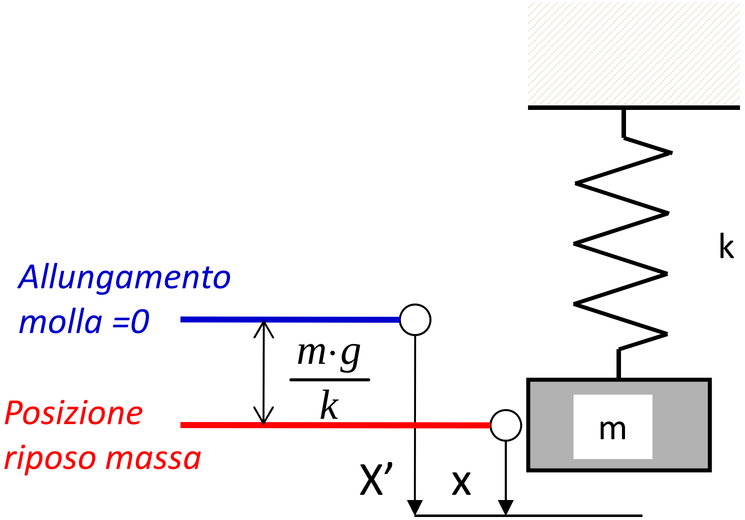
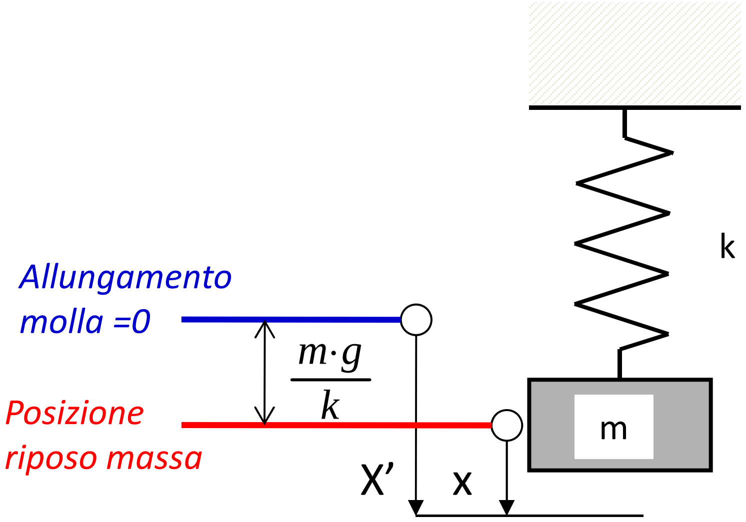
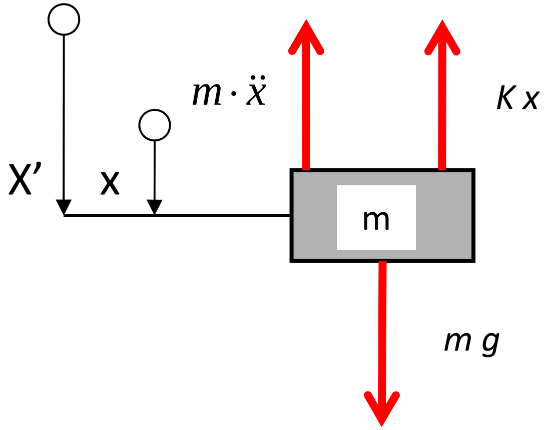
OSCILLAZIONE LIBERA SISTEMA 1 G.D.L. NON SMORZATO



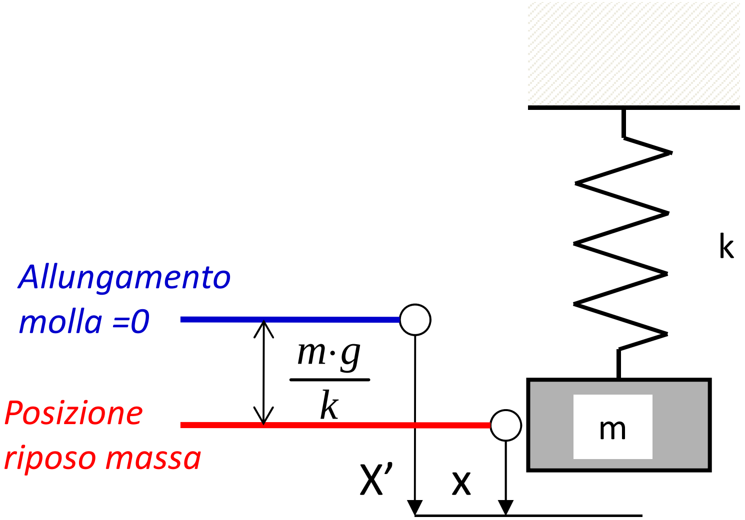
OSCILLAZIONE LIBERA SISTEMA 1 G.D.L. NON SMORZATO



Analisi delle forze agenti



OSCILLAZIONE LIBERA SISTEMA 1 G.D.L. NON SMORZATO

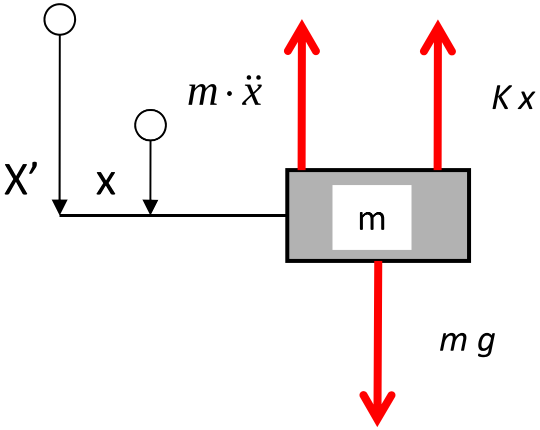


$$X' = x + \frac{mg}{k}$$

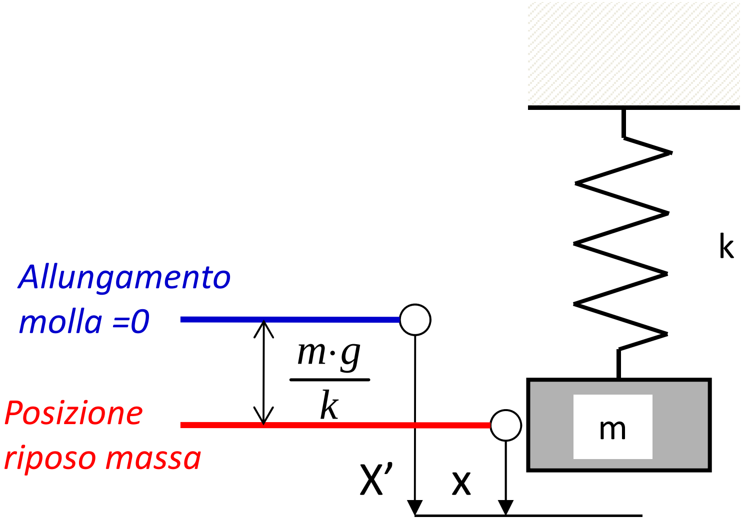
$$\ddot{X}' = \ddot{x}$$

$$-m\ddot{X}' - kX' + mg = 0$$

Analisi delle forze agenti



OSCILLAZIONE LIBERA SISTEMA 1 G.D.L. NON SMORZATO



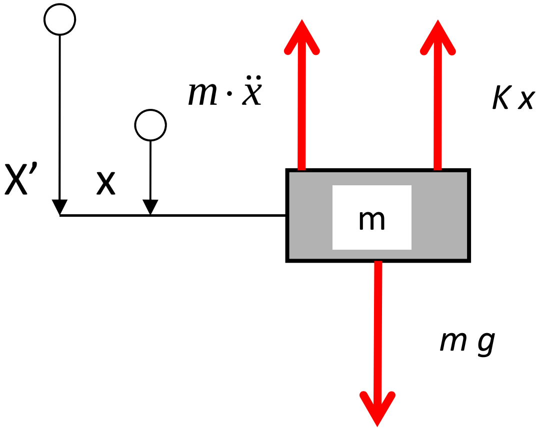
$$X' = x + \frac{mg}{k}$$

$$\ddot{X}' = \ddot{x}$$

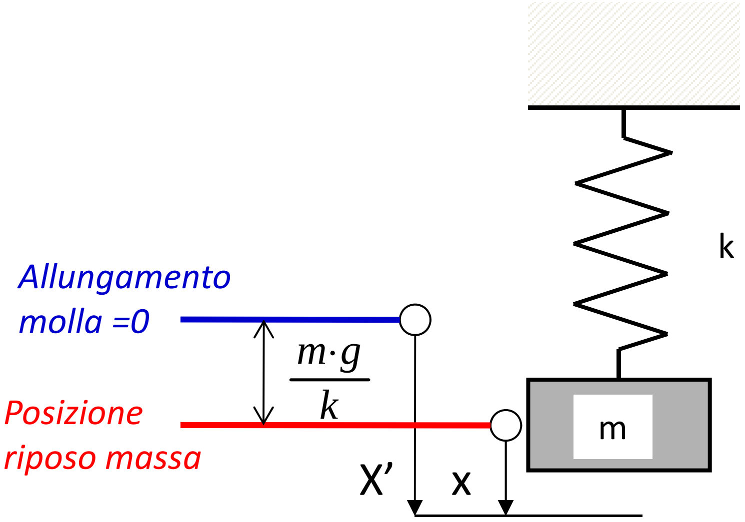
$$-m\ddot{X}' - kX' + mg = 0$$

$$-m\ddot{x} - k\left(x + \frac{mg}{k}\right) + mg = 0$$

Analisi delle forze agenti



OSCILLAZIONE LIBERA SISTEMA 1 G.D.L. NON SMORZATO



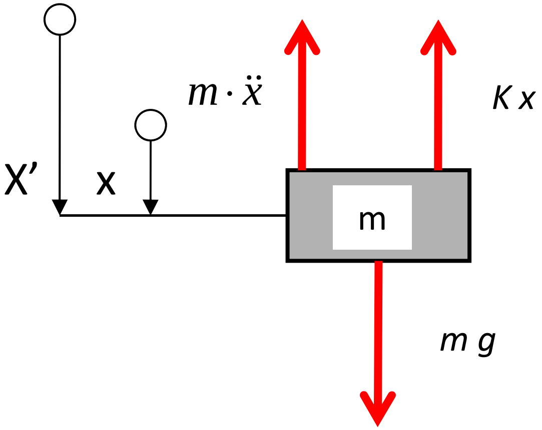
$$X' = x + \frac{mg}{k}$$

$$\ddot{X}' = \ddot{x}$$

$$-m\ddot{X}' - kX' + mg = 0$$

$$-m\ddot{x} - k\left(x + \frac{mg}{k}\right) + mg = 0$$

Analisi delle forze agenti



$$-m\ddot{x} - kx - mg + mg = 0$$

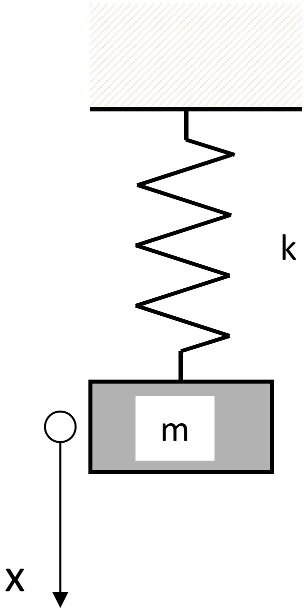
$$m\ddot{x} + kx = 0$$

Equazione del moto non influenzata dalla forza peso

OSCILLAZIONE LIBERA SISTEMA 1 G.D.L. NON SMORZATO

Sistema ad 1 g.d.l.

