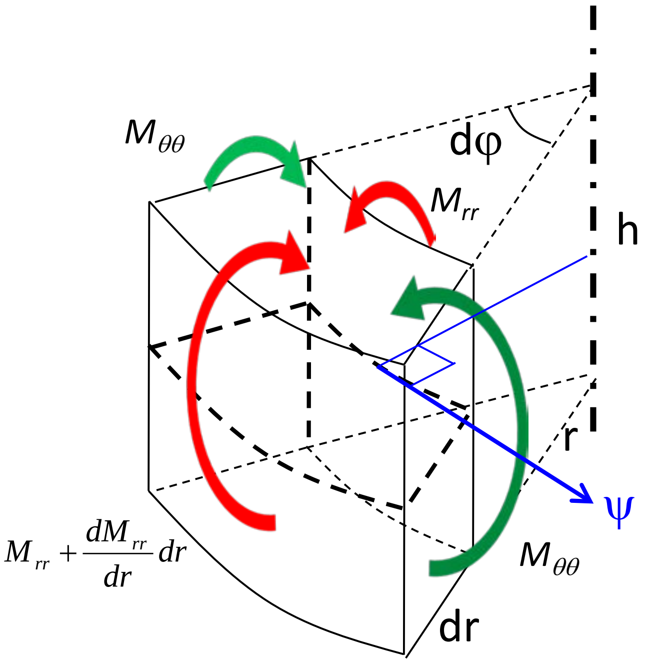
I valori massimi di tensione si verificano alle superfici inferiore e superiore dello spessore (z=±h/2). In valore assoluto i valori massimi di tensione sono :

$$\sigma_{rr\_\max} = \frac{6M_{rr}}{h^2}$$

$$\sigma_{\theta\theta\_\max} = \frac{6M_{\theta\theta}}{h^2}$$

Equazioni di equilibrio/1

Forze e momenti applicati all'elemento di volume:



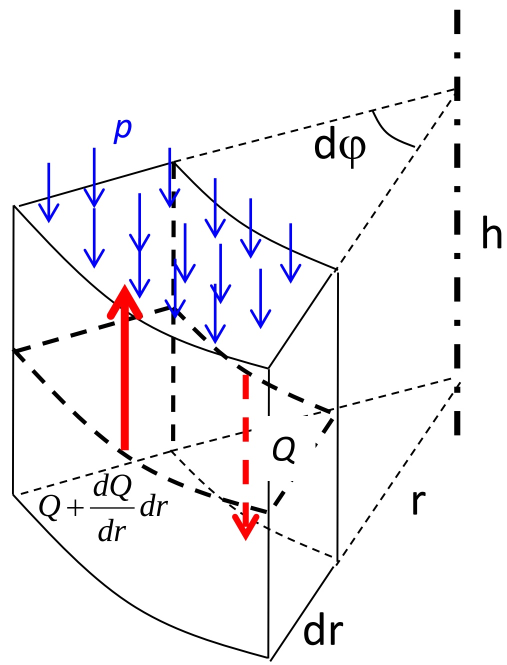
Equazioni di equilibrio identicamente soddisfatte:

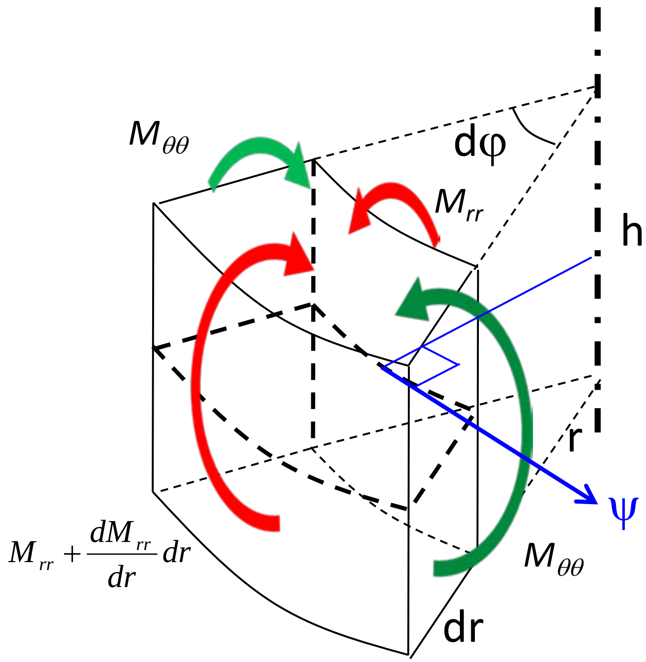
- Traslazione in direzione radiale e circonferenziale
- Rotazione attorno alla direzione radiale e assiale

Equazioni di equilibrio/2

Equilibrio in direzione «z» (assiale):

$$\left(Q+\frac{dQ}{dr}dr\right)(r+dr)d\varphi - Q\cdot r\cdot d\varphi - p\cdot r\cdot dr\cdot d\varphi = 0$$



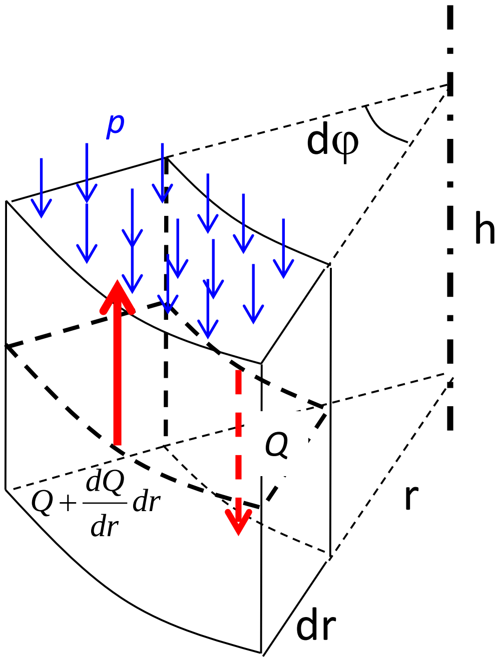


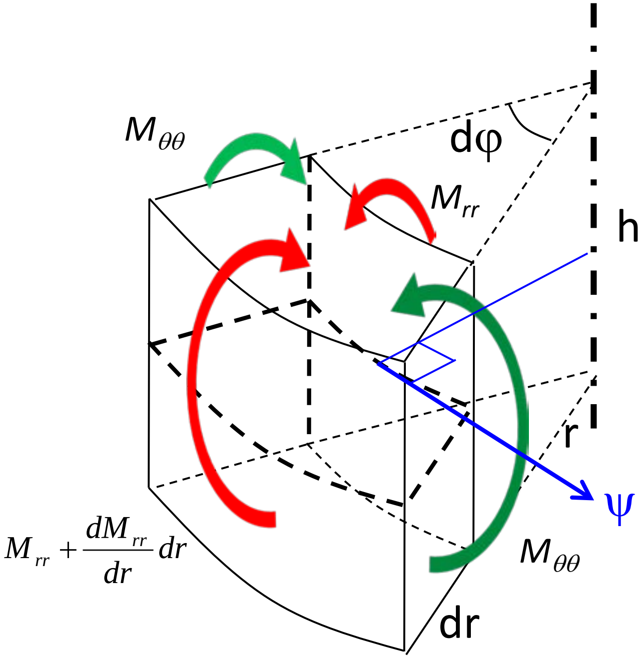
Equazioni di equilibrio/2

Equilibrio in direzione «z» (assiale):

$$\left(Q+\frac{dQ}{dr}dr\right)(r+dr)d\varphi-Q\cdot r\cdot d\varphi-p\cdot r\cdot dr\cdot d\varphi=0$$

$$Q\cdot r\cdot d\varphi+\frac{dQ}{dr}r\cdot dr\cdot d\varphi+Q\cdot dr\cdot d\varphi+\frac{dQ}{dr}dr^2\cdot d\varphi-Q\cdot r\cdot d\varphi-p\cdot r\cdot dr\cdot d\varphi=0$$





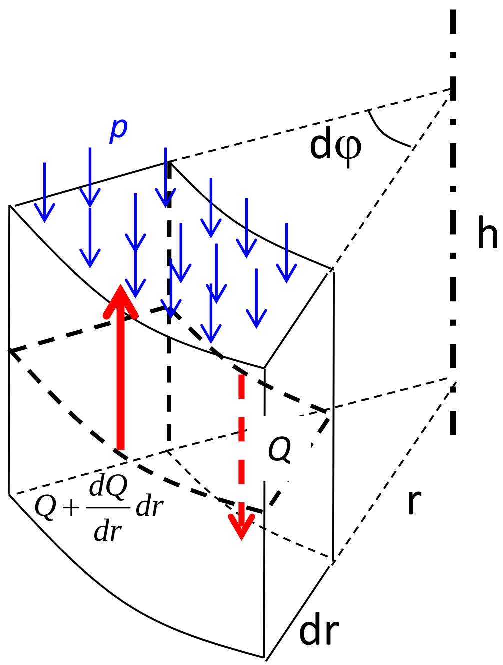
Equazioni di equilibrio/2

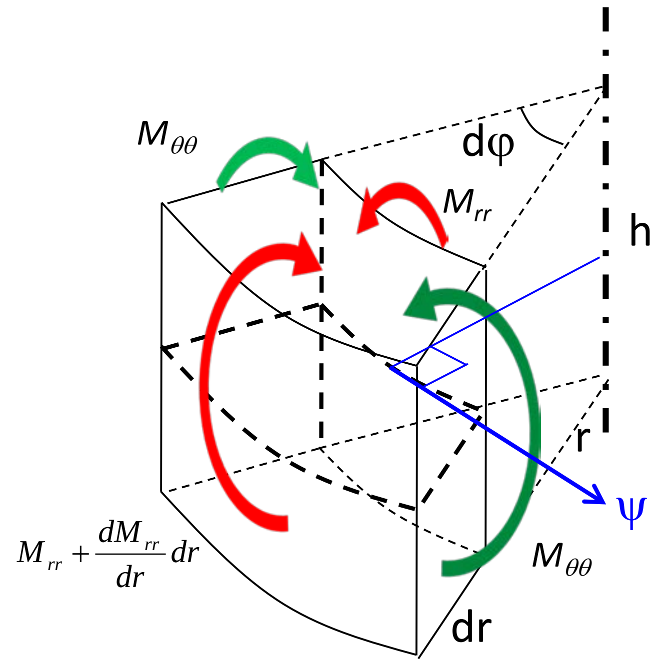
Equilibrio in direzione «z» (assiale):

Semplificando

$$\left(Q+\frac{dQ}{dr}dr\right)(r+dr)d\varphi - Q\cdot r\cdot d\varphi - p\cdot r\cdot dr\cdot d\varphi = 0$$

$$Q\cdot r\cdot d\varphi + \frac{dQ}{dr}r\cdot dr\cdot d\varphi + Q\cdot dr\cdot d\varphi + \frac{dQ}{dr}dr^2\cdot d\varphi - Q\cdot r\cdot d\varphi - p\cdot r\cdot dr\cdot d\varphi = 0$$





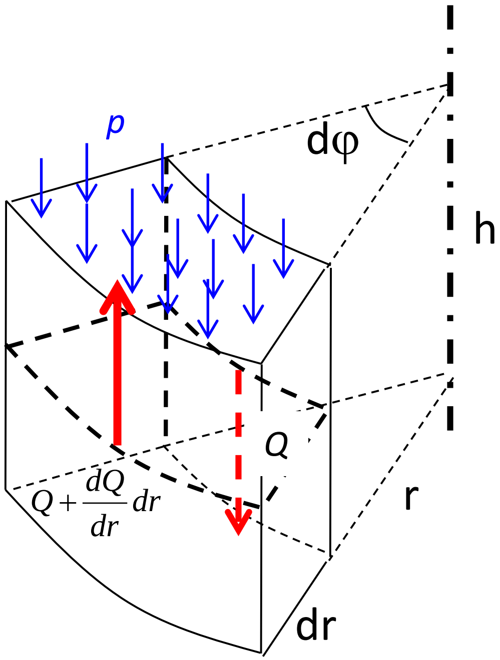
Equazioni di equilibrio/2

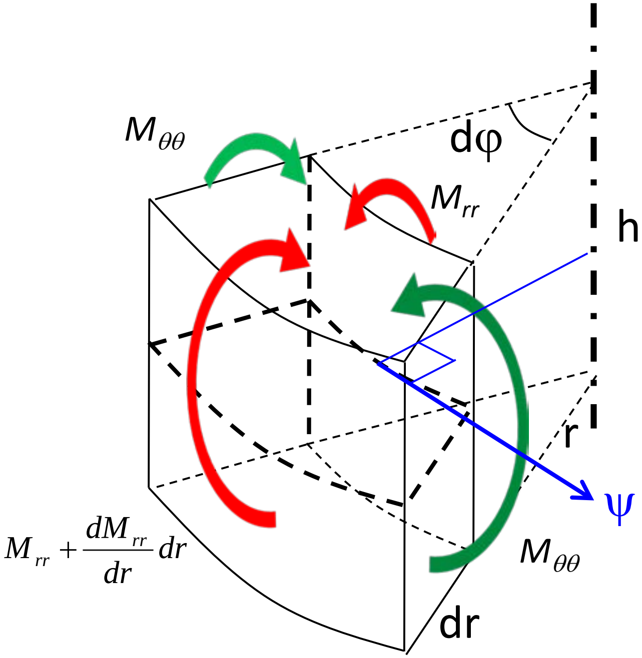
Equilibrio in direzione «z» (assiale):

$$\left(Q+\frac{dQ}{dr}dr\right)(r+dr)d\varphi-Q\cdot r\cdot d\varphi-p\cdot r\cdot dr\cdot d\varphi=0$$

$$\frac{dQ}{dr}r\cdot dr\cdot d\varphi+Q\cdot dr\cdot d\varphi+\frac{dQ}{dr}dr^2\cdot d\varphi-p\cdot r\cdot dr\cdot d\varphi=0$$