$$m\ddot{x} + kx = 0$$

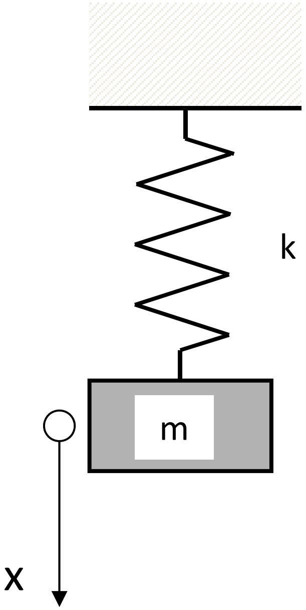


OSCILLAZIONE LIBERA SISTEMA 1 G.D.L. NON SMORZATO

Sistema ad 1 g.d.l.

$$m\ddot{x} + kx = 0$$

$$\ddot{x} + \frac{k}{m}x = \ddot{x} + \omega_n^2 x = 0$$



OSCILLAZIONE LIBERA SISTEMA 1 G.D.L. NON SMORZATO

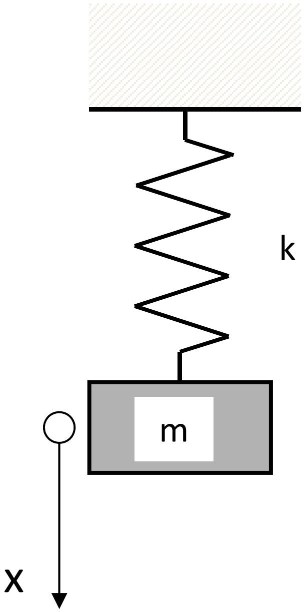
Sistema ad 1 g.d.l.

$$m\ddot{x} + kx = 0$$

$$\ddot{x} + \frac{k}{m}x = \ddot{x} + \omega_n^2 x = 0$$

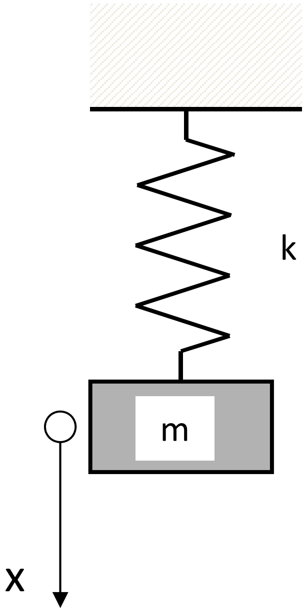
$$x(t) = C_1 e^{z_1 t} + C_2 e^{z_2 t}$$

$$z^2 + \omega_n^2 = 0 \qquad z_{1,2} = \pm i\omega_n$$



OSCILLAZIONE LIBERA SISTEMA 1 G.D.L. NON SMORZATO

Sistema ad 1 g.d.l.



$$m\ddot{x} + kx = 0$$

$$\ddot{x} + \frac{k}{m}x = \ddot{x} + \omega_n^2 x = 0$$

$$x(t) = C_1 e^{z_1 t} + C_2 e^{z_2 t}$$

$$z^2 + \omega_n^2 = 0 \qquad z_{1,2} = \pm i\omega_n$$

$$x(t) = C_1 e^{i\omega_n t} + C_2 e^{-i\omega_n t}$$

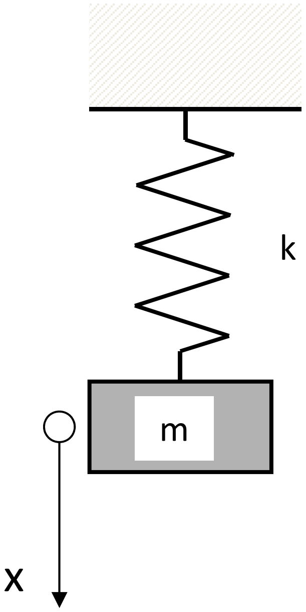
$$x(t) = C_3 \cos(\omega_n t) + C_4 \sin(\omega_n t)$$

$$x(t) = C_5 \cos(\omega_n t + \varphi_6)$$

$$x(t) = C_7 \cos(\omega_n t + \varphi_8)$$

OSCILLAZIONE LIBERA SISTEMA 1 G.D.L. NON SMORZATO

Sistema ad 1 g.d.l.

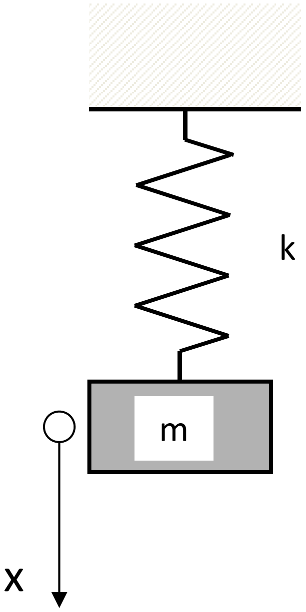


$$E_c = \frac{1}{2} m \cdot \dot{x}^2$$

$$E_p = \frac{1}{2} k \cdot x^2$$

OSCILLAZIONE LIBERA SISTEMA 1 G.D.L. NON SMORZATO

Sistema ad 1 g.d.l.



$$E_c = \frac{1}{2} m \cdot \dot{x}^2$$

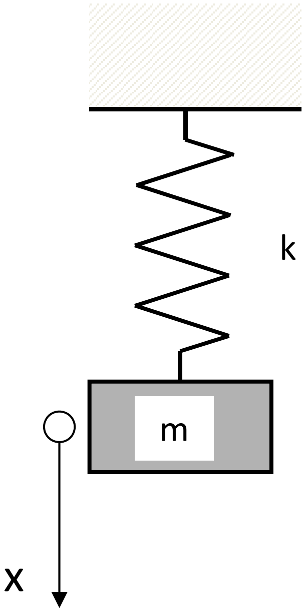
$$E_p = \frac{1}{2} k \cdot x^2$$

Energia totale

$$\Rightarrow \quad E = E_c + E_p = \frac{1}{2} m \cdot \dot{x}^2 + \frac{1}{2} k \cdot x^2$$

OSCILLAZIONE LIBERA SISTEMA 1 G.D.L. NON SMORZATO

Sistema ad 1 g.d.l.



$$E_c = \frac{1}{2} m \cdot \dot{x}^2$$

$$E_p = \frac{1}{2} k \cdot x^2$$

Energia totale

$$\Rightarrow \quad E = E_c + E_p = \frac{1}{2} m \cdot \dot{x}^2 + \frac{1}{2} k \cdot x^2$$

Soluzione trovata

$$x(t) = A\sin(\omega_n t)$$

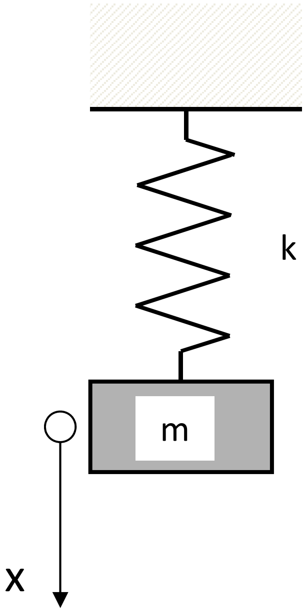
$$\dot{x}(t) = A\omega_n \cos(\omega_n t)$$

$$E = \frac{1}{2} m \cdot (A\omega_n \cos(\omega_n t))^2 + \frac{1}{2} k \cdot (A\sin(\omega_n t))^2 =$$

$$= \frac{A^2}{2}\left(m\,\omega_n^2 \cos^2(\omega_n t) + k\sin^2(\omega_n t)\right)$$

OSCILLAZIONE LIBERA SISTEMA 1 G.D.L. NON SMORZATO

Sistema ad 1 g.d.l.



$$E_c = \frac{1}{2} m \cdot \dot{x}^2$$

$$E_p = \frac{1}{2} k \cdot x^2$$

Energia totale

$$\Rightarrow \quad E = E_c + E_p = \frac{1}{2} m \cdot \dot{x}^2 + \frac{1}{2} k \cdot x^2$$

Soluzione trovata

$$x(t) = A \sin(\omega_n t)$$

$$\dot{x}(t) = A\omega_n \cos(\omega_n t)$$

$$E = \frac{1}{2} m \cdot (A\omega_n \cos(\omega_n t))^2 + \frac{1}{2} k \cdot (A \sin(\omega_n t))^2 =$$

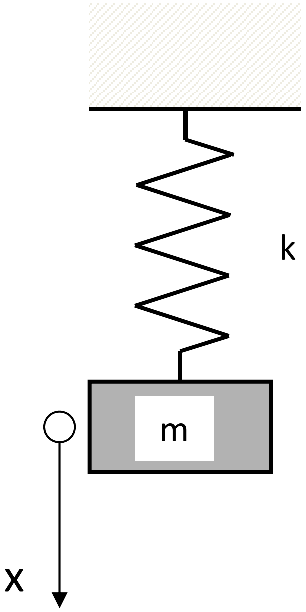
$$= \frac{A^2}{2} \left(m \omega_n^2 \cos^2(\omega_n t) + k \sin^2(\omega_n t)\right)$$

$$E = \text{cost} \quad \rightarrow \quad m\omega_n^2 = k$$

$$\omega_n = \sqrt{\frac{k}{m}}$$

OSCILLAZIONE LIBERA SISTEMA 1 G.D.L. NON SMORZATO

Sistema ad 1 g.d.l.



$$E_c = \frac{1}{2} m \cdot \dot{x}^2$$

$$E_p = \frac{1}{2} k \cdot x^2$$

Energia totale

$$\Rightarrow \quad E = E_c + E_p = \frac{1}{2} m \cdot \dot{x}^2 + \frac{1}{2} k \cdot x^2$$