Trascurando i termini di ordine superiore





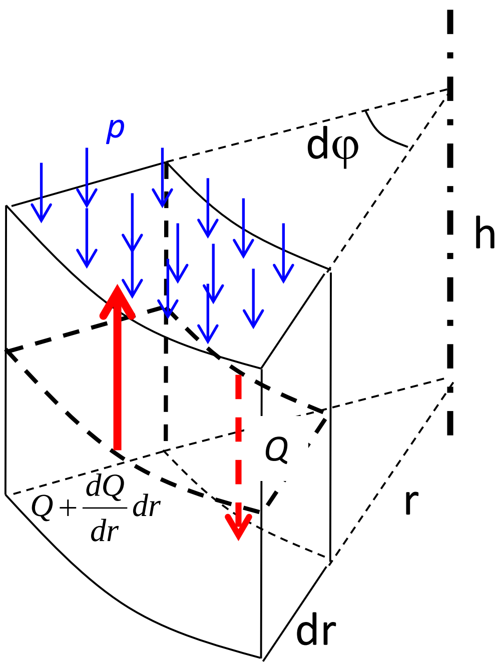
Equazioni di equilibrio/2

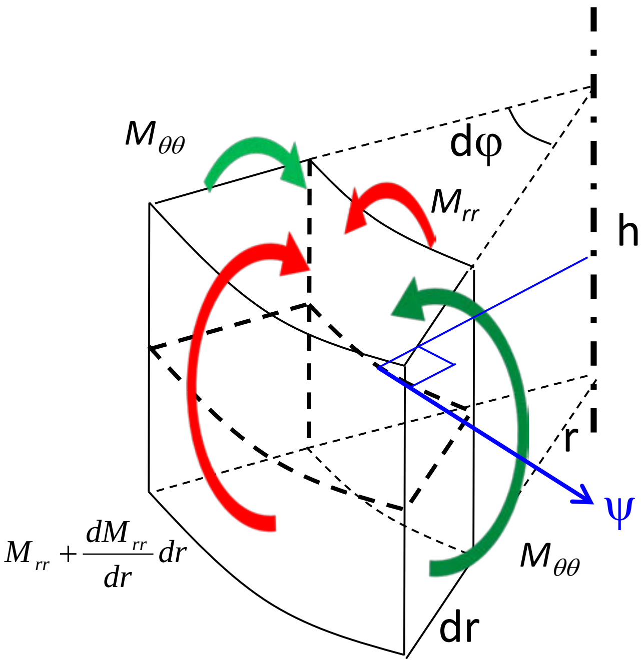
Equilibrio in direzione «z» (assiale):

$$\left(Q+\frac{dQ}{dr}dr\right)(r+dr)d\varphi-Q\cdot r\cdot d\varphi-p\cdot r\cdot dr\cdot d\varphi=0$$

Dividendo per il fattore comune

$$\frac{dQ}{dr}r\cdot dr\cdot d\varphi+Q\cdot dr\cdot d\varphi-p\cdot r\cdot dr\cdot d\varphi=0$$



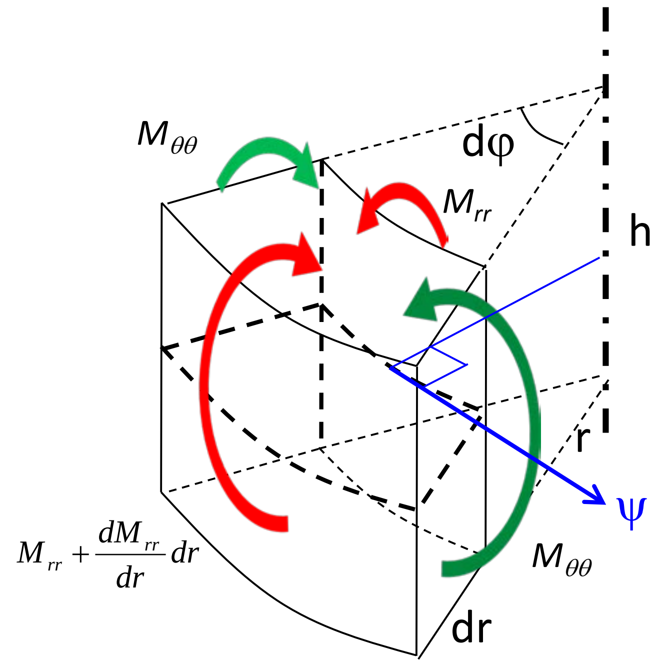


Equazioni di equilibrio/2

Equilibrio in direzione «z» (assiale):

$$\left(Q+\frac{dQ}{dr}dr\right)(r+dr)d\varphi-Q\cdot r\cdot d\varphi-p\cdot r\cdot dr\cdot d\varphi=0$$

$$\frac{dQ}{dr}\cdot r+Q-p\cdot r=0$$



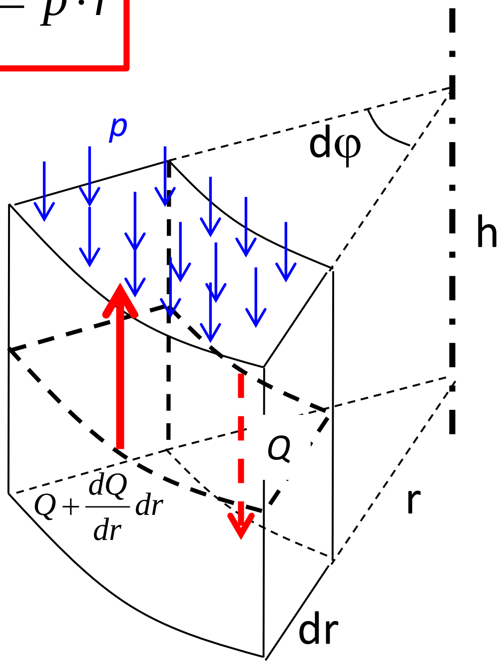
Equazioni di equilibrio/2

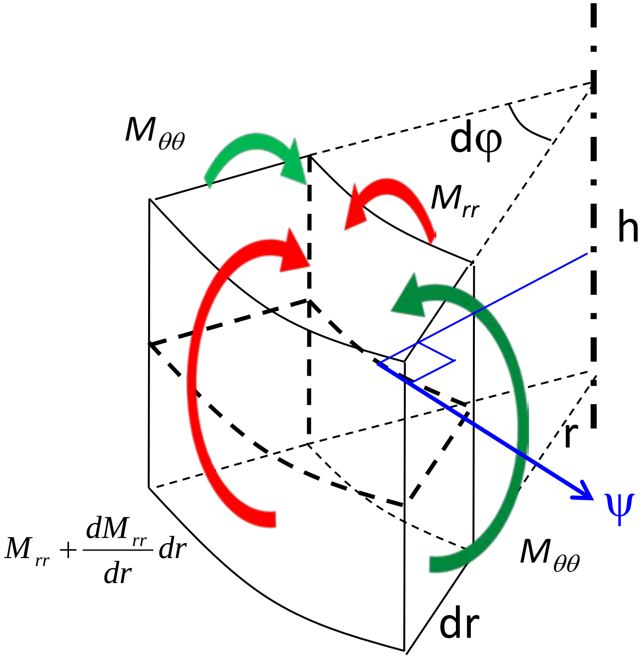
Equilibrio in direzione «z» (assiale):

$$\left(Q+\frac{dQ}{dr}dr\right)(r+dr)d\varphi-Q\cdot r\cdot d\varphi-p\cdot r\cdot dr\cdot d\varphi=0$$

$$\frac{dQ}{dr}\cdot r+Q-p\cdot r=0$$

$$\boxed{\frac{d(Q\cdot r)}{dr}=p\cdot r}$$

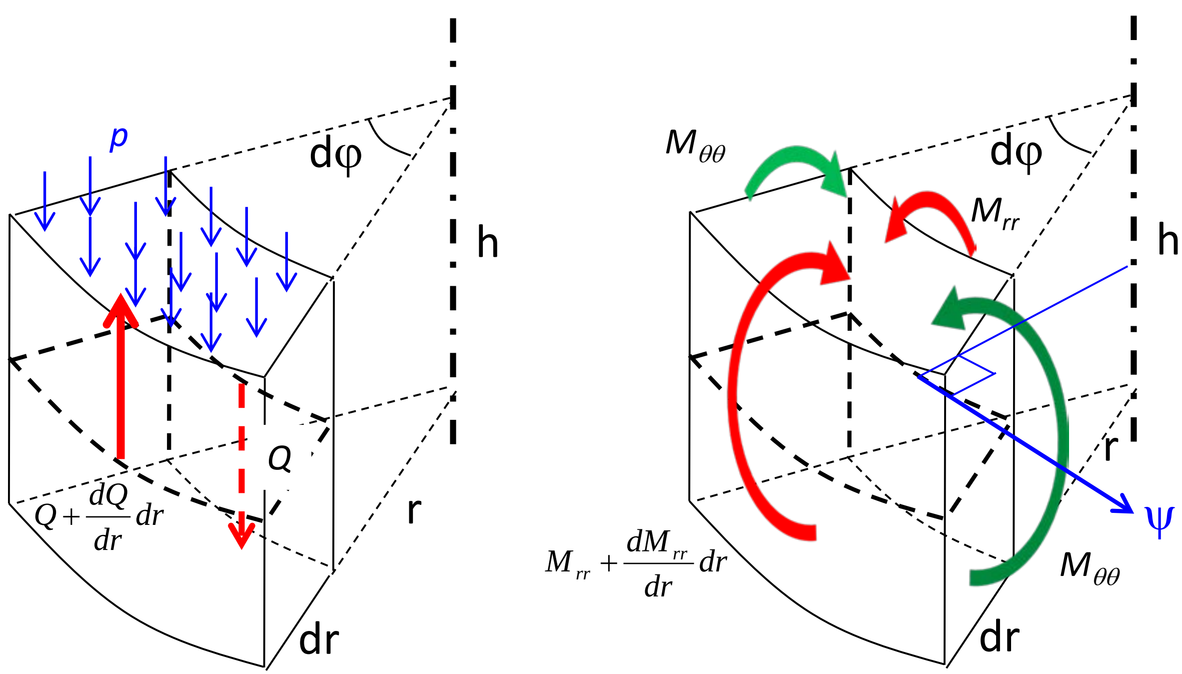




Equazioni di equilibrio/3

Rotazione attorno all’asse circonferenziale «φ»:

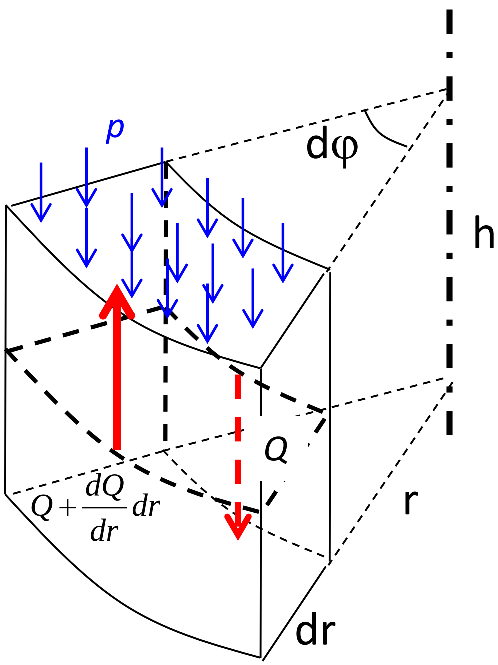
$$\left(M_{rr}+\frac{dM_{rr}}{dr}dr\right)(r+dr)d\varphi-M_{rr}\cdot r\cdot d\varphi+\left(Q+\frac{dQ}{dr}dr\right)(r+dr)d\varphi\cdot dr-p\cdot r\cdot\frac{dr^{2}}{2}\cdot d\varphi-M_{\theta\theta}\cdot dr\cdot d\varphi=0$$

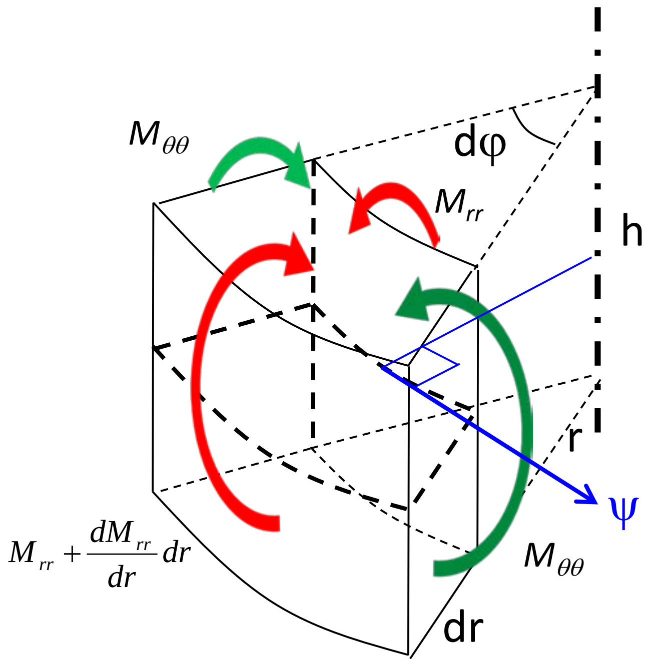


Equazioni di equilibrio/3

Rotazione attorno all'asse circonferenziale «φ»:

$$\left(M_{rr}+\frac{dM_{rr}}{dr}dr\right)(r+dr)d\varphi-M_{rr}\cdot r\cdot d\varphi+\left(Q+\frac{dQ}{dr}dr\right)(r+dr)d\varphi\cdot dr-p\cdot r\cdot\frac{dr^2}{2}\cdot d\varphi-M_{\theta\theta}\cdot dr\cdot d\varphi=0$$

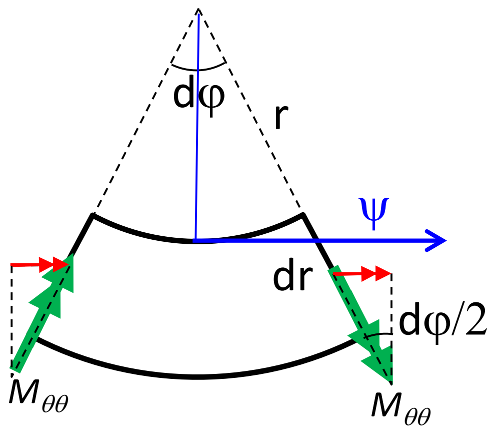




Equazioni di equilibrio/3

Rotazione attorno all'asse circonferenziale «φ»:

$$\left(M_{rr}+\frac{dM_{rr}}{dr}dr\right)(r+dr)d\varphi-M_{rr}\cdot r\cdot d\varphi+\left(Q+\frac{dQ}{dr}dr\right)(r+dr)d\varphi\cdot dr-p\cdot r\cdot\frac{dr^2}{2}\cdot d\varphi-M_{\theta\theta}\cdot dr\cdot d\varphi=0$$



$$2M_{\theta\theta}\cdot dr\cdot\sin\left(\frac{d\varphi}{2}\right)\approx M_{\theta\theta}\cdot dr\cdot d\varphi$$

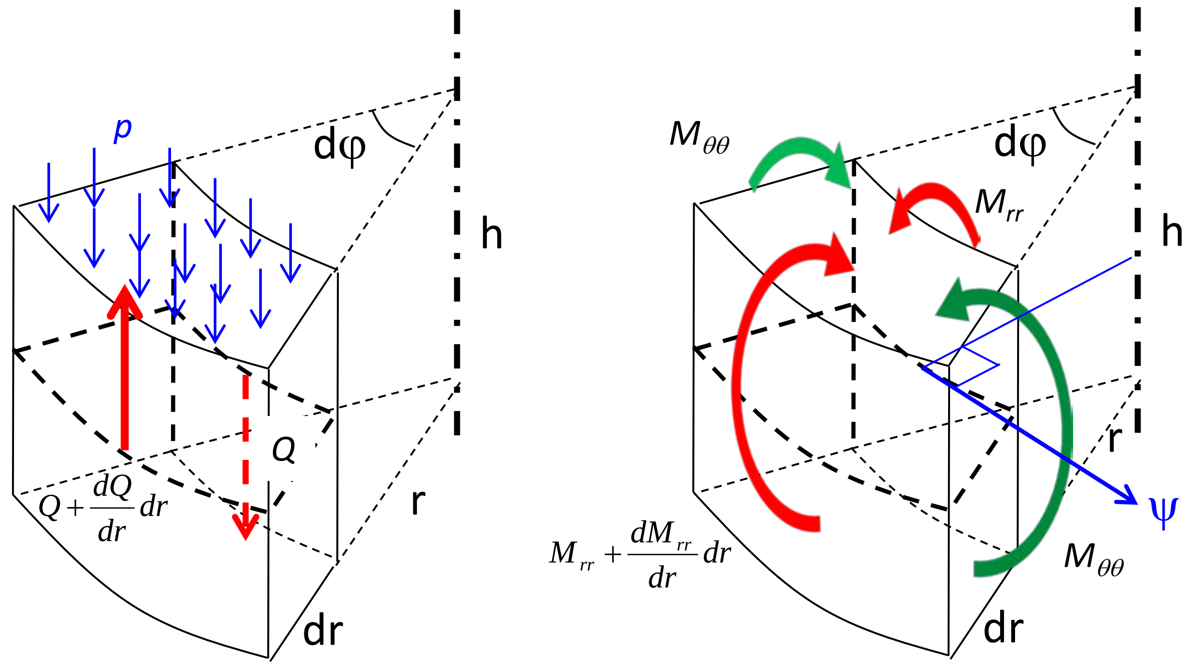
Equazioni di equilibrio/3

Rotazione attorno all’asse circonferenziale «φ»:

$$\left(M_{rr}+\frac{dM_{rr}}{dr}dr\right)(r+dr)d\varphi-M_{rr}\cdot r\cdot d\varphi+\left(Q+\frac{dQ}{dr}dr\right)(r+dr)d\varphi\cdot dr-p\cdot r\cdot\frac{dr^2}{2}\cdot d\varphi-M_{\theta\theta}\cdot dr\cdot d\varphi=0$$

$$M_{rr}\cdot r\cdot d\varphi+\frac{dM_{rr}}{dr}r\cdot dr\cdot d\varphi+M_{rr}\cdot dr\cdot d\varphi+\frac{dM_{rr}}{dr}dr^2\cdot d\varphi-M_{rr}\cdot r\cdot d\varphi+$$

$$Q\cdot r\cdot dr\cdot d\varphi+\frac{dQ}{dr}r\cdot dr^2\cdot d\varphi+Q\cdot dr^2\cdot d\varphi+\frac{dQ}{dr}dr^3\cdot d\varphi-p\cdot r\cdot\frac{dr^2}{2}\cdot d\varphi-M_{\theta\theta}\cdot dr\cdot d\varphi=0$$



Equazioni di equilibrio/3

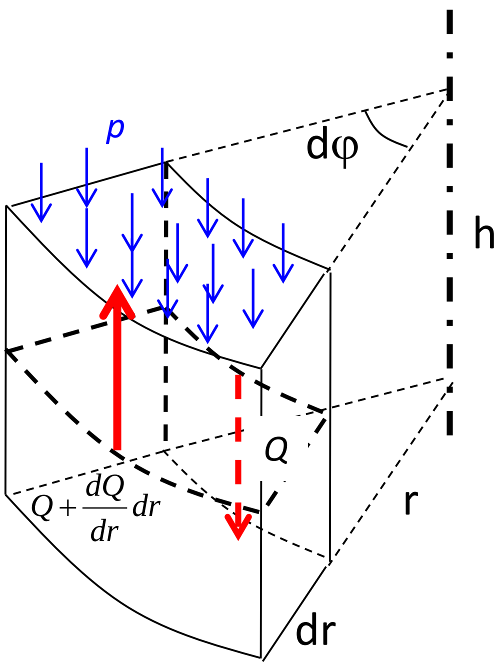
Rotazione attorno all'asse circonferenziale «φ»:

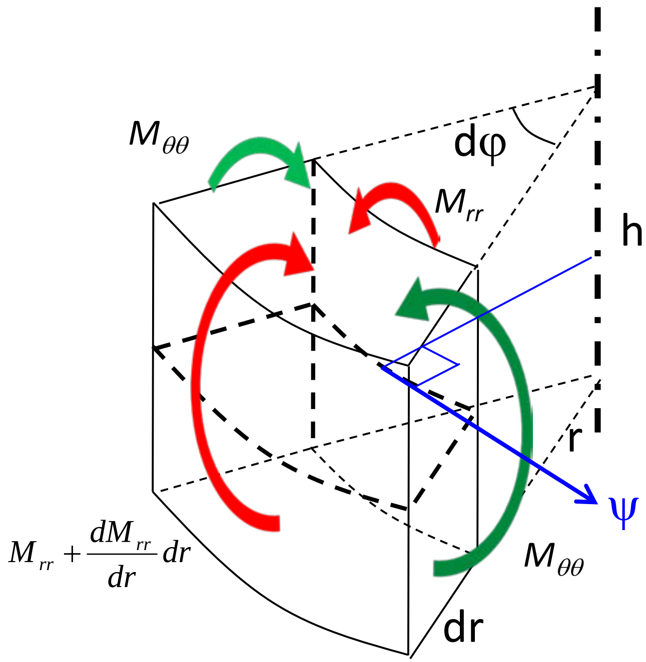
$$\left(M_{rr}+\frac{dM_{rr}}{dr}dr\right)(r+dr)d\varphi-M_{rr}\cdot r\cdot d\varphi+\left(Q+\frac{dQ}{dr}dr\right)(r+dr)d\varphi\cdot dr-p\cdot r\cdot\frac{dr^2}{2}\cdot d\varphi-M_{\theta\theta}\cdot dr\cdot d\varphi=0$$

$$M_{rr}\cdot r\cdot d\varphi+\frac{dM_{rr}}{dr}r\cdot dr\cdot d\varphi+M_{rr}\cdot dr\cdot d\varphi+\frac{dM_{rr}}{dr}dr^2\cdot d\varphi-M_{rr}\cdot r\cdot d\varphi+$$

Semplificando

$$Q\cdot r\cdot dr\cdot d\varphi+\frac{dQ}{dr}r\cdot dr^2\cdot d\varphi+Q\cdot dr^2\cdot d\varphi+\frac{dQ}{dr}dr^3\cdot d\varphi-p\cdot r\cdot\frac{dr^2}{2}\cdot d\varphi-M_{\theta\theta}\cdot dr\cdot d\varphi=0$$





Equazioni di equilibrio/3

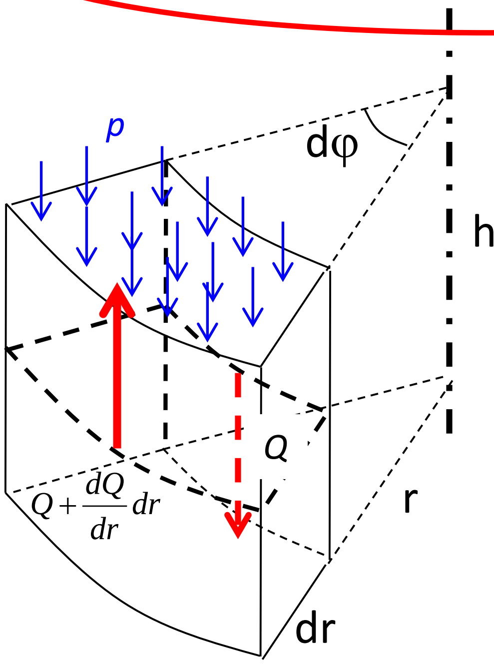
Rotazione attorno all'asse circonferenziale «φ»:

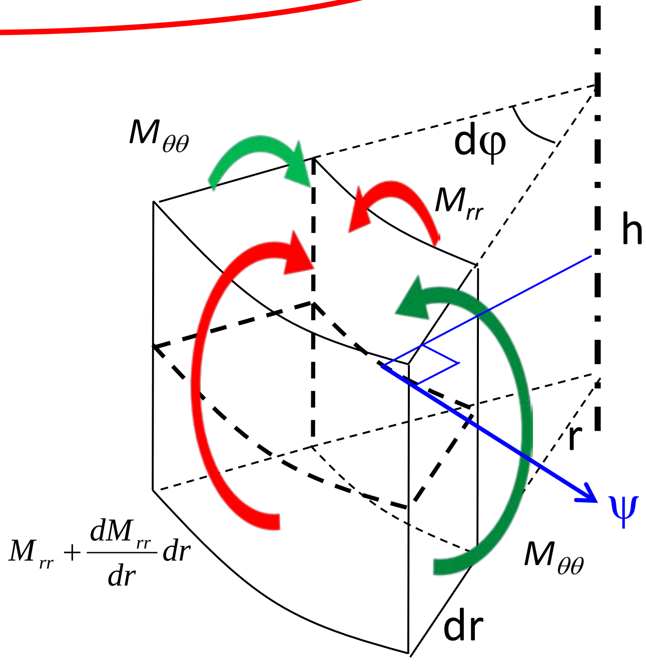
$$\left(M_{rr}+\frac{dM_{rr}}{dr}dr\right)(r+dr)d\varphi-M_{rr}\cdot r\cdot d\varphi+\left(Q+\frac{dQ}{dr}dr\right)(r+dr)d\varphi\cdot dr-p\cdot r\cdot\frac{dr^2}{2}\cdot d\varphi-M_{\theta\theta}\cdot dr\cdot d\varphi=0$$

$$\frac{dM_{rr}}{dr}r\cdot dr\cdot d\varphi+M_{rr}\cdot dr\cdot d\varphi+\frac{dM_{rr}}{dr}dr^2\cdot d\varphi+$$

$$Q\cdot r\cdot dr\cdot d\varphi+\frac{dQ}{dr}r\cdot dr^2\cdot d\varphi+Q\cdot dr^2\cdot d\varphi+\frac{dQ}{dr}dr^3\cdot d\varphi-p\cdot r\cdot\frac{dr^2}{2}\cdot d\varphi-M_{\theta\theta}\cdot dr\cdot d\varphi=0$$

Trascurando i termini di ordine superiore





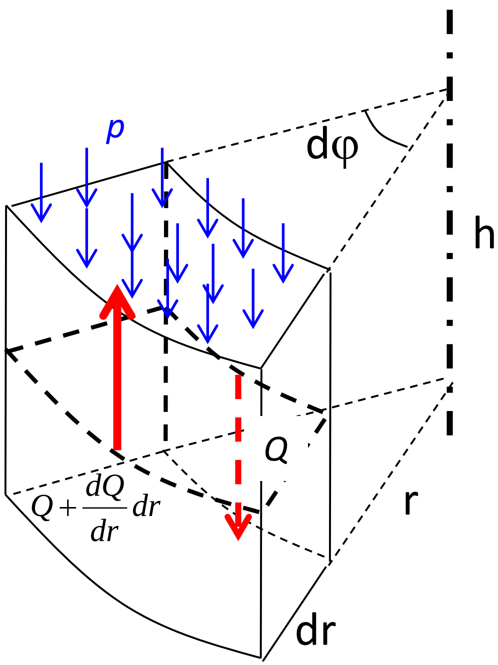
Equazioni di equilibrio/3

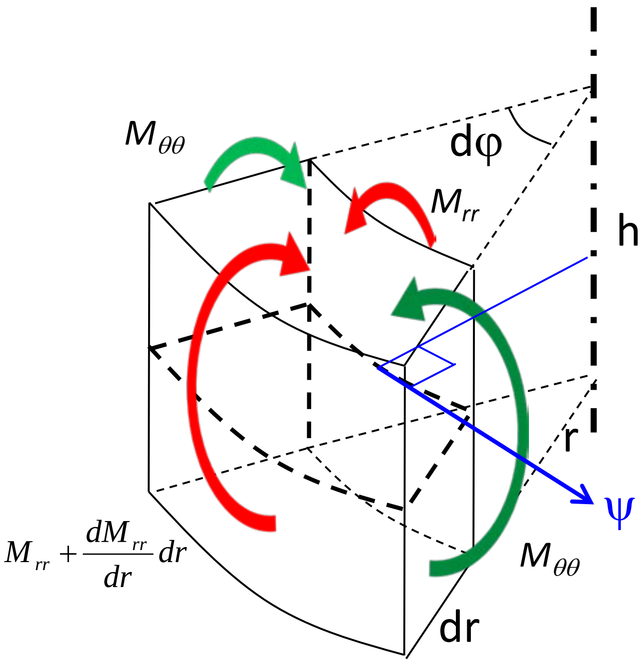
Rotazione attorno all'asse circonferenziale «φ»:

$$\left(M_{rr}+\frac{dM_{rr}}{dr}dr\right)(r+dr)d\varphi-M_{rr}\cdot r\cdot d\varphi+\left(Q+\frac{dQ}{dr}dr\right)(r+dr)d\varphi\cdot dr-p\cdot r\cdot\frac{dr^2}{2}\cdot d\varphi-M_{\theta\theta}\cdot dr\cdot d\varphi=0$$

Dividendo per il fattore comune

$$\frac{dM_{rr}}{dr}r\cdot dr\cdot d\varphi+M_{rr}\cdot dr\cdot d\varphi+Q\cdot r\cdot dr\cdot d\varphi-M_{\theta\theta}\cdot dr\cdot d\varphi=0$$