Soluzione trovata

$$x(t) = A\sin(\omega_n t)$$

$$\dot{x}(t) = A\omega_n \cos(\omega_n t)$$

$$E = \frac{1}{2} m \cdot (A\omega_n \cos(\omega_n t))^2 + \frac{1}{2} k \cdot (A\sin(\omega_n t))^2 =$$

$$= \frac{A^2}{2}\left(m\,\omega_n^2 \cos^2(\omega_n t) + k\sin^2(\omega_n t)\right)$$

$$E = \text{cost} \;\rightarrow\; m\omega_n^2 = k$$

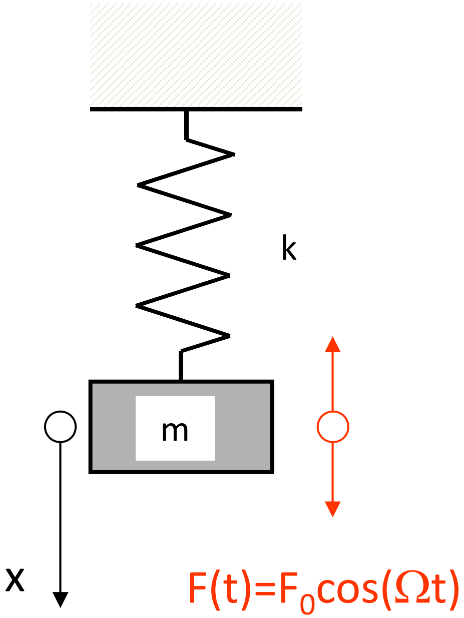
$$\omega_n = \sqrt{\frac{k}{m}}$$

$$E = \frac{A^2}{2}\left(m\frac{k}{m}\cos^2(\omega_n t) + k\sin^2(\omega_n t)\right) =$$

$$= \frac{A^2 k}{2}\left(\cos^2(\omega_n t) + \sin^2(\omega_n t)\right) = \frac{A^2 k}{2} = \text{cost}$$

OSCILLAZIONE FORZATA SISTEMA 1 G.D.L. NON SMORZATO

Sistema ad 1 g.d.l. $m\ddot{x}+kx=F_0e^{i\Omega t}=F_0\cos(\Omega t)$ $(\Omega\neq\omega_n)$

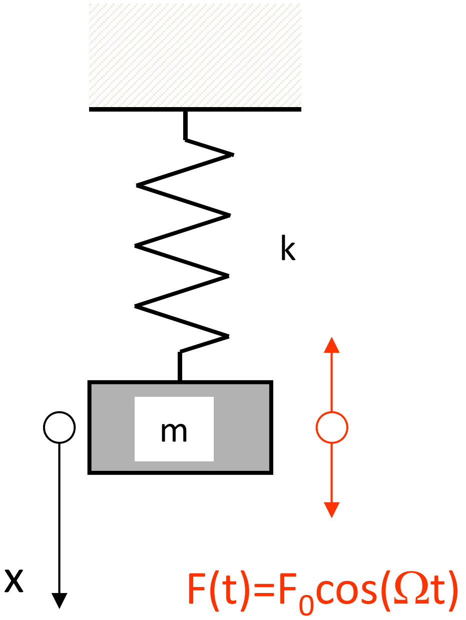


OSCILLAZIONE FORZATA SISTEMA 1 G.D.L. NON SMORZATO

Sistema ad 1 g.d.l.

$$m\ddot{x} + kx = F_0 e^{i\Omega t} = F_0 \cos(\Omega t) \qquad (\Omega \neq \omega_n)$$

$$x(t) = C_1 e^{z_1 t} + C_2 e^{z_2 t} + X e^{i\Omega t}$$



OSCILLAZIONE FORZATA SISTEMA 1 G.D.L. NON SMORZATO

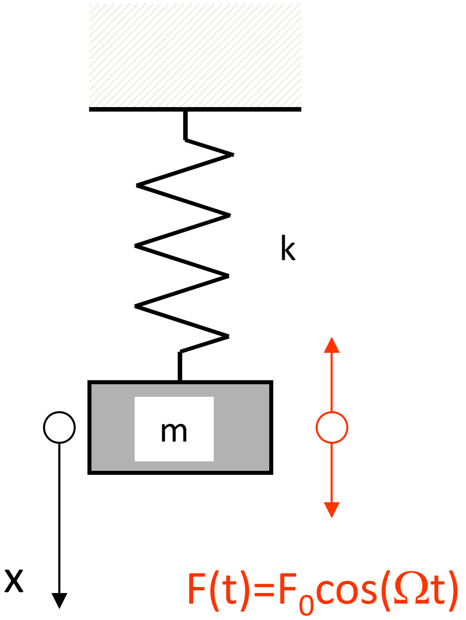
Sistema ad 1 g.d.l.

$$m\ddot{x} + kx = F_0 e^{i\Omega t} = F_0 \cos(\Omega t) \qquad (\Omega \neq \omega_n)$$

$$x(t) = \boxed{C_1 e^{z_1 t} + C_2 e^{z_2 t}} + \boxed{X e^{i\Omega t}}$$

Integrale generale omogenea associata

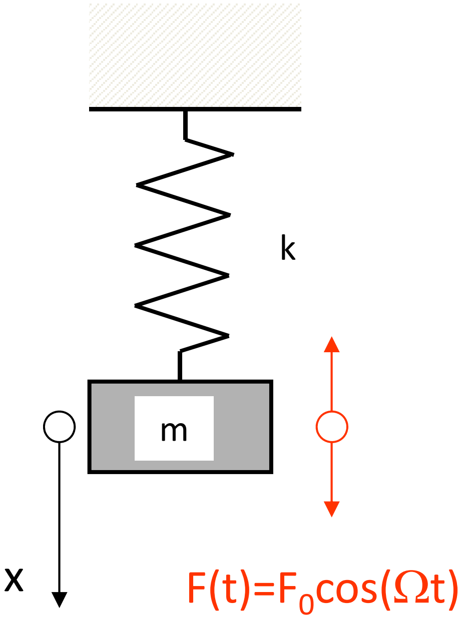
Integrale particolare non omogenea



OSCILLAZIONE FORZATA SISTEMA 1 G.D.L. NON SMORZATO

Sistema ad 1 g.d.l.

$$m\ddot{x} + kx = F_0 e^{i\Omega t} = F_0 \cos(\Omega t) \qquad (\Omega \neq \omega_n)$$



F(t)=F_0cos(Ωt)

$$x(t) = \boxed{C_1 e^{z_1 t} + C_2 e^{z_2 t}} + \boxed{X e^{i\Omega t}}$$

Integrale generale omogenea associata

Integrale particolare non omogenea

Verifica validità integrale particolare non omogenea:

$$x(t) = X e^{i\Omega t}$$

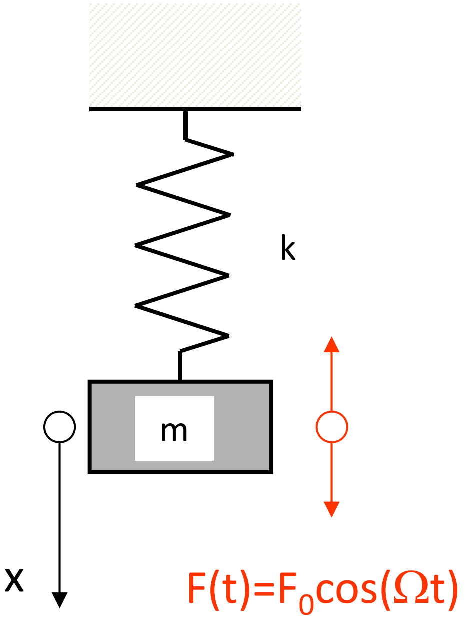
$$\dot{x}(t) = i\Omega X e^{i\Omega t}$$

$$\ddot{x}(t) = -\Omega^2 X e^{i\Omega t}$$

OSCILLAZIONE FORZATA SISTEMA 1 G.D.L. NON SMORZATO

Sistema ad 1 g.d.l.

$$m\ddot{x} + kx = F_0 e^{i\Omega t} = F_0 \cos(\Omega t) \qquad (\Omega \neq \omega_n)$$



F(t)=F_0cos(Ωt)

$$x(t) = \boxed{C_1 e^{z_1 t} + C_2 e^{z_2 t}} + \boxed{X e^{i\Omega t}}$$

Integrale generale omogenea associata

Integrale particolare non omogenea

Verifica validità integrale particolare non omogenea:

$$x(t) = X e^{i\Omega t} \qquad \dot{x}(t) = i\Omega X e^{i\Omega t}$$

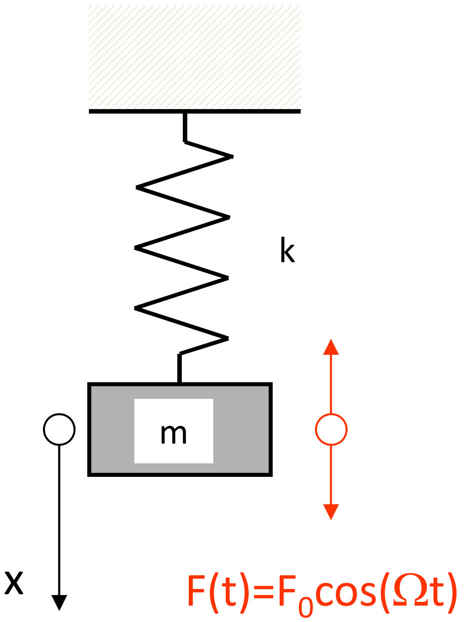
$$\ddot{x}(t) = -\Omega^2 X e^{i\Omega t}$$

$$-m\Omega^2 X e^{i\Omega t} + kX e^{i\Omega t} = F_0 e^{i\Omega t}$$

OSCILLAZIONE FORZATA SISTEMA 1 G.D.L. NON SMORZATO

Sistema ad 1 g.d.l.

$$m\ddot{x} + kx = F_0 e^{i\Omega t} = F_0 \cos(\Omega t) \qquad (\Omega \neq \omega_n)$$



$$x(t) = \boxed{C_1 e^{z_1 t} + C_2 e^{z_2 t}} + \boxed{X e^{i\Omega t}}$$

Integrale generale omogenea associata

Integrale particolare non omogenea

Verifica validità integrale particolare non omogenea:

$$x(t) = X e^{i\Omega t} \qquad \dot{x}(t) = i\Omega X e^{i\Omega t}$$

$$\ddot{x}(t) = -\Omega^2 X e^{i\Omega t}$$

$$-m\Omega^2 X e^{i\Omega t} + kX e^{i\Omega t} = F_0 e^{i\Omega t}$$

$$X = \frac{F_0}{k - m\Omega^2} = \frac{F_0}{k} \frac{1}{1 - \frac{m\Omega^2}{k}} = \frac{F_0}{k} \frac{1}{1 - \left(\frac{\Omega}{\omega_n}\right)^2}$$