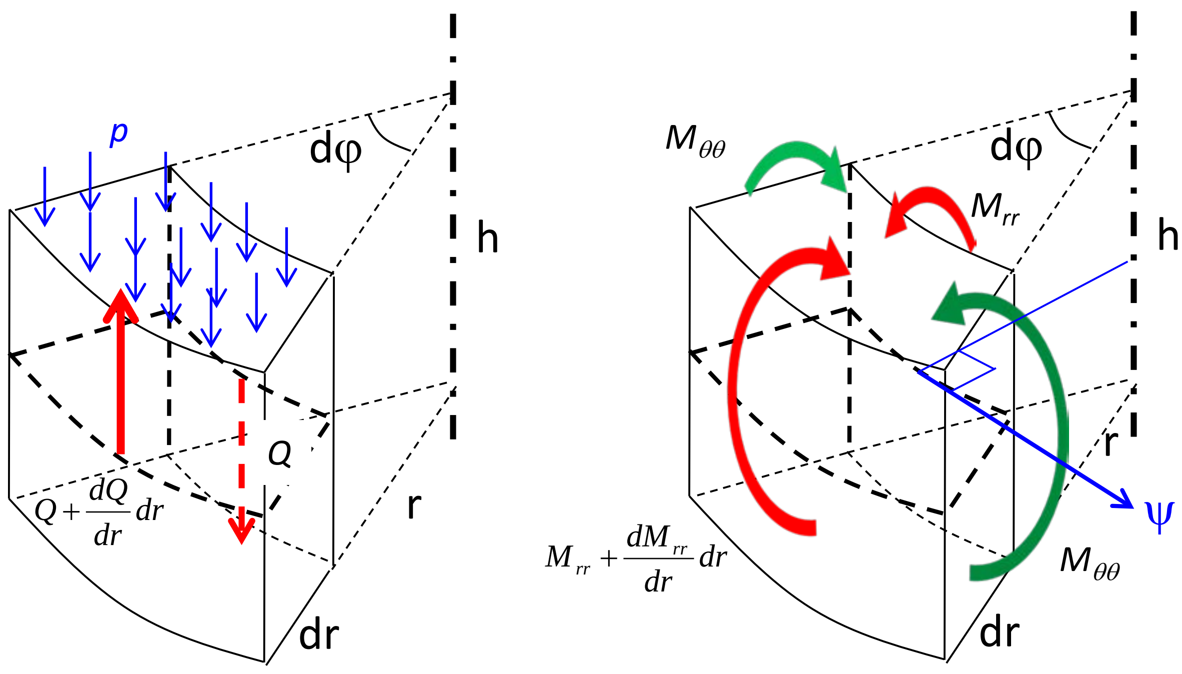


Equazioni di equilibrio/3

Rotazione attorno all’asse circonferenziale «φ»:

$$\left(M_{rr}+\frac{dM_{rr}}{dr}dr\right)(r+dr)d\varphi-M_{rr}\cdot r\cdot d\varphi+\left(Q+\frac{dQ}{dr}dr\right)(r+dr)d\varphi\cdot dr-p\cdot r\cdot\frac{dr^{2}}{2}\cdot d\varphi-M_{\theta\theta}\cdot dr\cdot d\varphi=0$$

$$\frac{dM_{rr}}{dr}r+M_{rr}+Q\cdot r-M_{\theta\theta}=0$$



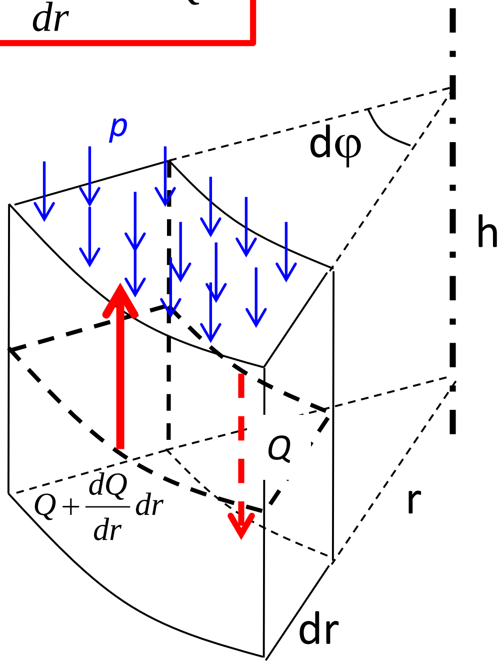
Equazioni di equilibrio/3

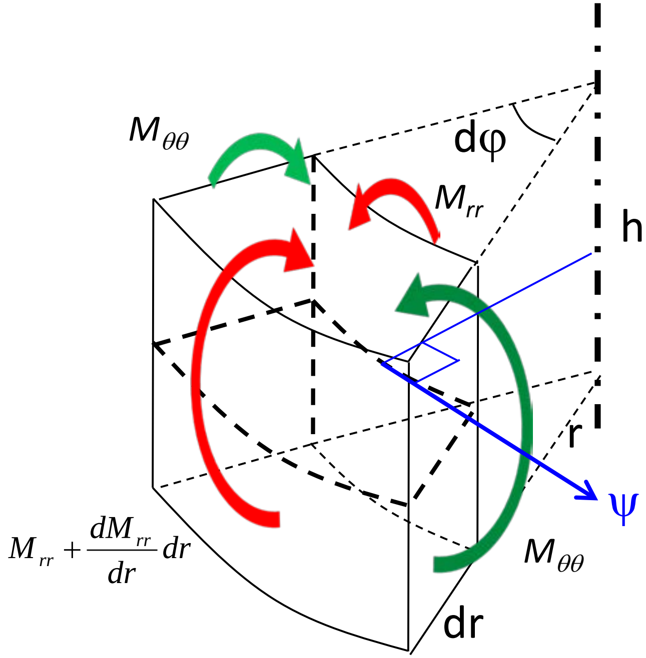
Rotazione attorno all'asse circonferenziale «φ»:

$$\left(M_{rr}+\frac{dM_{rr}}{dr}dr\right)(r+dr)d\varphi-M_{rr}\cdot r\cdot d\varphi+\left(Q+\frac{dQ}{dr}dr\right)(r+dr)d\varphi\cdot dr-p\cdot r\cdot\frac{dr^2}{2}\cdot d\varphi-M_{\theta\theta}\cdot dr\cdot d\varphi=0$$

$$\frac{dM_{rr}}{dr}r+M_{rr}+Q\cdot r-M_{\theta\theta}=0$$

$$M_{\theta\theta}-\frac{d(M_{rr}\cdot r)}{dr}=Q\cdot r$$





Equazioni risolventi/1

$$M_{\theta\theta} - \frac{d\left(M_{rr} \cdot r\right)}{dr} = Q \cdot r$$

$$M_{rr} = D\left(\frac{d\theta}{dr} + \nu \frac{\theta}{r}\right)$$

$$M_{\theta\theta} = D\left(\frac{\theta}{r} + \nu \frac{d\theta}{dr}\right)$$

Equazioni risolventi/1

$$M_{rr} = D\left(\frac{d\theta}{dr} + \nu\frac{\theta}{r}\right)$$

$$M_{\theta\theta} - \frac{d\left(M_{rr}\cdot r\right)}{dr} = Q\cdot r$$

$$M_{\theta\theta} = D\left(\frac{\theta}{r} + \nu\frac{d\theta}{dr}\right)$$

$$D\left(\frac{\theta}{r} + \nu\frac{d\theta}{dr}\right) - D\frac{d}{dr}\left(\frac{d\theta}{dr}r + \nu\theta\right) = Q\cdot r$$

Equazioni risolventi/1

$$M_{\theta\theta} - \frac{d(M_{rr} \cdot r)}{dr} = Q \cdot r$$

$$M_{rr} = D\left(\frac{d\theta}{dr} + \nu \frac{\theta}{r}\right)$$

$$M_{\theta\theta} = D\left(\frac{\theta}{r} + \nu \frac{d\theta}{dr}\right)$$

$$D\left(\frac{\theta}{r} + \nu \frac{d\theta}{dr}\right) - D\frac{d}{dr}\left(\frac{d\theta}{dr} r + \nu\theta\right) = Q \cdot r$$

Semplificando

$$\left(\frac{\theta}{r} + \nu \frac{d\theta}{dr}\right) - \frac{d}{dr}\left(\frac{d\theta}{dr} r + \nu\theta\right) = \frac{Q \cdot r}{D}$$

$$\frac{\theta}{r} + \nu \frac{d\theta}{dr} - \frac{d^2\theta}{dr^2} r - \frac{d\theta}{dr} - \nu \frac{d\theta}{dr} = \frac{Q \cdot r}{D}$$

Equazioni risolventi/1

$$M_{rr} = D\left(\frac{d\theta}{dr} + \nu\frac{\theta}{r}\right)$$

$$M_{\theta\theta} = D\left(\frac{\theta}{r} + \nu\frac{d\theta}{dr}\right)$$

$$M_{\theta\theta} - \frac{d\left(M_{rr}\cdot r\right)}{dr} = Q\cdot r$$

$$D\left(\frac{\theta}{r} + \nu\frac{d\theta}{dr}\right) - D\frac{d}{dr}\left(\frac{d\theta}{dr}r + \nu\theta\right) = Q\cdot r$$

$$\left(\frac{\theta}{r} + \nu\frac{d\theta}{dr}\right) - \frac{d}{dr}\left(\frac{d\theta}{dr}r + \nu\theta\right) = \frac{Q\cdot r}{D}$$

$$\frac{\theta}{r} + \nu\frac{d\theta}{dr} - \frac{d^2\theta}{dr^2}r - \frac{d\theta}{dr} - \nu\frac{d\theta}{dr} = \frac{Q\cdot r}{D}$$

$$\frac{d^2\theta}{dr^2}r + \frac{d\theta}{dr} - \frac{\theta}{r} = -\frac{Q\cdot r}{D}$$

Dividendo per «r»

Equazioni risolventi/1

$$M_{rr} = D\left(\frac{d\theta}{dr} + \nu\frac{\theta}{r}\right)$$

$$M_{\theta\theta} = D\left(\frac{\theta}{r} + \nu\frac{d\theta}{dr}\right)$$

$$M_{\theta\theta} - \frac{d(M_{rr}\cdot r)}{dr} = Q\cdot r$$

$$D\left(\frac{\theta}{r} + \nu\frac{d\theta}{dr}\right) - D\frac{d}{dr}\left(\frac{d\theta}{dr}r + \nu\theta\right) = Q\cdot r$$

$$\left(\frac{\theta}{r} + \nu\frac{d\theta}{dr}\right) - \frac{d}{dr}\left(\frac{d\theta}{dr}r + \nu\theta\right) = \frac{Q\cdot r}{D}$$

$$\frac{\theta}{r} + \nu\frac{d\theta}{dr} - \frac{d^2\theta}{dr^2}r - \frac{d\theta}{dr} - \nu\frac{d\theta}{dr} = \frac{Q\cdot r}{D}$$

$$\frac{d^2\theta}{dr^2}r + \frac{d\theta}{dr} - \frac{\theta}{r} = -\frac{Q\cdot r}{D}$$

$$\frac{d^2\theta}{dr^2} + \frac{1}{r}\frac{d\theta}{dr} - \frac{\theta}{r^2} = -\frac{Q}{D}$$

La relazione può essere scritta come segue

Equazioni risolventi/1

$$M_{rr} = D\left(\frac{d\theta}{dr} + \nu\frac{\theta}{r}\right)$$

$$M_{\theta\theta} - \frac{d\left(M_{rr}\cdot r\right)}{dr} = Q\cdot r$$

$$M_{\theta\theta} = D\left(\frac{\theta}{r} + \nu\frac{d\theta}{dr}\right)$$

$$D\left(\frac{\theta}{r} + \nu\frac{d\theta}{dr}\right) - D\frac{d}{dr}\left(\frac{d\theta}{dr}r + \nu\theta\right) = Q\cdot r$$

$$\left(\frac{\theta}{r} + \nu\frac{d\theta}{dr}\right) - \frac{d}{dr}\left(\frac{d\theta}{dr}r + \nu\theta\right) = \frac{Q\cdot r}{D}$$

$$\frac{\theta}{r} + \nu\frac{d\theta}{dr} - \frac{d^2\theta}{dr^2}r - \frac{d\theta}{dr} - \nu\frac{d\theta}{dr} = \frac{Q\cdot r}{D}$$

$$\frac{d^2\theta}{dr^2}r + \frac{d\theta}{dr} - \frac{\theta}{r} = -\frac{Q\cdot r}{D}$$

$$\frac{d^2\theta}{dr^2} + \frac{1}{r}\frac{d\theta}{dr} - \frac{\theta}{r^2} = -\frac{Q}{D}$$

$$\boxed{\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}\left(\theta\cdot r\right)\right] = -\frac{Q}{D}}$$

Relazione 1

Equazioni risolventi/2

Volendo una relazione nella quale compaia esplicitamente il carico esterno

$$\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}\left(\theta\cdot r\right)\right]=-\frac{Q}{D}$$

Moltiplicando per «r»

Equazioni risolventi/2

Volendo una relazione nella quale compaia esplicitamente il carico esterno

$$\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}(\theta \cdot r)\right] = -\frac{Q}{D}$$

$$r\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}(\theta \cdot r)\right] = -\frac{Q \cdot r}{D}$$

Derivando rispetto a «r»

Equazioni risolventi/2

Volendo una relazione nella quale compaia esplicitamente il carico esterno

$$\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}(\theta\cdot r)\right]=-\frac{Q}{D}$$

$$r\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}(\theta\cdot r)\right]=-\frac{Q\cdot r}{D}$$

$$\frac{d}{dr}\left\{r\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}(\theta\cdot r)\right]\right\}=-\frac{1}{D}\frac{d(Q\cdot r)}{dr}$$

1° equazione di equilibrio

$$\frac{d(Q\cdot r)}{dr}=p\cdot r$$

Equazioni risolventi/2

Volendo una relazione nella quale compaia esplicitamente il carico esterno

$$\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}(\theta\cdot r)\right]=-\frac{Q}{D}$$

$$r\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}(\theta\cdot r)\right]=-\frac{Q\cdot r}{D}$$

$$\frac{d}{dr}\left\{r\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}(\theta\cdot r)\right]\right\}=-\frac{1}{D}\frac{d(Q\cdot r)}{dr}$$

1° equazione di equilibrio

$$\frac{d(Q\cdot r)}{dr}=p\cdot r$$

Relazione 2

$$\frac{d}{dr}\left\{r\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}(\theta\cdot r)\right]\right\}=-\frac{p\cdot r}{D}$$