OSCILLAZIONE FORZATA SISTEMA 1 G.D.L. NON SMORZATO

$$X=\frac{F_0}{k}\frac{1}{1-\left(\frac{\Omega}{\omega_n}\right)^2}$$

OSCILLAZIONE FORZATA SISTEMA 1 G.D.L. NON SMORZATO

$$X = \boxed{\frac{F_0}{k}} \frac{1}{1-\left(\frac{\Omega}{\omega_n}\right)^2}$$

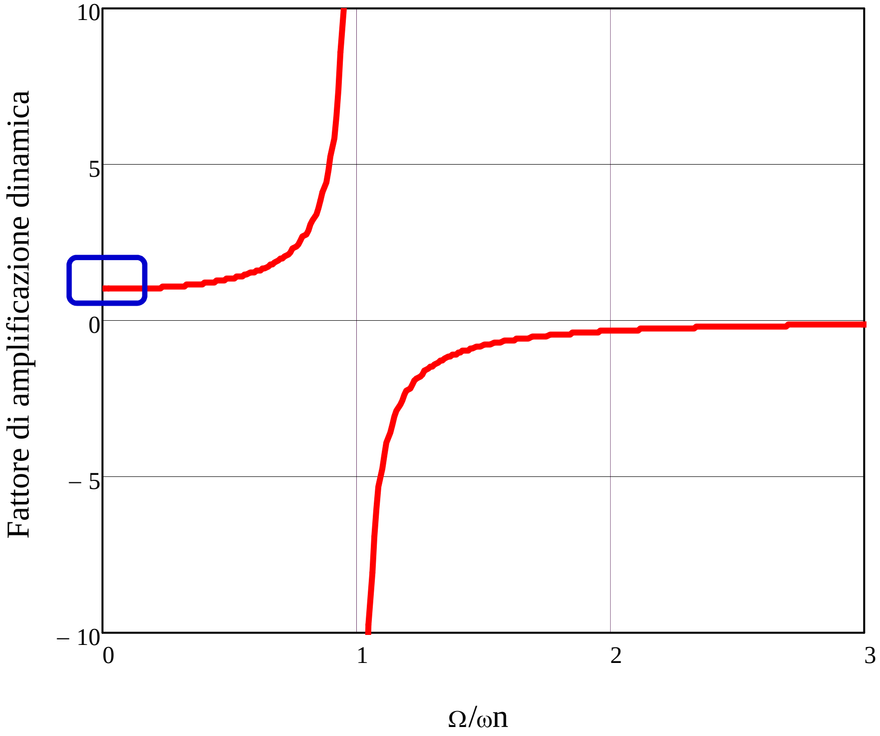
Freccia statica

OSCILLAZIONE FORZATA SISTEMA 1 G.D.L. NON SMORZATO

$$X = \boxed{\frac{F_0}{k}}\boxed{\frac{1}{1-\left(\frac{\Omega}{\omega_n}\right)^2}}$$

Freccia statica

Fattore di amplificazione dinamica

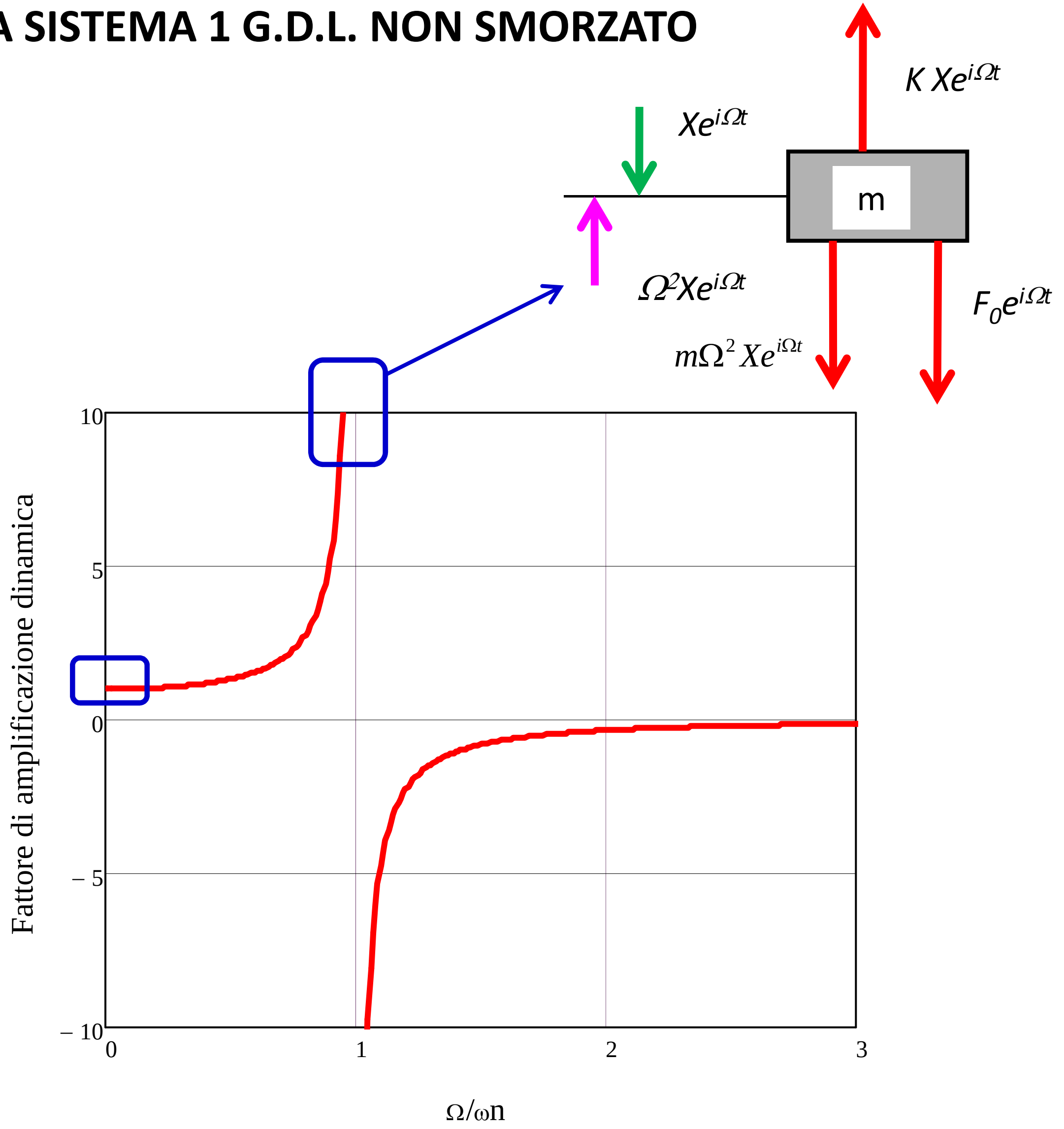


OSCILLAZIONE FORZATA SISTEMA 1 G.D.L. NON SMORZATO

$$X = \boxed{\frac{F_0}{k}}\boxed{\frac{1}{1-\left(\frac{\Omega}{\omega_n}\right)^2}}$$

Freccia statica

Fattore di amplificazione dinamica

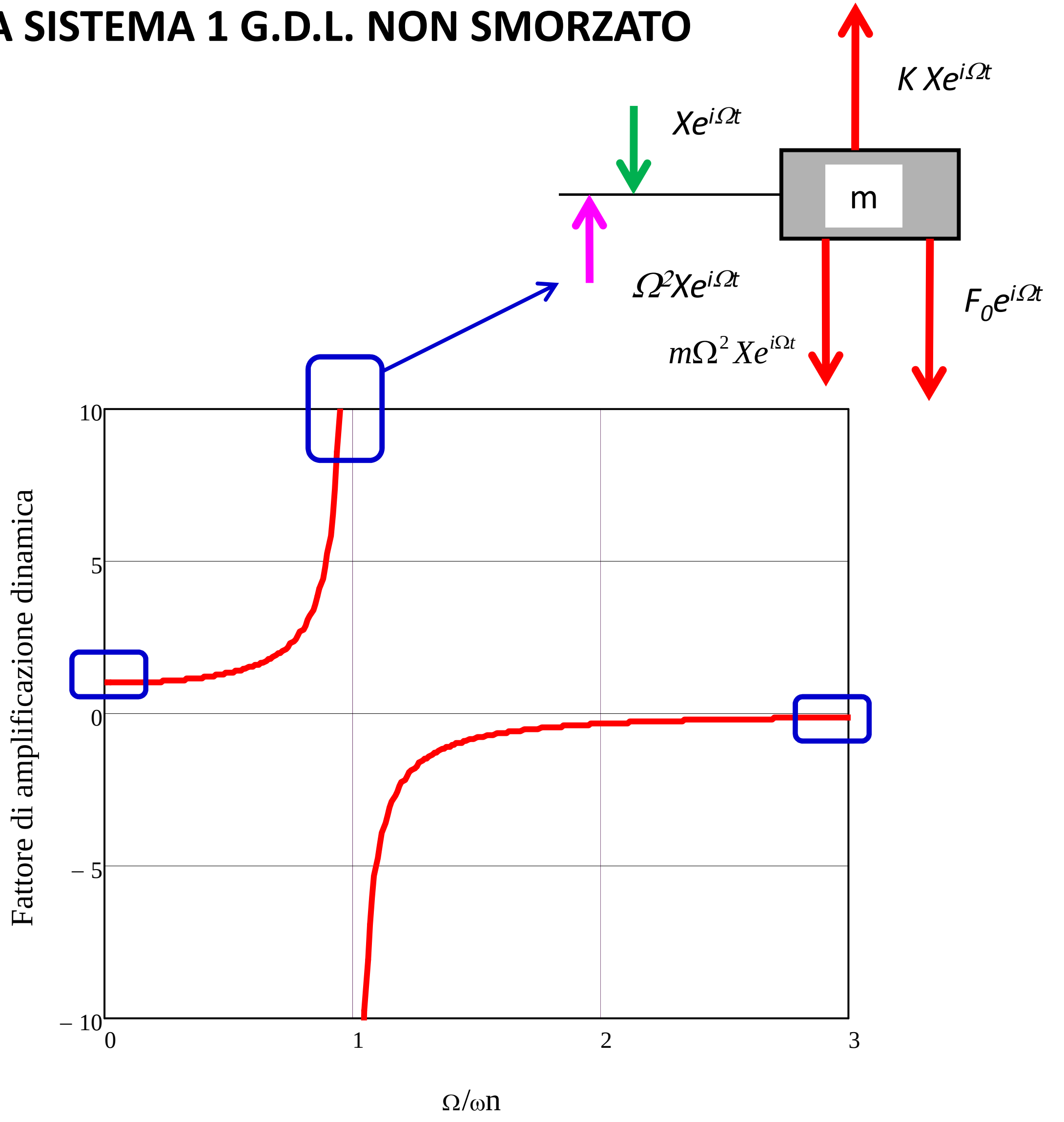


OSCILLAZIONE FORZATA SISTEMA 1 G.D.L. NON SMORZATO

$$X = \boxed{\frac{F_0}{k}}\boxed{\frac{1}{1-\left(\frac{\Omega}{\omega_n}\right)^2}}$$

Freccia statica

Fattore di amplificazione dinamica

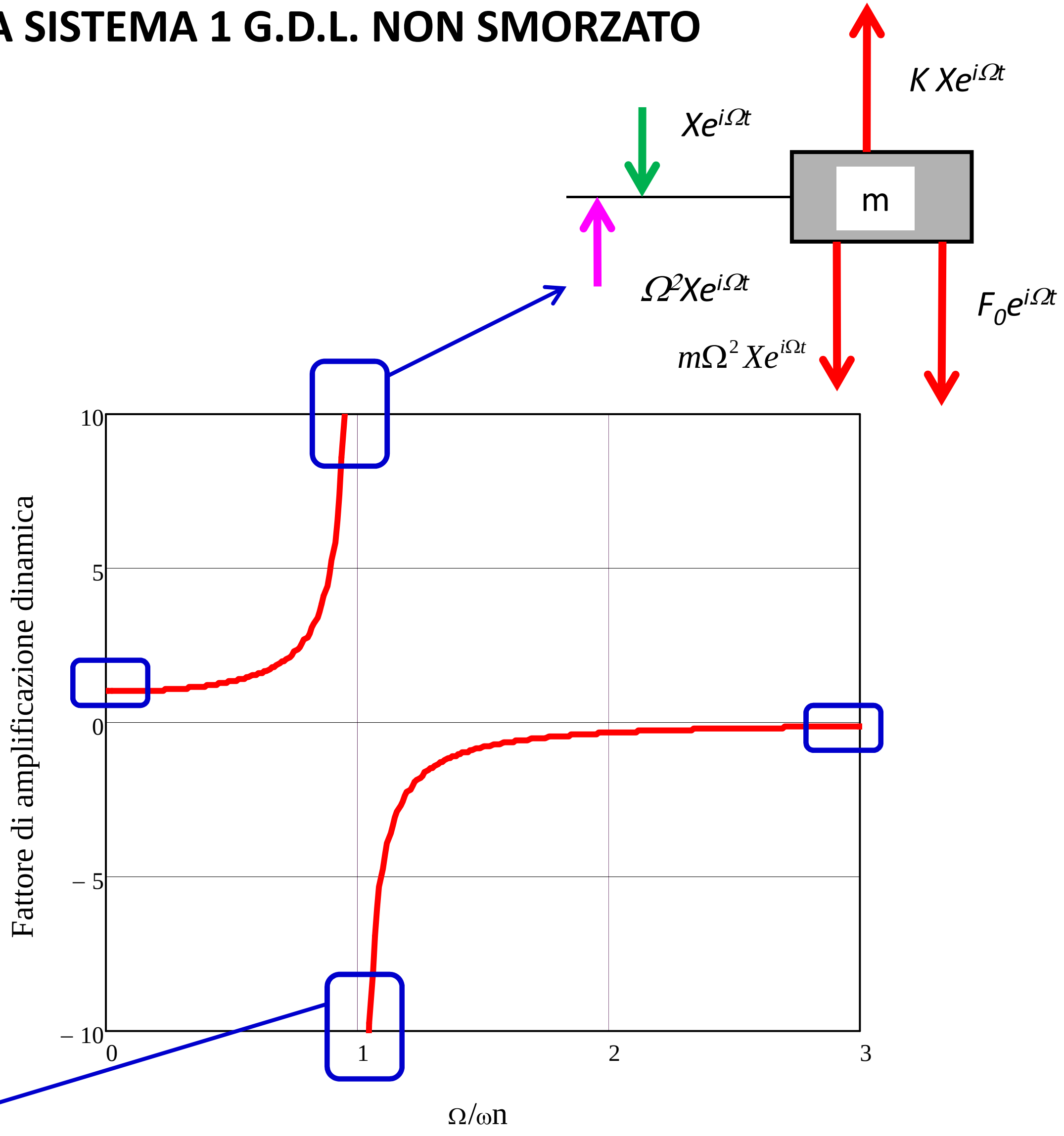


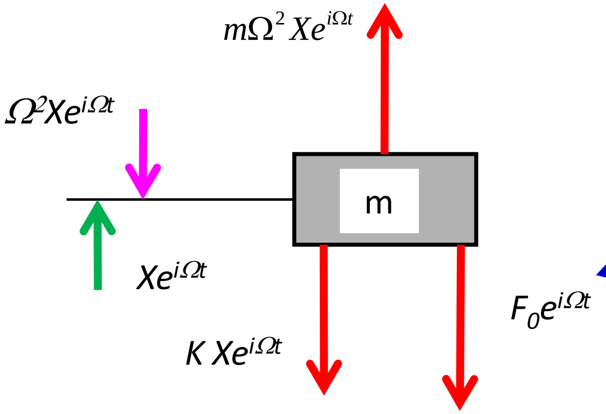
OSCILLAZIONE FORZATA SISTEMA 1 G.D.L. NON SMORZATO

$$X = \boxed{\frac{F_0}{k}}\,\boxed{\frac{1}{1-\left(\frac{\Omega}{\omega_n}\right)^2}}$$

Freccia statica

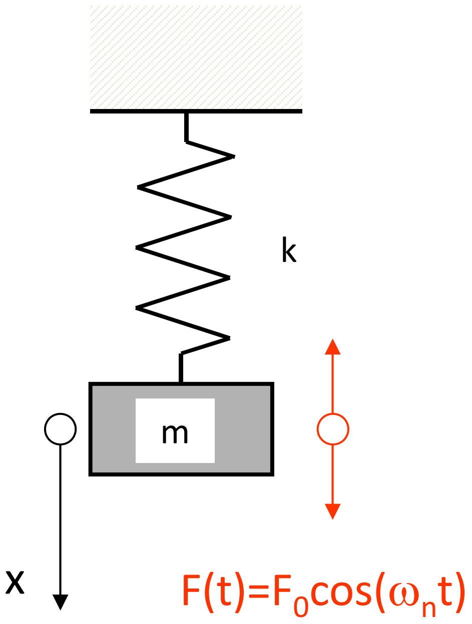
Fattore di amplificazione dinamica





OSCILLAZIONE FORZATA IN RISONANZA SISTEMA 1 G.D.L. NON SMORZATO

Sistema ad 1 g.d.l. $$m\ddot{x} + kx = F_0 e^{i\omega_n t}$$

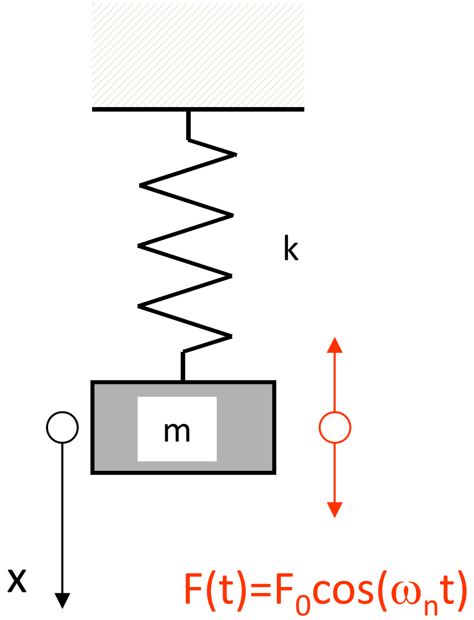


OSCILLAZIONE FORZATA IN RISONANZA SISTEMA 1 G.D.L. NON SMORZATO

Sistema ad 1 g.d.l.

$$m\ddot{x} + kx = F_0 e^{i\omega_n t}$$

$$\ddot{x} + \omega_n^2 x = \frac{F_0}{m} e^{i\omega_n t}$$

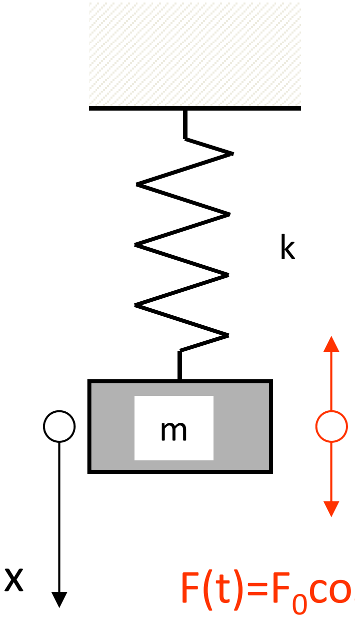


OSCILLAZIONE FORZATA IN RISONANZA SISTEMA 1 G.D.L. NON SMORZATO

Sistema ad 1 g.d.l.

$$m\ddot{x} + kx = F_0 e^{i\omega_n t}$$

$$\ddot{x} + \omega_n^2 x = \frac{F_0}{m} e^{i\omega_n t}$$



$F(t)=F_0\cos(\omega_n t)$

$$x(t) = \boxed{Xte^{i\omega_n t}}$$

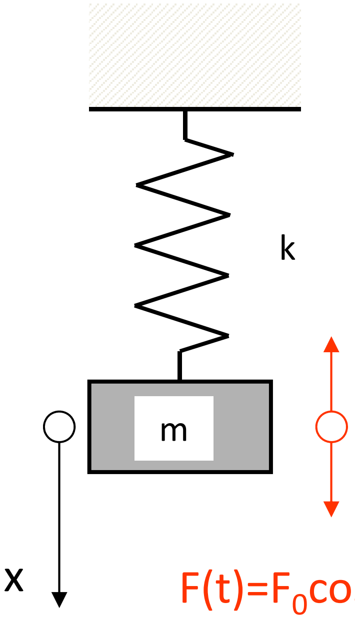
Integrale particolare
non omogenea

OSCILLAZIONE FORZATA IN RISONANZA SISTEMA 1 G.D.L. NON SMORZATO

Sistema ad 1 g.d.l.

$$m\ddot{x} + kx = F_0 e^{i\omega_n t}$$

$$\ddot{x} + \omega_n^2 x = \frac{F_0}{m} e^{i\omega_n t}$$



$F(t)=F_0\cos(\omega_n t)$

$$x(t) = \boxed{Xte^{i\omega_n t}}$$

Integrale particolare non omogenea

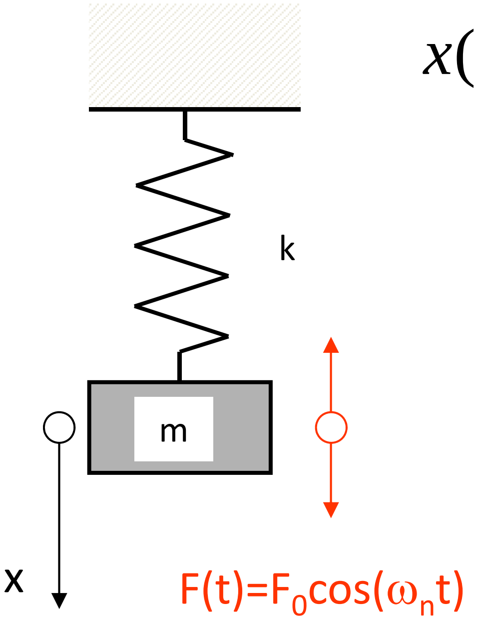
$$\dot{x}(t) = Xe^{i\omega_n t}\left(1 + i\omega_n t\right)$$

$$\ddot{x}(t) = \omega_n Xe^{i\omega_n t}\left(2i - \omega_n t\right)$$

OSCILLAZIONE FORZATA IN RISONANZA SISTEMA 1 G.D.L. NON SMORZATO

Sistema ad 1 g.d.l.