Equazioni risolventi/3

Volendo una relazione nella quale compaia esplicitamente il carico esterno

Relazione 1
$$\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}(\theta \cdot r)\right] = -\frac{Q}{D}$$

Relazione 2
$$\frac{d}{dr}\left\{r\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}(\theta \cdot r)\right]\right\} = -\frac{p \cdot r}{D}$$

$$\theta = -\frac{dw}{dr}$$

Equazioni risolvente 1
$$\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right)\right] = \frac{Q}{D}$$

Equazioni risolvente 2
$$\frac{d}{dr}\left\{r\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right)\right]\right\} = \frac{p \cdot r}{D}$$

Momenti e tensioni in funzione della w(r)

$$M_{rr} = D\left(\frac{d\theta}{dr} + \nu\frac{\theta}{r}\right) = -D\left(\frac{d^2w}{dr^2} + \nu\frac{1}{r}\frac{dw}{dr}\right)$$

$$M_{\theta\theta} = D\left(\frac{\theta}{r} + \nu\frac{d\theta}{dr}\right) = -D\left(\frac{1}{r}\frac{dw}{dr} + \nu\frac{d^2w}{dr^2}\right)$$

$$\sigma_{rr} = \frac{E\cdot z}{1-\nu^2}\left(\frac{d\theta}{dr} + \nu\frac{\theta}{r}\right) = -\frac{E\cdot z}{1-\nu^2}\left(\frac{d^2w}{dr^2} + \nu\frac{1}{r}\frac{dw}{dr}\right)$$

$$\sigma_{\theta\theta} = \frac{E\cdot z}{1-\nu^2}\left(\frac{\theta}{r} + \nu\frac{d\theta}{dr}\right) = -\frac{E\cdot z}{1-\nu^2}\left(\frac{1}{r}\frac{dw}{dr} + \nu\frac{d^2w}{dr^2}\right)$$

Integrali generali/1

Le principali condizioni che possono verificarsi in pratica sono:

- piastra libera da carichi
- piastra soggetta a pressione uniforme p_0
- piastra soggetta a carico costante P_0

Integrali generali/1

Le principali condizioni che possono verificarsi in pratica sono:

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- piastra soggetta a pressione uniforme p_0
- piastra soggetta a carico costante P_0

Piastra libera da carichi

Equazioni risolvente 1 $Q = 0$

$$\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right)\right] = 0$$

Integrali generali/1

Le principali condizioni che possono verificarsi in pratica sono:

- piastra libera da carichi
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Piastra libera da carichi

Equazioni risolvente 1 $Q = 0$

$$\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right)\right] = 0$$

Integrando

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right) = C_1$$

Integrali generali/1

Le principali condizioni che possono verificarsi in pratica sono:

- piastra libera da carichi
- piastra soggetta a pressione uniforme p_0
- piastra soggetta a carico costante P_0

Piastra libera da carichi

Equazioni risolvente 1 $Q = 0$

$$\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right)\right] = 0$$

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right) = C_1$$

Moltiplicando per «r»

$$\frac{d}{dr}\left(r\frac{dw}{dr}\right) = C_1 \cdot r$$

Integrali generali/1

Le principali condizioni che possono verificarsi in pratica sono:

- piastra libera da carichi
- piastra soggetta a pressione uniforme p_0
- piastra soggetta a carico costante P_0

Piastra libera da carichi

Equazioni risolvente 1 $Q = 0$

$$\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right)\right] = 0$$

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right) = C_1$$

$$\frac{d}{dr}\left(r\frac{dw}{dr}\right) = C_1 \cdot r$$

Integrando

$$r\frac{dw}{dr} = C_1 \cdot \frac{r^2}{2} + C_2$$

Integrali generali/1

Le principali condizioni che possono verificarsi in pratica sono:

- piastra libera da carichi
- piastra soggetta a pressione uniforme p_0
- piastra soggetta a carico costante P_0

Piastra libera da carichi

Equazioni risolvente 1 $Q = 0$

$$\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right)\right] = 0$$

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right) = C_1$$

$$\frac{d}{dr}\left(r\frac{dw}{dr}\right) = C_1 \cdot r$$

$$r\frac{dw}{dr} = C_1 \cdot \frac{r^2}{2} + C_2$$

Dividendo per «r»

$$\frac{dw}{dr} = C_1 \cdot \frac{r}{2} + \frac{C_2}{r}$$

Integrali generali/1

Le principali condizioni che possono verificarsi in pratica sono:

- piastra libera da carichi
- piastra soggetta a pressione uniforme p_0
- piastra soggetta a carico costante P_0

Piastra libera da carichi

Equazioni risolvente 1 $Q = 0$

$$\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right)\right] = 0$$

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right) = C_1$$

$$\frac{d}{dr}\left(r\frac{dw}{dr}\right) = C_1 \cdot r$$

$$r\frac{dw}{dr} = C_1 \cdot \frac{r^2}{2} + C_2$$

$$\frac{dw}{dr} = C_1 \cdot \frac{r}{2} + \frac{C_2}{r}$$

Integrando

$$w(r) = C_1 \cdot \frac{r^2}{4} + C_2 \ln\left(\frac{r}{R}\right) + C_3$$

Integrali generali/1

Le principali condizioni che possono verificarsi in pratica sono:

- piastra libera da carichi
- piastra soggetta a pressione uniforme p_0
- piastra soggetta a carico costante P_0

Piastra libera da carichi

Equazioni risolvente 1 $Q = 0$

$$\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right)\right] = 0$$

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right) = C_1$$

$$\frac{d}{dr}\left(r\frac{dw}{dr}\right) = C_1 \cdot r$$

$$r\frac{dw}{dr} = C_1 \cdot \frac{r^2}{2} + C_2$$

$$\frac{dw}{dr} = C_1 \cdot \frac{r}{2} + \frac{C_2}{r}$$

$$w(r) = C_1 \cdot \frac{r^2}{4} + C_2 \ln\left(\frac{r}{R}\right) + C_3$$

$$w(r) = C_1 \cdot \frac{r^2}{4} + C_2 \ln\left(\frac{r}{R}\right) + C_3$$

$$\frac{dw(r)}{dr} = C_1 \cdot \frac{r}{2} + \frac{C_2}{r}$$

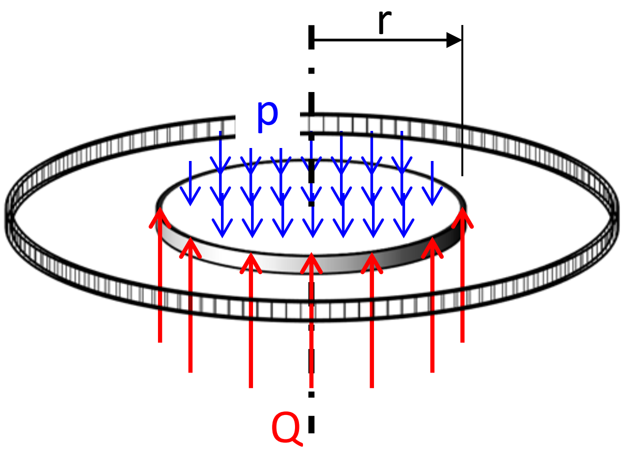
$$\frac{d^2 w(r)}{dr^2} = C_1 - \frac{C_2}{r^2}$$

Integrali generali/2

Piastra soggetta a pressione uniforme p_0

$$Q \cdot 2\pi \cdot r = p_0 \cdot \pi \cdot r^2$$

$$Q = \frac{p_0 \cdot r}{2}$$



Equazioni risolvente 1

$$\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right)\right] = \frac{p_0 r}{2D}$$

Integrali generali/2

Piastra soggetta a pressione uniforme p_0

$$\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right)\right]=\frac{p_0 r}{2D}$$

Integrando

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right)=\frac{p_0 r^2}{4D}+C_1$$

Integrali generali/2

Piastra soggetta a pressione uniforme p_0

$$\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right)\right]=\frac{p_0 r}{2D}$$

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right)=\frac{p_0 r^2}{4D}+C_1$$

Moltiplicando per «r»

$$\frac{d}{dr}\left(r\frac{dw}{dr}\right)=\frac{p_0 r^3}{4D}+C_1 r$$

Integrali generali/2

Piastra soggetta a pressione uniforme p_0

$$\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right)\right]=\frac{p_0 r}{2D}$$

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right)=\frac{p_0 r^2}{4D}+C_1$$

$$\frac{d}{dr}\left(r\frac{dw}{dr}\right)=\frac{p_0 r^3}{4D}+C_1 r$$

Integrando

$$r\frac{dw}{dr}=\frac{p_0 r^4}{16D}+C_1\frac{r^2}{2}+C_2$$

Integrali generali/2

Piastra soggetta a pressione uniforme p_0

$$\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right)\right]=\frac{p_0 r}{2D}$$

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right)=\frac{p_0 r^2}{4D}+C_1$$

$$\frac{d}{dr}\left(r\frac{dw}{dr}\right)=\frac{p_0 r^3}{4D}+C_1 r$$

$$r\frac{dw}{dr}=\frac{p_0 r^4}{16D}+C_1\frac{r^2}{2}+C_2$$

Dividendo per «r»

$$\frac{dw}{dr}=\frac{p_0 r^3}{16D}+C_1\frac{r}{2}+\frac{C_2}{r}$$

Integrali generali/2

Piastra soggetta a pressione uniforme p_0

$$\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right)\right]=\frac{p_0 r}{2D}$$

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right)=\frac{p_0 r^2}{4D}+C_1$$

$$\frac{d}{dr}\left(r\frac{dw}{dr}\right)=\frac{p_0 r^3}{4D}+C_1 r$$

$$r\frac{dw}{dr}=\frac{p_0 r^4}{16D}+C_1\frac{r^2}{2}+C_2$$

$$\frac{dw}{dr}=\frac{p_0 r^3}{16D}+C_1\frac{r}{2}+\frac{C_2}{r}$$

Integrando

$$w(r)=\frac{p_0 r^4}{64D}+C_1\frac{r^2}{4}+C_2\ln\left(\frac{r}{R}\right)+C_3$$

Integrali generali/2

Piastra soggetta a pressione uniforme p_0

$$\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right)\right]=\frac{p_0 r}{2D}$$

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right)=\frac{p_0 r^2}{4D}+C_1$$

$$\frac{d}{dr}\left(r\frac{dw}{dr}\right)=\frac{p_0 r^3}{4D}+C_1 r$$

$$r\frac{dw}{dr}=\frac{p_0 r^4}{16D}+C_1\frac{r^2}{2}+C_2$$

$$\frac{dw}{dr}=\frac{p_0 r^3}{16D}+C_1\frac{r}{2}+\frac{C_2}{r}$$

$$w(r)=\frac{p_0 r^4}{64D}+C_1\frac{r^2}{4}+C_2\ln\left(\frac{r}{R}\right)+C_3$$

$$w(r)=\frac{p_0 r^4}{64D}+C_1\cdot\frac{r^2}{4}+C_2\ln\left(\frac{r}{R}\right)+C_3$$

$$\frac{dw(r)}{dr}=\frac{p_0 r^3}{16D}+C_1\cdot\frac{r}{2}+\frac{C_2}{r}$$

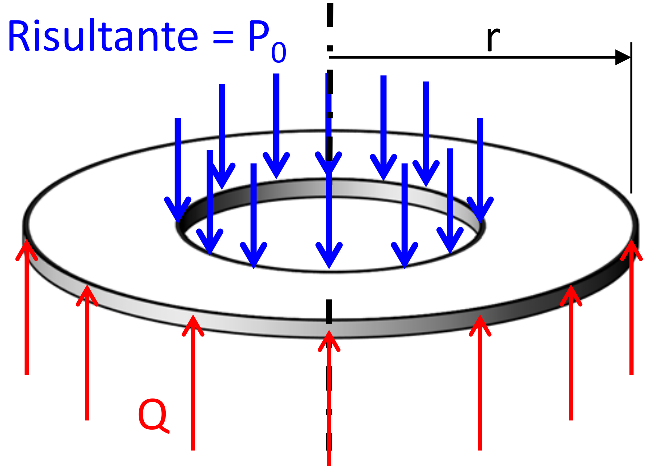
$$\frac{d^2w(r)}{dr^2}=\frac{3p_0 r^2}{16D}+\frac{C_1}{2}-\frac{C_2}{r^2}$$

Integrali generali/3

Piastra soggetta a carico costante P_0

$$Q \cdot 2\pi \cdot r = P_0$$

$$Q = \frac{P_0}{2\pi \cdot r}$$



Equazioni risolvente 1 $$\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right)\right] = \frac{P_0}{2\pi D \cdot r}$$

Integrali generali/3 *Piastra soggetta a carico costante P_0*

$$\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right)\right]=\frac{P_0}{2\pi D\cdot r}$$

Integrando

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right)=\frac{P_0}{2\pi D}\ln\left(\frac{r}{R}\right)+C_1$$

Integrali generali/3 *Piastra soggetta a carico costante P_0*

$$\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right)\right]=\frac{P_0}{2\pi D\cdot r}$$

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right)=\frac{P_0}{2\pi D}\ln\left(\frac{r}{R}\right)+C_1$$

Moltiplicando per «r»

$$\frac{d}{dr}\left(r\frac{dw}{dr}\right)=\frac{P_0}{2\pi D}r\ln\left(\frac{r}{R}\right)+C_1\cdot r$$

<u>**Integrali generali/3** *Piastra soggetta a carico costante P_0*</u>

$$\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right)\right]=\frac{P_0}{2\pi D\cdot r}$$

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right)=\frac{P_0}{2\pi D}\ln\left(\frac{r}{R}\right)+C_1$$

$$\frac{d}{dr}\left(r\frac{dw}{dr}\right)=\frac{P_0}{2\pi D}r\ln\left(\frac{r}{R}\right)+C_1\cdot r$$

Integrando

$$r\frac{dw}{dr}=\frac{P_0}{4\pi D}r^2\ln\left(\frac{r}{R}\right)-\frac{P_0}{16\pi D}r^2+C_1\cdot\frac{r^2}{2}+C_2$$

Integrali generali/3 *Piastra soggetta a carico costante* P_0

$$\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right)\right]=\frac{P_0}{2\pi D\cdot r}$$

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right)=\frac{P_0}{2\pi D}\ln\left(\frac{r}{R}\right)+C_1$$

$$\frac{d}{dr}\left(r\frac{dw}{dr}\right)=\frac{P_0}{2\pi D}r\ln\left(\frac{r}{R}\right)+C_1\cdot r$$

$$r\frac{dw}{dr}=\frac{P_0}{4\pi D}r^2\ln\left(\frac{r}{R}\right)-\frac{P_0}{16\pi D}r^2+C_1\cdot\frac{r^2}{2}+C_2$$

Dividendo per «r»

$$\frac{dw}{dr}=\frac{P_0}{4\pi D}r\ln\left(\frac{r}{R}\right)-\frac{P_0}{16\pi D}r+C_1\cdot\frac{r}{2}+\frac{C_2}{r}$$

Integrali generali/3 *Piastra soggetta a carico costante P_0*

$$\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right)\right]=\frac{P_0}{2\pi D\cdot r}$$

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right)=\frac{P_0}{2\pi D}\ln\left(\frac{r}{R}\right)+C_1$$

$$\frac{d}{dr}\left(r\frac{dw}{dr}\right)=\frac{P_0}{2\pi D}r\ln\left(\frac{r}{R}\right)+C_1\cdot r$$

$$r\frac{dw}{dr}=\frac{P_0}{4\pi D}r^2\ln\left(\frac{r}{R}\right)-\frac{P_0}{16\pi D}r^2+C_1\cdot\frac{r^2}{2}+C_2$$

$$\frac{dw}{dr}=\frac{P_0}{4\pi D}r\ln\left(\frac{r}{R}\right)-\frac{P_0}{16\pi D}r+C_1\cdot\frac{r}{2}+\frac{C_2}{r}$$

Integrando

$$w(r)=\frac{P_0}{8\pi D}r^2\ln\left(\frac{r}{R}\right)-\frac{P_0}{32\pi D}r^2-\frac{P_0}{32\pi D}r^2+C_1\cdot\frac{r^2}{4}+C_2\ln\left(\frac{r}{R}\right)+C_3$$

Integrali generali/3 - *Piastra soggetta a carico costante P_0*

$$\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right)\right]=\frac{P_0}{2\pi D\cdot r}$$

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right)=\frac{P_0}{2\pi D}\ln\left(\frac{r}{R}\right)+C_1$$

$$\frac{d}{dr}\left(r\frac{dw}{dr}\right)=\frac{P_0}{2\pi D}r\ln\left(\frac{r}{R}\right)+C_1\cdot r$$

$$r\frac{dw}{dr}=\frac{P_0}{4\pi D}r^2\ln\left(\frac{r}{R}\right)-\frac{P_0}{16\pi D}r^2+C_1\cdot\frac{r^2}{2}+C_2$$

$$\frac{dw}{dr}=\frac{P_0}{4\pi D}r\ln\left(\frac{r}{R}\right)-\frac{P_0}{16\pi D}r+C_1\cdot\frac{r}{2}+\frac{C_2}{r}$$

$$w(r)=\frac{P_0}{8\pi D}r^2\ln\left(\frac{r}{R}\right)-\frac{P_0}{32\pi D}r^2-\frac{P_0}{32\pi D}r^2+C_1\cdot\frac{r^2}{4}+C_2\ln\left(\frac{r}{R}\right)+C_3$$

Semplificando

$$w(r)=\frac{P_0}{8\pi D}r^2\ln\left(\frac{r}{R}\right)-\frac{P_0}{16\pi D}r^2+C_1\cdot\frac{r^2}{4}+C_2\ln\left(\frac{r}{R}\right)+C_3$$

Integrali generali/3 - *Piastra soggetta a carico costante P_0*

$$\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right)\right]=\frac{P_0}{2\pi D\cdot r}$$

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right)=\frac{P_0}{2\pi D}\ln\left(\frac{r}{R}\right)+C_1$$

$$\frac{d}{dr}\left(r\frac{dw}{dr}\right)=\frac{P_0}{2\pi D}r\ln\left(\frac{r}{R}\right)+C_1\cdot r$$

$$r\frac{dw}{dr}=\frac{P_0}{4\pi D}r^2\ln\left(\frac{r}{R}\right)-\frac{P_0}{16\pi D}r^2+C_1\cdot\frac{r^2}{2}+C_2$$

$$\frac{dw}{dr}=\frac{P_0}{4\pi D}r\ln\left(\frac{r}{R}\right)-\frac{P_0}{16\pi D}r+C_1\cdot\frac{r}{2}+\frac{C_2}{r}$$

$$w(r)=\frac{P_0}{8\pi D}r^2\ln\left(\frac{r}{R}\right)-\frac{P_0}{32\pi D}r^2-\frac{P_0}{32\pi D}r^2+C_1\cdot\frac{r^2}{4}+C_2\ln\left(\frac{r}{R}\right)+C_3$$

$$w(r)=\frac{P_0}{8\pi D}r^2\ln\left(\frac{r}{R}\right)-\frac{P_0}{16\pi D}r^2+C_1\cdot\frac{r^2}{4}+C_2\ln\left(\frac{r}{R}\right)+C_3$$

Conglobando in C_1

$$w(r)=\frac{P_0}{8\pi D}r^2\ln\left(\frac{r}{R}\right)+C_1\cdot\frac{r^2}{4}+C_2\ln\left(\frac{r}{R}\right)+C_3$$

Integrali generali/3

Piastra soggetta a carico costante P_0

$$\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right)\right]=\frac{P_0}{2\pi D\cdot r}$$

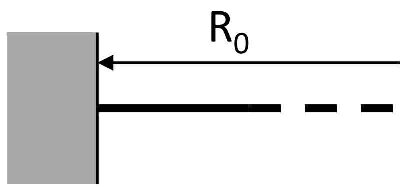
$$w(r)=\frac{P_0}{8\pi D}r^2\ln\left(\frac{r}{R}\right)+C_1\cdot\frac{r^2}{4}+C_2\ln\left(\frac{r}{R}\right)+C_3$$

$$\frac{dw(r)}{dr}=\frac{P_0}{4\pi D}r\ln\left(\frac{r}{R}\right)+\frac{P_0}{8\pi D}r+C_1\cdot\frac{r}{2}+\frac{C_2}{r}$$

$$\frac{d^2w(r)}{dr^2}=\frac{P_0}{4\pi D}\ln\left(\frac{r}{R}\right)+\frac{3P_0}{8\pi D}+\frac{C_1}{2}-\frac{C_2}{r^2}$$

Condizioni al contorno/1

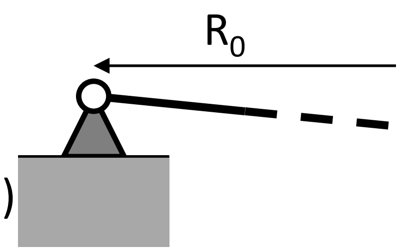
Tipiche CC

(a)


Incastro

$$w(R) = 0$$

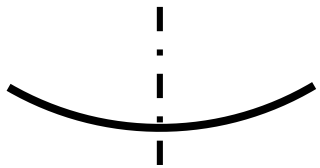
$$\left(\frac{dw(r)}{dr}\right)_{r=R_0} = 0$$

(b)


Appoggio

$$w(R_0) = 0$$

$$\left(\frac{d^2w(r)}{dr^2} + \frac{\nu}{r}\frac{dw(r)}{dr}\right)_{r=R_0} = 0$$

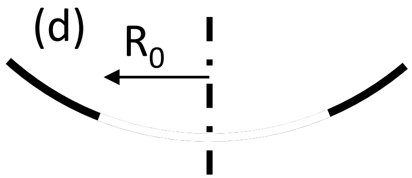
(c)


Asse simmetria

$$\left(\frac{dw(r)}{dr}\right)_{r=0} = 0$$

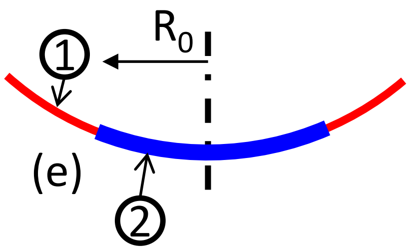
Condizioni al contorno/2

Tipiche CC



Estremo libero al raggio R_0

$$\left(\frac{d^2w(r)}{dr^2}+\frac{\nu}{r}\frac{dw(r)}{dr}\right)_{r=R}=0$$

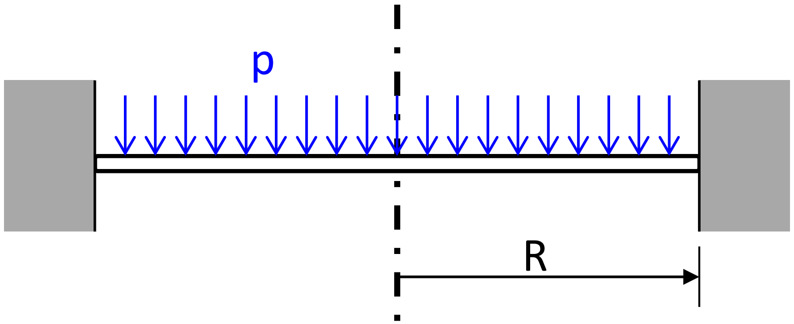


Confine tra piastre diverse al raggio R_0

$$w_1(R_0)=w_2(R_0)$$

$$\left(\frac{dw_1(r)}{dr}\right)_{r=R_0}=\left(\frac{dw_2(r)}{dr}\right)_{r=R_0}$$

Piastra circolare incastrata al bordo soggetta a pressione uniforme p_0/1



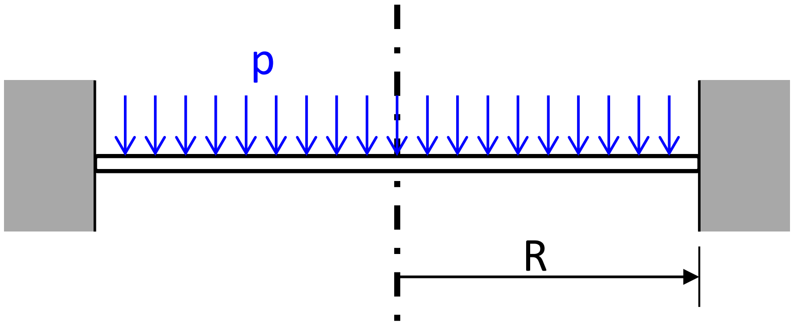
$$CC\begin{cases} w(R)=0 \\ \left(\dfrac{dw(r)}{r}\right)_{r=R}=0 \\ \left(\dfrac{dw(r)}{r}\right)_{r=0}=0 \end{cases}$$

$$w(r)=\frac{p_0 r^4}{64D}+C_1\cdot\frac{r^2}{4}+C_2\ln\left(\frac{r}{R}\right)+C_3$$

$$\frac{dw(r)}{dr}=\frac{p_0 r^3}{16D}+C_1\cdot\frac{r}{2}+\frac{C_2}{r}$$

$$CC\begin{cases} \dfrac{p_0R^4}{64D}+C_1\cdot\dfrac{R^2}{4}+C_3=0 \\ \dfrac{p_0R^3}{16D}+C_1\cdot\dfrac{R}{2}+\dfrac{C_2}{R}=0 \\ \dfrac{C_2}{0}=0 \end{cases} \rightarrow \begin{cases} C_1=-\dfrac{p_0R^2}{8D} \\ C_2=0 \\ C_3=\dfrac{p_0R^4}{64D} \end{cases}$$

Piastra circolare incastrata al bordo soggetta a pressione uniforme $p_0/2$



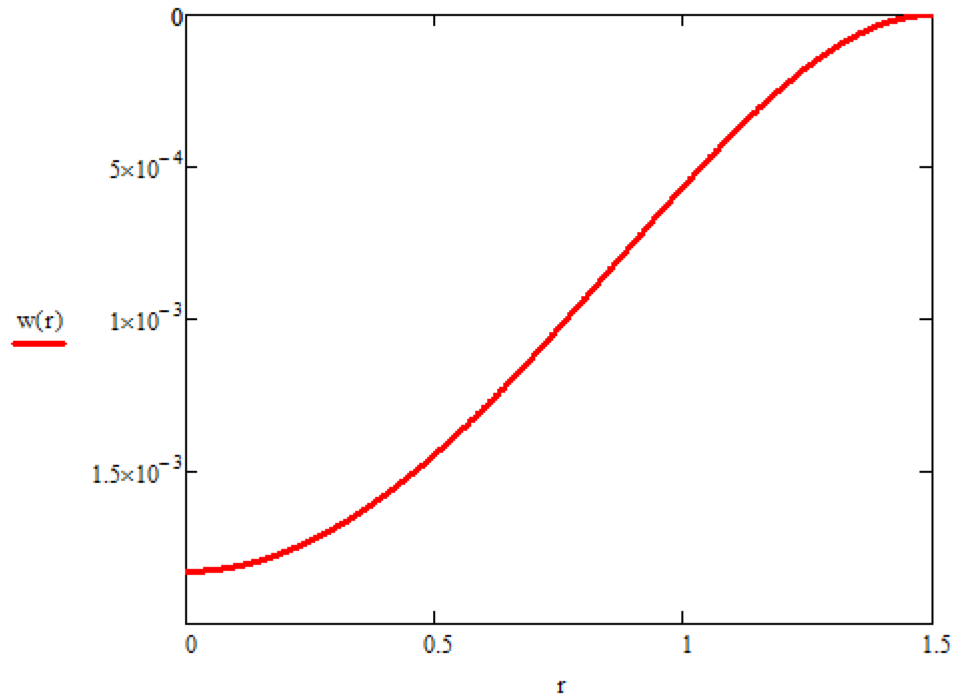
$$R = 1.5\,m \quad p_0 = 1500\,Pa \quad h = 15\,mm$$

$$E = 210000\,MPa \quad \nu = 0.3$$

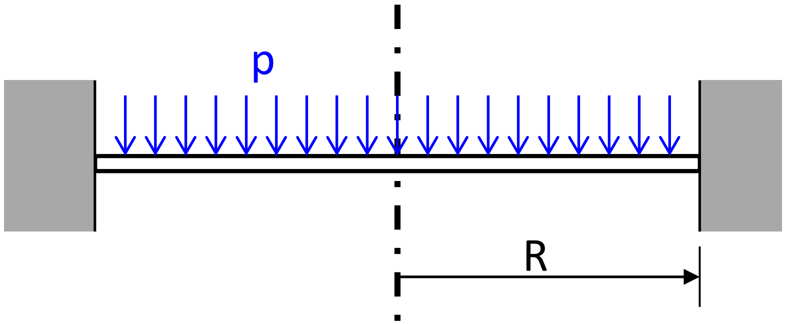
$$w(r) = \frac{p_0 r^4}{64D} - \frac{p_0 R^2}{32D} r^2 + \frac{p_0 R^4}{64D}$$