$$m\ddot{x} + kx = F_0 e^{i\omega_n t} \qquad \ddot{x} + \omega_n^2 x = \frac{F_0}{m} e^{i\omega_n t}$$



$$x(t) = \boxed{Xte^{i\omega_n t}}$$

Integrale particolare
non omogenea

$$\dot{x}(t) = Xe^{i\omega_n t}\left(1 + i\omega_n t\right)$$

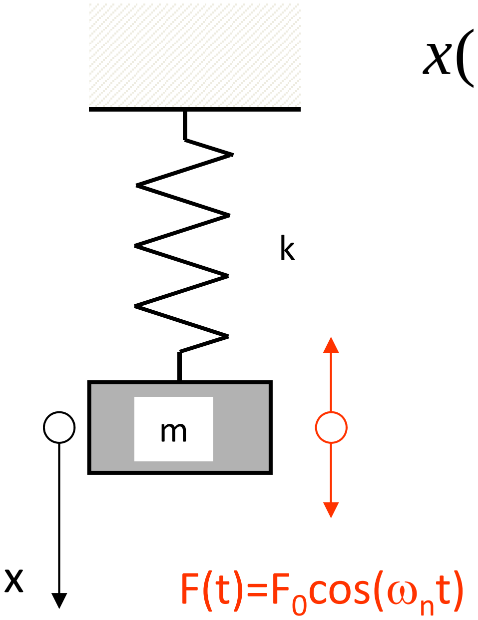
$$\ddot{x}(t) = \omega_n Xe^{i\omega_n t}\left(2i - \omega_n t\right)$$

$$\omega_n Xe^{i\omega_n t}\left(2i - \omega_n t\right) + \omega_n^2 Xte^{i\omega_n t} = \frac{F_0}{m} e^{i\omega_n t}$$

OSCILLAZIONE FORZATA IN RISONANZA SISTEMA 1 G.D.L. NON SMORZATO

Sistema ad 1 g.d.l.

$$m\ddot{x} + kx = F_0 e^{i\omega_n t} \qquad \ddot{x} + \omega_n^2 x = \frac{F_0}{m} e^{i\omega_n t}$$



$$x(t) = \boxed{Xte^{i\omega_n t}}$$

Integrale particolare
non omogenea

$$\dot{x}(t) = Xe^{i\omega_n t}\left(1 + i\omega_n t\right)$$

$$\ddot{x}(t) = \omega_n X e^{i\omega_n t}\left(2i - \omega_n t\right)$$

$$\omega_n X e^{i\omega_n t}\left(2i - \omega_n t\right) + \omega_n^2 Xte^{i\omega_n t} = \frac{F_0}{m} e^{i\omega_n t}$$

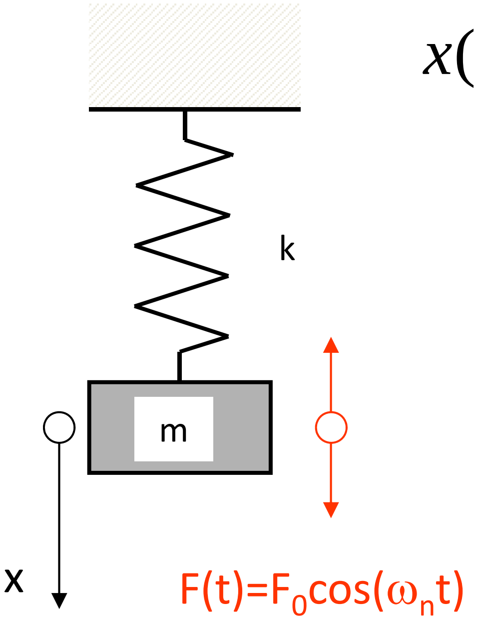
$$2i\omega_n X - \omega_n^2 Xt + \omega_n^2 Xt = \frac{F_0}{m}$$

OSCILLAZIONE FORZATA IN RISONANZA SISTEMA 1 G.D.L. NON SMORZATO

Sistema ad 1 g.d.l.

$$m\ddot{x}+kx=F_0e^{i\omega_n t}$$

$$\ddot{x}+\omega_n^2x=\frac{F_0}{m}e^{i\omega_n t}$$



F(t)=F$_0$cos(ω$_n$t)

$$x(t)=\boxed{Xte^{i\omega_n t}}$$

Integrale particolare
non omogenea

$$\dot{x}(t)=Xe^{i\omega_n t}\left(1+i\omega_n t\right)$$

$$\ddot{x}(t)=\omega_n Xe^{i\omega_n t}\left(2i-\omega_n t\right)$$

$$\omega_n Xe^{i\omega_n t}\left(2i-\omega_n t\right)+\omega_n^2 Xte^{i\omega_n t}=\frac{F_0}{m}e^{i\omega_n t}$$

$$2i\omega_n X-\omega_n^2Xt+\omega_n^2Xt=\frac{F_0}{m}$$

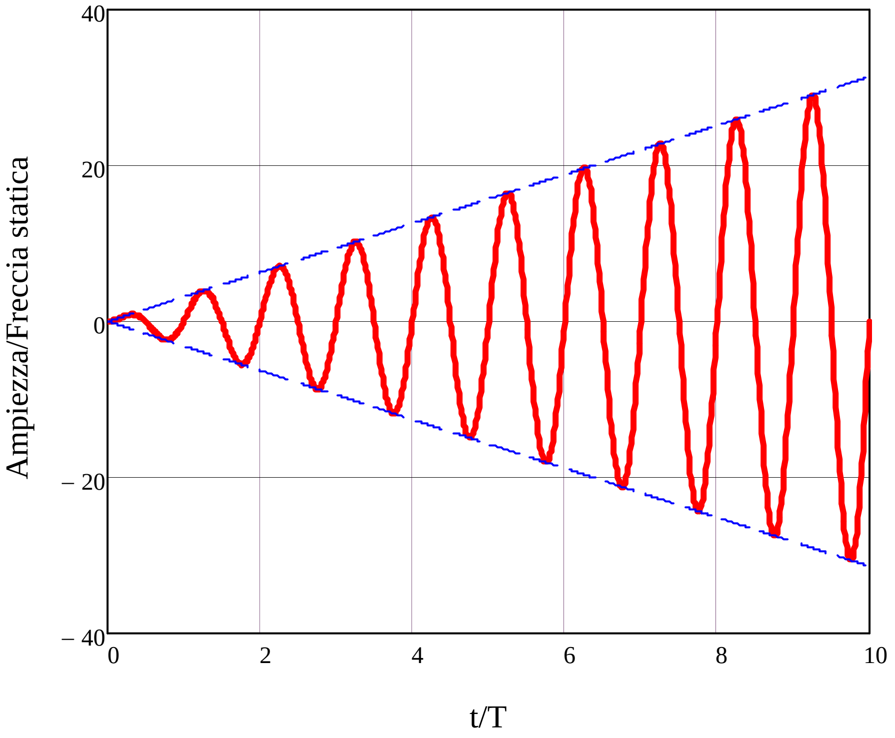
$$X=\frac{F_0}{m}\frac{1}{2i\omega_n}=\frac{F_0}{k}\frac{\omega_n}{2i}$$

OSCILLAZIONE FORZATA IN RISONANZA SISTEMA 1 G.D.L. NON SMORZATO

$$x(t) = \frac{F_0}{k}\frac{\omega_n}{2i} t e^{i\omega_n t} = -i\pi \frac{F_0}{k}\frac{t}{T} e^{i\frac{2\pi}{T}t}$$

OSCILLAZIONE FORZATA IN RISONANZA SISTEMA 1 G.D.L. NON SMORZATO

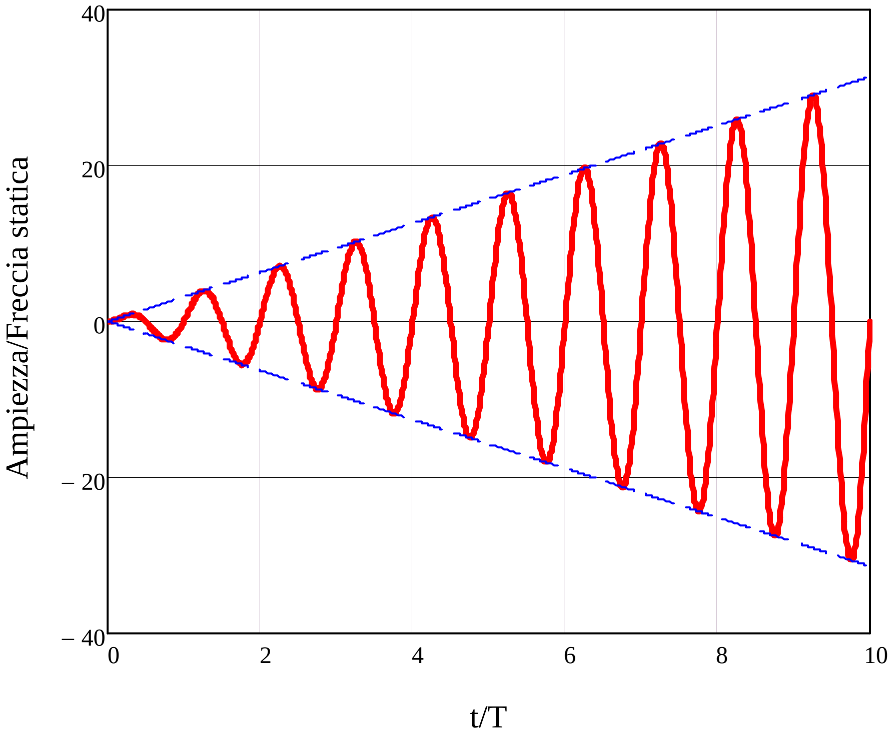
$$x(t) = \frac{F_0}{k}\frac{\omega_n}{2i}te^{i\omega_n t} = -i\pi\frac{F_0}{k}\frac{t}{T}e^{i\frac{2\pi}{T}t}$$



OSCILLAZIONE FORZATA IN RISONANZA SISTEMA 1 G.D.L. NON SMORZATO

$$x(t) = \frac{F_0}{k}\frac{\omega_n}{2i} t e^{i\omega_n t} = -i\pi \frac{F_0}{k}\frac{t}{T} e^{i\frac{2\pi}{T}t}$$

Dato che, in corrispondenza di ω_n, il sistema è in grado di oscillare senza cedere energia all’esterno, tutto il lavoro fatto dalla forza applicata si tasforma in aumento del suo contenuto enegetico.