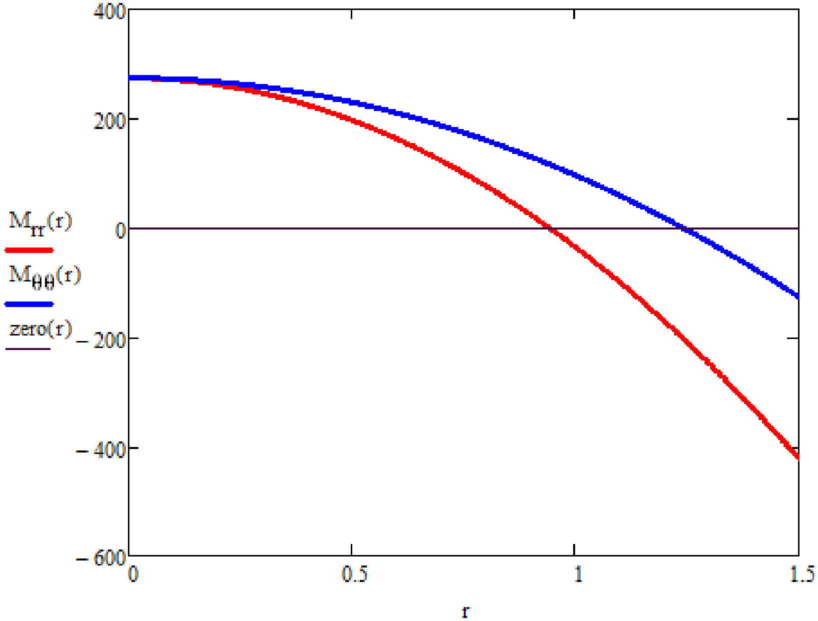
$$w(r) = \frac{p_0}{64D}\left(R^2 - r^2\right)^2$$

$$w_{max} = w(0) = \frac{p_0 R^4}{64D}$$



Piastra circolare incastrata al bordo soggetta a pressione uniforme $p_0/2$





$$\frac{dw}{dr} = \frac{p_0 r^3}{16D} - \frac{p_0 R^2}{16D} r$$

$$\frac{d^2 w}{dr^2} = \frac{3 p_0 r^2}{16D} - \frac{p_0 R^2}{16D}$$

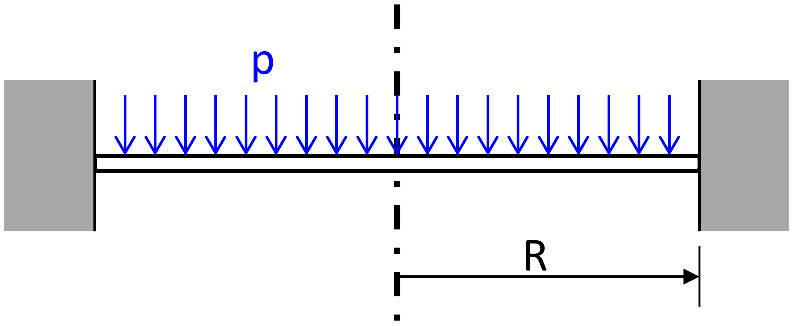
$$M_{rr} = -D\left(\frac{d^2 w}{dr^2} + \nu \frac{1}{r}\frac{dw}{dr}\right) = \left[\frac{(1+\nu)p_0 R^2}{16} - \frac{(3+\nu)p_0 r^2}{16}\right]$$

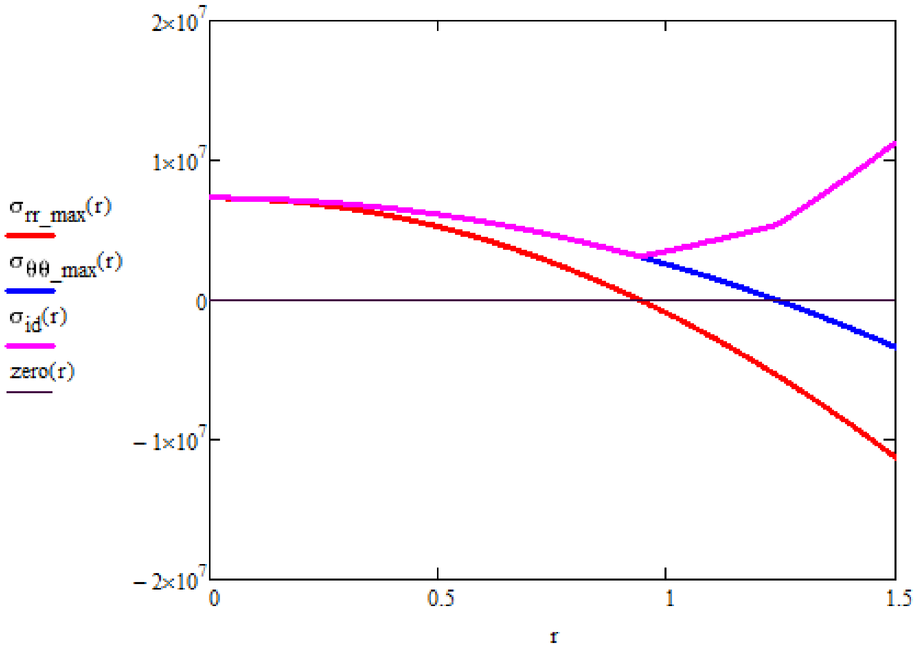
$$M_{\theta\theta} = -D\left(\frac{1}{r}\frac{dw}{dr} + \nu \frac{d^2 w}{dr^2}\right) = \left[\frac{(1+\nu)p_0 R^2}{16} - \frac{(1+3\nu)p_0 r^2}{16}\right]$$

$$M_{rr}(0) = M_{\theta\theta}(0) = \frac{(1+\nu)p_0 R^2}{16}$$

$$M_{\max} = |M_{rr}(R)| = \frac{p_0 R^2}{8}$$

Piastra circolare incastrata al bordo soggetta a pressione uniforme $p_0/3$





$$\sigma_{rr\_\max}(r) = \frac{6M_{rr}(r)}{h^2}$$

$$\sigma_{\theta\theta\_\max}(r) = \frac{6M_{\theta\theta}(r)}{h^2}$$

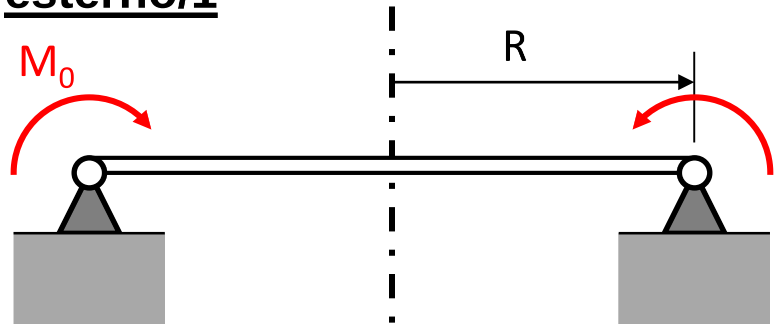
Valori massimi sullo spessore

$$\sigma_{id}(r) = \max\left(\left|\sigma_{rr\_\max}\right|, \left|\sigma_{\theta\theta\_\max}\right|\right)$$

$$\sigma_{id\_\max} = \frac{3p_0R^2}{4 \cdot h^2}$$

Valore massimo sull'intera piastra

Piastra circolare appoggiata e caricata con momento radiale M_0 al bordo esterno/1



$$CC\begin{cases} w(R)=0 \\ M_{rr}(R)=M_0 \\ \left(\dfrac{dw(r)}{r}\right)_{r=0}=0 \end{cases}$$

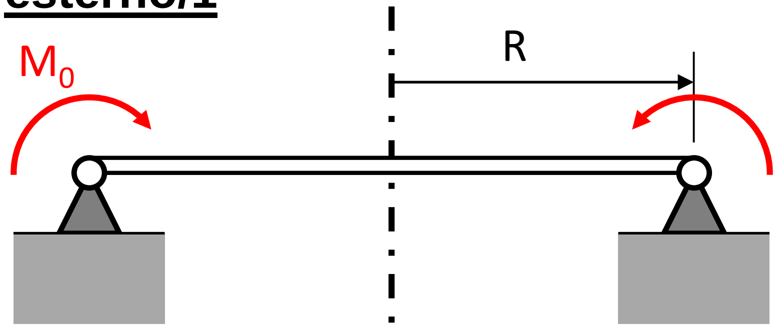
$$w(r)=C_1\cdot\frac{r^2}{4}+C_2\ln\left(\frac{r}{R}\right)+C_3$$

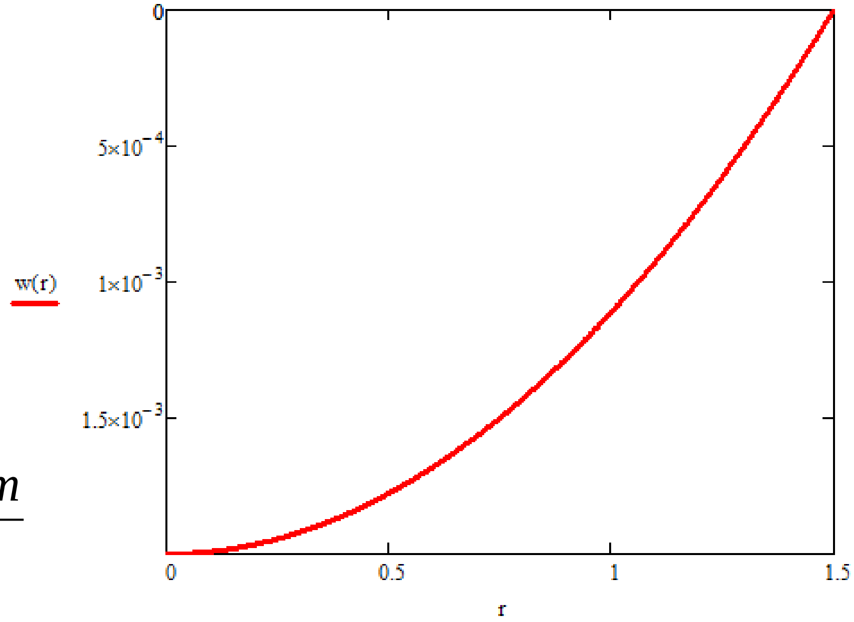
$$\frac{dw(r)}{dr}=C_1\cdot\frac{r}{2}+\frac{C_2}{r}$$

$$\frac{d^2w(r)}{dr^2}=\frac{C_1}{2}-\frac{C_2}{r^2}$$

$$CC\begin{cases} C_1\cdot\dfrac{R^2}{4}+C_3=0 \\ -D\left(\dfrac{C_1}{2}-\dfrac{C_2}{R^2}+\nu\cdot\dfrac{C_1}{2}+\nu\dfrac{C_2}{R^2}\right)=M_0 \\ \dfrac{C_2}{0}=0 \end{cases}\rightarrow\begin{cases} C_1=-\dfrac{2M_0}{(1+\nu)D} \\ C_2=0 \\ C_3=\dfrac{M_0R^2}{2(1+\nu)D} \end{cases}$$

Piastra circolare appoggiata e caricata con momento radiale M_0 al bordo esterno/1





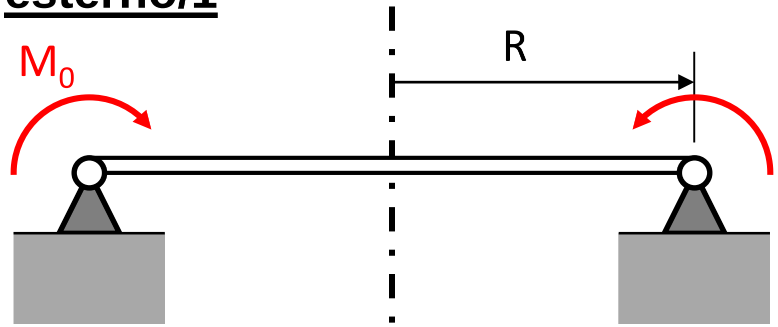
$$R = 1.5\,m \quad p_0 = 1500\,Pa \quad h = 15\,mm$$

$$E = 210000\,MPa \quad \nu = 0.3 \quad M_0 = 150\,\frac{N \cdot m}{m}$$

$$w(r) = -\frac{M_0}{2(1+\nu)D} r^2 + \frac{M_0}{2(1+\nu)D} R^2 = \frac{M_0}{2(1+\nu)D}\left(R^2 - r^2\right)$$

$$w_{\max} = \frac{M_0 R^2}{2(1+\nu)D}$$

Piastra circolare appoggiata e caricata con momento radiale M_0 al bordo esterno/1



$$\frac{dw}{dr} = -\frac{M_0}{(1+\nu)D} r$$

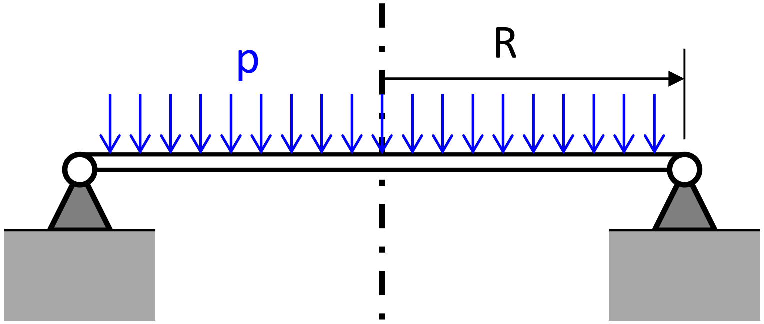
$$\frac{d^2w}{dr^2} = -\frac{M_0}{(1+\nu)D}$$

$$M_{rr} = -D\left(\frac{d^2w}{dr^2} + \nu \frac{1}{r}\frac{dw}{dr}\right) = \left[\frac{M_0}{(1+\nu)} + \nu \frac{M_0}{(1+\nu)}\right] = M_0$$

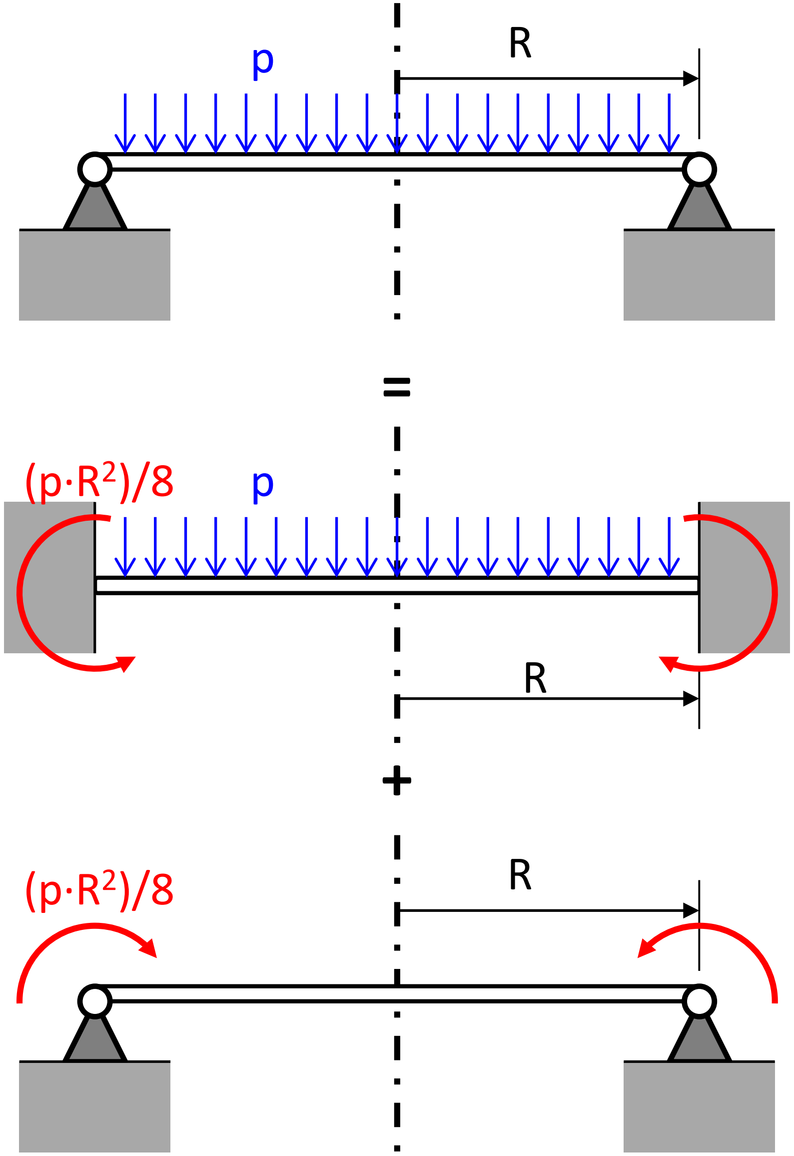
$$M_{\theta\theta} = -D\left(\frac{1}{r}\frac{dw}{dr} + \nu \frac{d^2w}{dr^2}\right) = M_0$$

Momenti (e tensioni) costanti su tutta la piastra

Piastra circolare appoggiata al bordo soggetta a pressione uniforme p_0/1

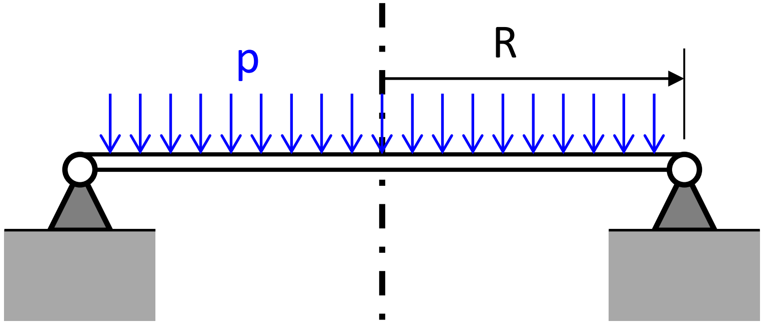


Piastra circolare appoggiata al bordo soggetta a pressione uniforme p_0/1



Sovrapposizione di due problemi di cui si conosce la soluzione

Piastra circolare appoggiata al bordo soggetta a pressione uniforme p_0/1



$$w(r)=\frac{p_0}{64D}\left(R^2-r^2\right)^2$$

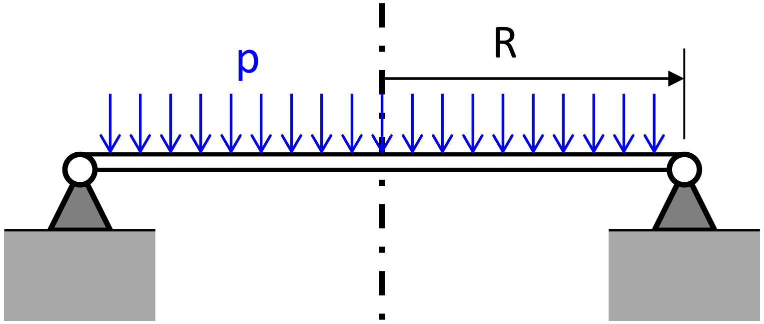
$$w(r)=\frac{p_0R^2}{16(1+\nu)D}\left(R^2-r^2\right)$$

$$w(r)=\frac{p_0}{64D}\left(R^2-r^2\right)^2+\frac{p_0R^2}{16(1+\nu)D}\left(R^2-r^2\right)$$

$$M_{rr}=\left[\frac{(1+\nu)p_0R^2}{16}-\frac{(3+\nu)p_0r^2}{16}\right]+\frac{p_0R^2}{8}$$

$$M_{\theta\theta}=\left[\frac{(1+\nu)p_0R^2}{16}-\frac{(1+3\nu)p_0r^2}{16}\right]+\frac{p_0R^2}{8}$$

Piastra circolare appoggiata al bordo soggetta a pressione uniforme p_0/1



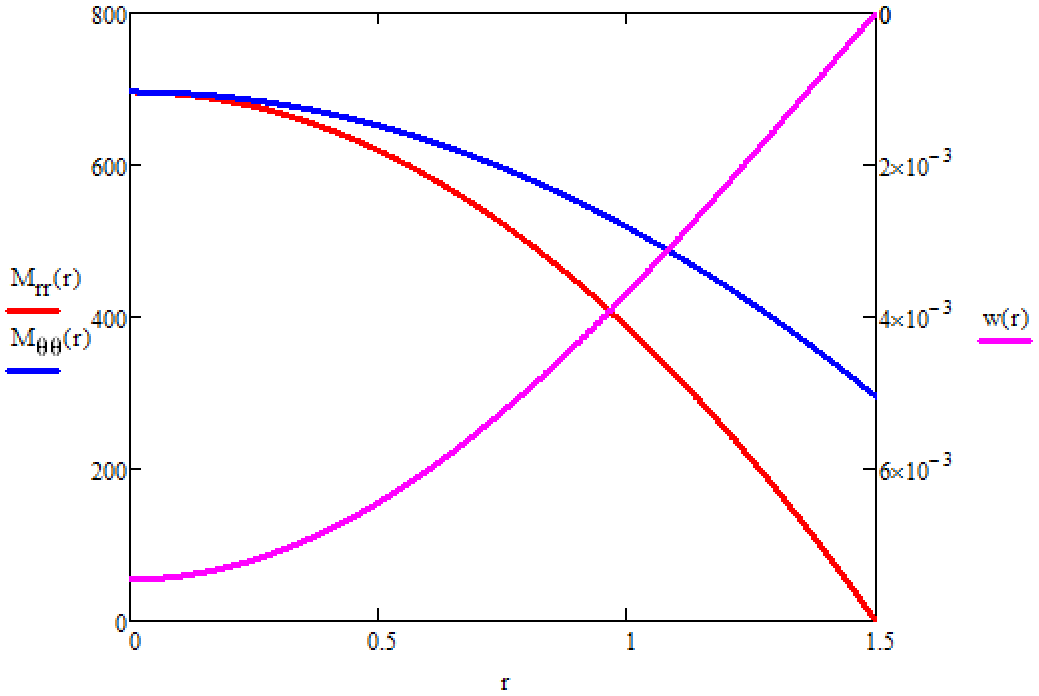
$$w_{\max} = \frac{p_0 R^4}{64D}\left(\frac{5+\nu}{1+\nu}\right)$$

$$\left(\frac{5+\nu}{1+\nu}\right) = 4.07$$

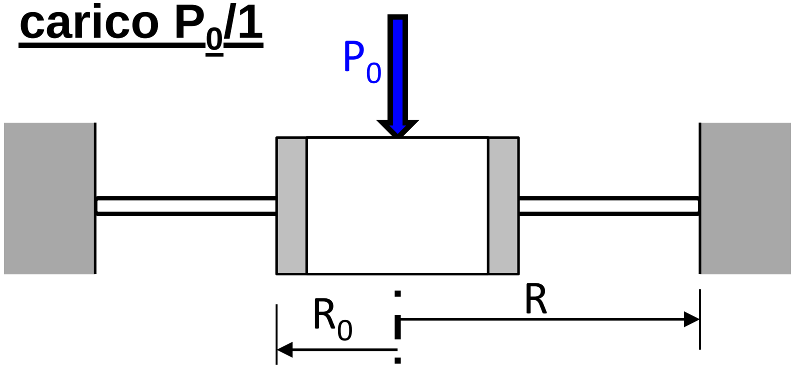
$$M_{\theta\theta\_\max} = \frac{p_0 R^2}{8}\left(\frac{3+\nu}{2}\right)$$

$$\sigma_{id\_\max} = \frac{3 p_0 R^2}{4 \cdot h^2}\left(\frac{3+\nu}{2}\right)$$

$$\left(\frac{3+\nu}{2}\right) = 1.65$$



Piastra circolare incastrata al bordo con rinforzo centrale rigido soggetto a carico P_0/1



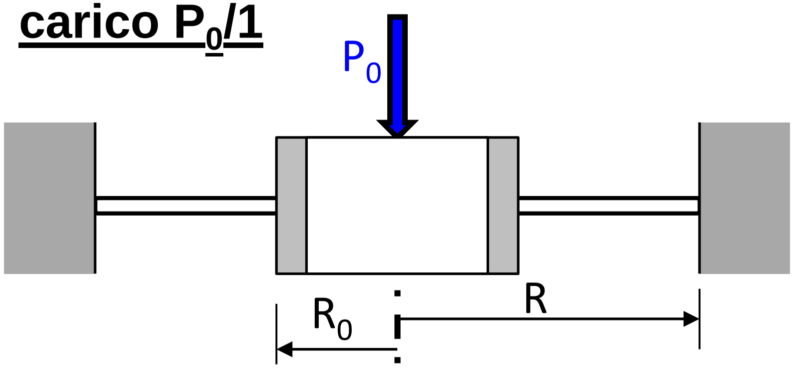
$$CC\begin{cases} w(R)=0 \\ \left(\dfrac{dw(r)}{r}\right)_{r=R}=0 \\ \left(\dfrac{dw(r)}{r}\right)_{r=R_0}=0 \end{cases}$$

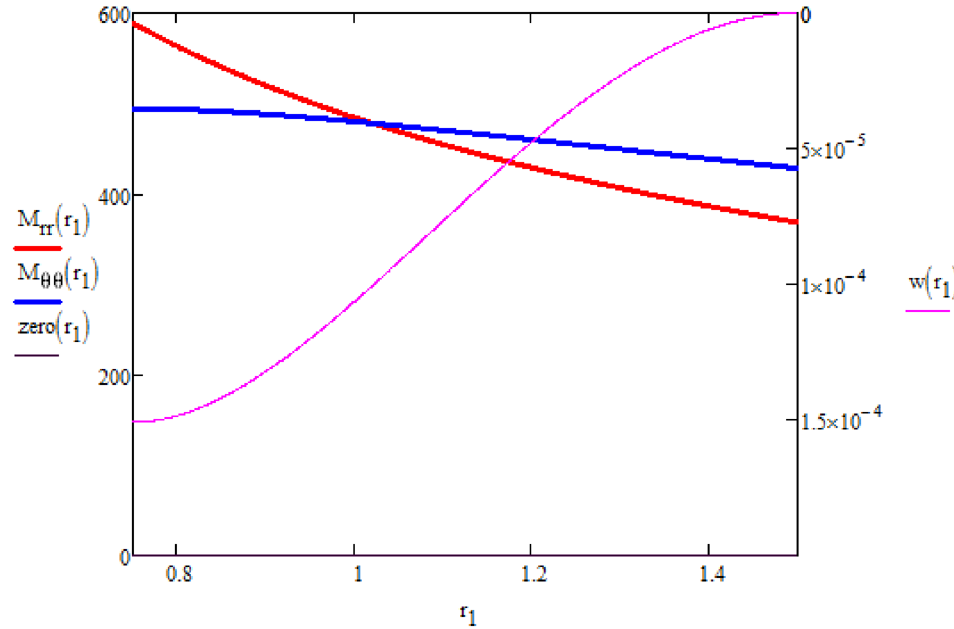
$$w(r)=\frac{P_0}{8\pi D}r^2\ln\left(\frac{r}{R}\right)+C_1\cdot\frac{r^2}{4}+C_2\ln\left(\frac{r}{R}\right)+C_3$$

$$\frac{dw(r)}{dr}=\frac{P_0}{4\pi D}r\ln\left(\frac{r}{R}\right)+\frac{P_0}{8\pi D}r+C_1\cdot\frac{r}{2}+\frac{C_2}{r}$$

$$CC\begin{cases} C_1\cdot\dfrac{R^2}{4}+C_3=0 \\ \dfrac{P_0}{8\pi D}R+C_1\cdot\dfrac{R}{2}+\dfrac{C_2}{R}=0 \\ \dfrac{P_0}{4\pi D}R_0\ln\left(\dfrac{R_0}{R}\right)+\dfrac{P_0}{8\pi D}R_0+C_1\cdot\dfrac{R_0}{2}+\dfrac{C_2}{R_0}=0 \end{cases}$$

Piastra circolare incastrata al bordo con rinforzo centrale rigido soggetto a carico P_0/1





$$C_1 = \frac{P_0}{4\pi D}\frac{R_0^2}{R_0^2 - R^2}\left[\frac{R^2 - R_0^2}{R_0^2} - 2\cdot\ln\left(\frac{R_0}{R}\right)\right]$$

$$C_2 = -\frac{P_0 R^2}{8\pi D} - \frac{P_0}{8\pi D}\frac{R^2 R_0^2}{R_0^2 - R^2}\left[\frac{R^2 - R_0^2}{R_0^2} - 2\cdot\ln\left(\frac{R_0}{R}\right)\right]$$

$$C_3 = -\frac{P_0}{16\pi D}\frac{R_0^2 R^2}{R_0^2 - R^2}\left[\frac{R^2 - R_0^2}{R_0^2} - 2\cdot\ln\left(\frac{R_0}{R}\right)\right]$$