OSCILLAZIONE FORZATA IN RISONANZA SISTEMA 1 G.D.L. NON SMORZATO

$$\ddot{x} + \omega_n^2 x = \frac{F_0}{m} e^{i\omega_n t}$$

$$x(t) = -i \frac{F_0}{2k} \omega_n t e^{i\frac{2\pi}{T}t}$$

OSCILLAZIONE FORZATA IN RISONANZA SISTEMA 1 G.D.L. NON SMORZATO

$$\ddot{x}+\omega_n^2 x=\frac{F_0}{m}e^{i\omega_n t}$$

$$x(t)=-i\frac{F_0}{2k}\omega_n t e^{i\frac{2\pi}{T}t}$$

$$\frac{F_0}{k}\omega_n^2 e^{i\omega_n t}(1+i\frac{\omega_n}{2}t)-i\frac{F_0}{2k}\omega_n^3 t e^{i\omega_n t}=\frac{F_0}{k}\omega_n^2 e^{i\omega_n t}$$

OSCILLAZIONE FORZATA IN RISONANZA SISTEMA 1 G.D.L. NON SMORZATO

$$\boxed{\ddot{x}} + \omega_n^2 x = \frac{F_0}{m} e^{i\omega_n t}$$

$$x(t) = -i\frac{F_0}{2k}\omega_n t e^{i\frac{2\pi}{T}t}$$

Forza d'inerzia

$$\boxed{\frac{F_0}{k}\omega_n^2 e^{i\omega_n t}\left(1+i\frac{\omega_n}{2}t\right)} - i\frac{F_0}{2k}\omega_n^3 t e^{i\omega_n t} = \frac{F_0}{k}\omega_n^2 e^{i\omega_n t}$$

OSCILLAZIONE FORZATA IN RISONANZA SISTEMA 1 G.D.L. NON SMORZATO

$$\boxed{\ddot{x}} + \boxed{\omega_n^2 x} = \frac{F_0}{m} e^{i\omega_n t}$$

$$x(t) = -i\frac{F_0}{2k}\omega_n t e^{i\frac{2\pi}{T}t}$$

Forza d'inerzia

Forza elastica

$$\boxed{\frac{F_0}{k}\omega_n^2 e^{i\omega_n t}(1+i\frac{\omega_n}{2}t)} - \boxed{i\frac{F_0}{2k}\omega_n^3 t e^{i\omega_n t}} = \frac{F_0}{k}\omega_n^2 e^{i\omega_n t}$$

OSCILLAZIONE FORZATA IN RISONANZA SISTEMA 1 G.D.L. NON SMORZATO

$$\boxed{\ddot{x}} + \boxed{\omega_n^2 x} = \boxed{\frac{F_0}{m} e^{i\omega_n t}}$$

$$x(t) = -i\frac{F_0}{2k}\omega_n t e^{i\frac{2\pi}{T}t}$$

Forza d’inerzia

Forza elastica

Forza esterna

$$\boxed{\frac{F_0}{k}\omega_n^2 e^{i\omega_n t}(1+i\frac{\omega_n}{2}t)} - \boxed{i\frac{F_0}{2k}\omega_n^3 t e^{i\omega_n t}} = \boxed{\frac{F_0}{k}\omega_n^2 e^{i\omega_n t}}$$

OSCILLAZIONE FORZATA IN RISONANZA SISTEMA 1 G.D.L. NON SMORZATO

$$\boxed{\ddot{x}} + \boxed{\omega_n^2 x} = \boxed{\frac{F_0}{m} e^{i\omega_n t}}$$

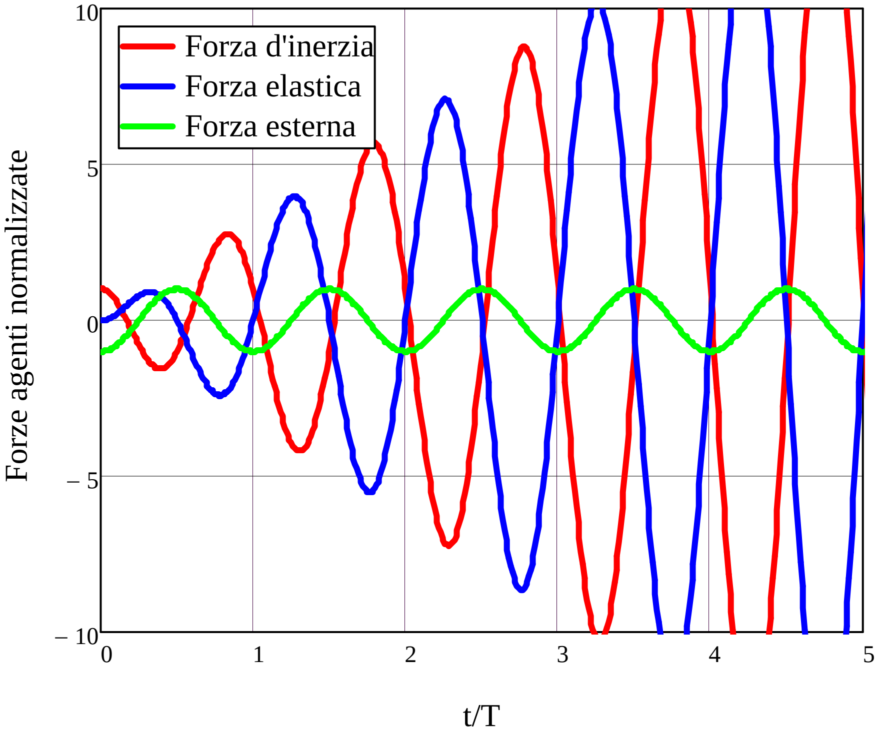
$$x(t) = -i\frac{F_0}{2k}\omega_n t e^{i\frac{2\pi}{T}t}$$

Forza d'inerzia

Forza elastica

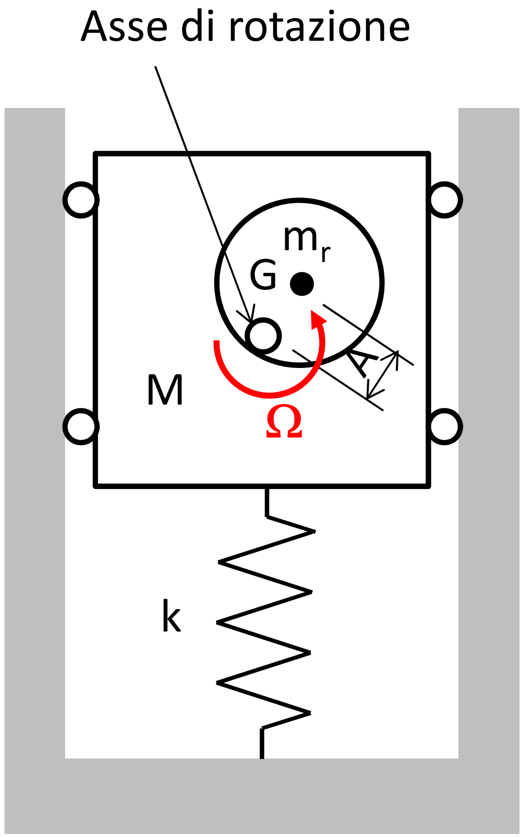
Forza esterna

$$\boxed{\frac{F_0}{k}\omega_n^2 e^{i\omega_n t}\left(1+i\frac{\omega_n}{2}t\right)} - \boxed{i\frac{F_0}{2k}\omega_n^3 t e^{i\omega_n t}} = \boxed{\frac{F_0}{k}\omega_n^2 e^{i\omega_n t}}$$



OSCILLAZIONE FORZATA SISTEMA 1 G.D.L. NON SMORZATO

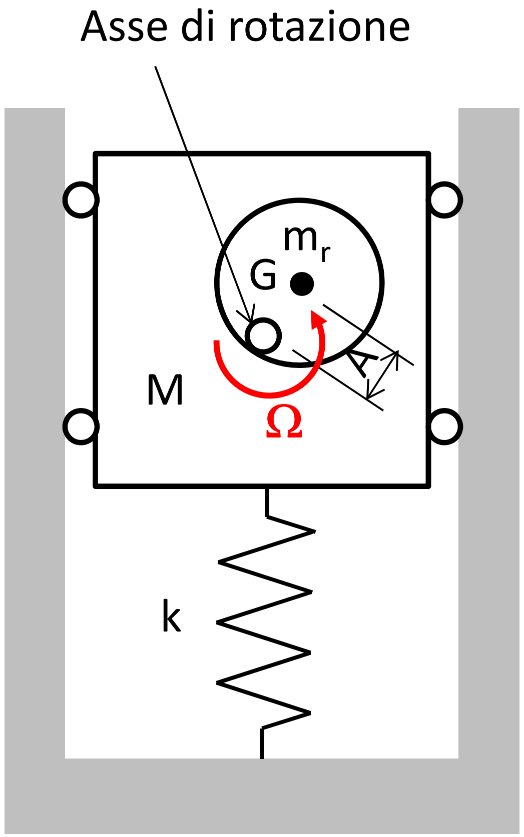
Forzante armonica di ampiezza proporzionale al quadrato della pulsazione



$$M\ddot{x}+kx=m_r A\Omega^2 e^{i\Omega t}$$

OSCILLAZIONE FORZATA SISTEMA 1 G.D.L. NON SMORZATO

Forzante armonica di ampiezza proporzionale al quadrato della pulsazione

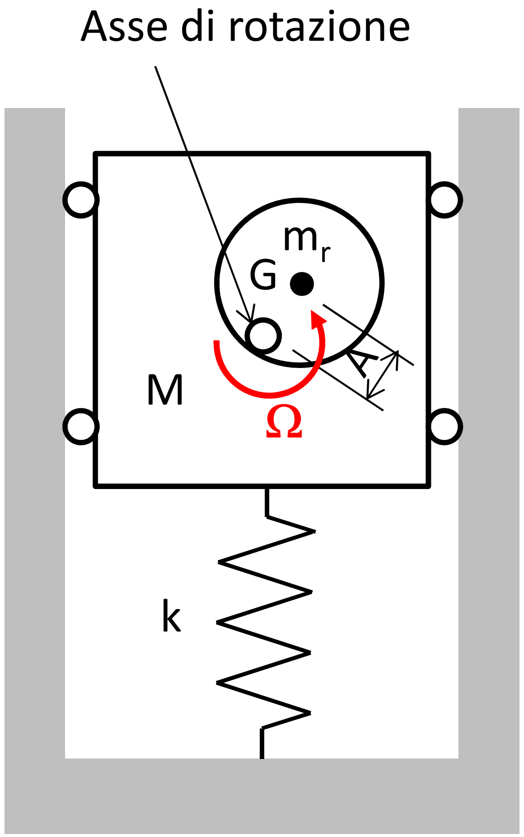


$$M\ddot{x} + kx = m_r A\Omega^2 e^{i\Omega t}$$

$$\ddot{x} + \omega_n^2 x = \frac{m_r A}{M}\Omega^2 e^{i\Omega t}$$

OSCILLAZIONE FORZATA SISTEMA 1 G.D.L. NON SMORZATO

Forzante armonica di ampiezza proporzionale al quadrato della pulsazione



$$M\ddot{x} + kx = m_r A \Omega^2 e^{i\Omega t}$$

$$\ddot{x} + \omega_n^2 x = \frac{m_r A}{M} \Omega^2 e^{i\Omega t}$$

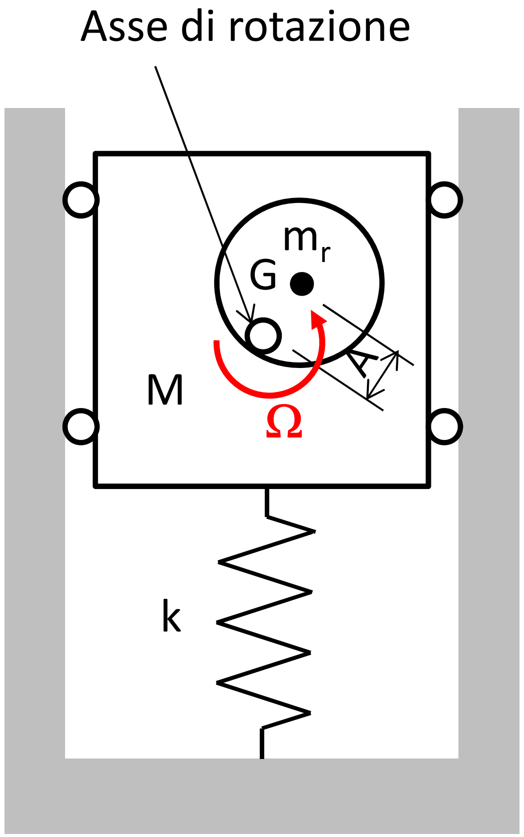
$$x(t) = X e^{i\Omega t}$$

$$\dot{x}(t) = i\Omega X e^{i\Omega t}$$

$$\ddot{x}(t) = -\Omega^2 X e^{i\Omega t}$$

OSCILLAZIONE FORZATA SISTEMA 1 G.D.L. NON SMORZATO

Forzante armonica di ampiezza proporzionale al quadrato della pulsazione



$$M\ddot{x} + kx = m_r A\Omega^2 e^{i\Omega t}$$

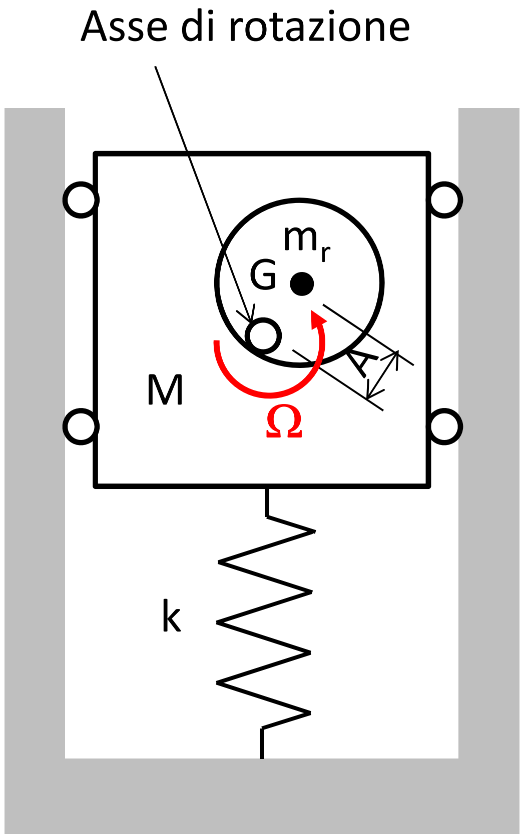
$$\ddot{x} + \omega_n^2 x = \frac{m_r A}{M}\Omega^2 e^{i\Omega t}$$

$$x(t) = Xe^{i\Omega t} \qquad \dot{x}(t) = i\Omega Xe^{i\Omega t}$$

$$\ddot{x}(t) = -\Omega^2 Xe^{i\Omega t}$$

$$-\Omega^2 Xe^{i\Omega t} + \omega_n^2 Xe^{i\Omega t} = \frac{m_r A}{M}\Omega^2 e^{i\Omega t}$$

OSCILLAZIONE FORZATA SISTEMA 1 G.D.L. NON SMORZATO
Forzante armonica di ampiezza proporzionale al quadrato della pulsazione



$$M\ddot{x} + kx = m_r A \Omega^2 e^{i\Omega t}$$

$$\ddot{x} + \omega_n^2 x = \frac{m_r A}{M} \Omega^2 e^{i\Omega t}$$

$$x(t) = X e^{i\Omega t} \qquad \dot{x}(t) = i\Omega X e^{i\Omega t}$$

$$\ddot{x}(t) = -\Omega^2 X e^{i\Omega t}$$

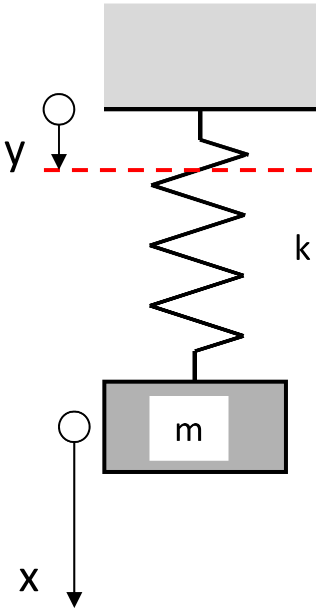
$$-\Omega^2 X e^{i\Omega t} + \omega_n^2 X e^{i\Omega t} = \frac{m_r A}{M} \Omega^2 e^{i\Omega t}$$

$$X = \frac{\dfrac{m_r A}{M} \Omega^2}{\omega_n^2 - \Omega^2} = \frac{\dfrac{m_r A}{M} \left(\dfrac{\Omega}{\omega_n} \right)^2}{1 - \left(\dfrac{\Omega}{\omega_n} \right)^2}$$

OSCILLAZIONE FORZATA SISTEMA 1 G.D.L. NON SMORZATO

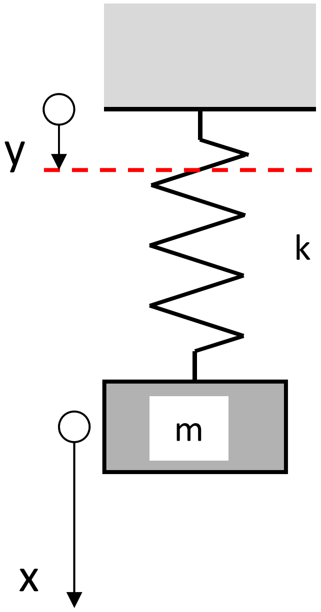
Eccitazione per moto del supporto

$$m\ddot{x} + k(x - y) = 0$$



OSCILLAZIONE FORZATA SISTEMA 1 G.D.L. NON SMORZATO

Eccitazione per moto del supporto

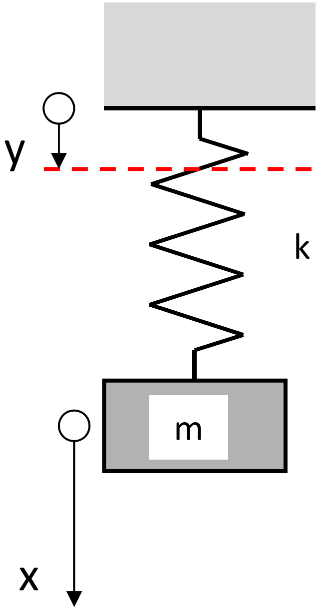


$$m\ddot{x}+k\left(x-y\right)=0$$

$$m\ddot{x}+kx=ky=kYe^{i\Omega t}$$

OSCILLAZIONE FORZATA SISTEMA 1 G.D.L. NON SMORZATO

Eccitazione per moto del supporto

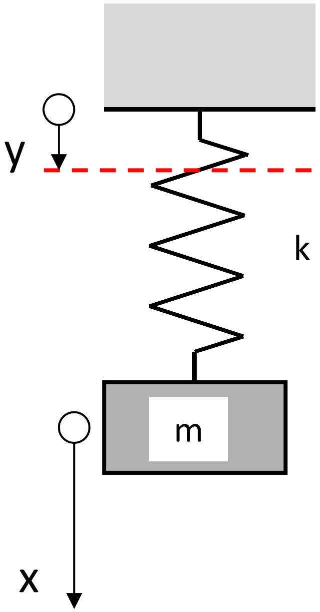


$$m\ddot{x}+k\left(x-y\right)=0$$

$$m\ddot{x}+kx=ky=kYe^{i\Omega t} \qquad \ddot{x}+\omega_n^2 x=\frac{k}{m}Ye^{i\Omega t}=\omega_n^2 Ye^{i\Omega t}$$

OSCILLAZIONE FORZATA SISTEMA 1 G.D.L. NON SMORZATO

Eccitazione per moto del supporto



$$m\ddot{x} + k(x - y) = 0$$

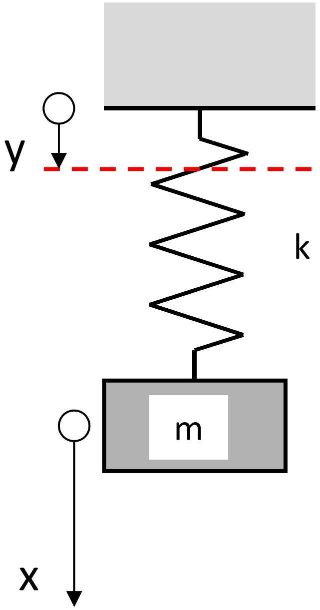
$$m\ddot{x} + kx = ky = kYe^{i\Omega t} \qquad \ddot{x} + \omega_n^2 x = \frac{k}{m} Ye^{i\Omega t} = \omega_n^2 Ye^{i\Omega t}$$

$$x(t) = Xe^{i\Omega t} \qquad \dot{x}(t) = i\Omega Xe^{i\Omega t}$$

$$\ddot{x}(t) = -\Omega^2 Xe^{i\Omega t}$$

OSCILLAZIONE FORZATA SISTEMA 1 G.D.L. NON SMORZATO

Eccitazione per moto del supporto



$$m\ddot{x}+k\left(x-y\right)=0$$

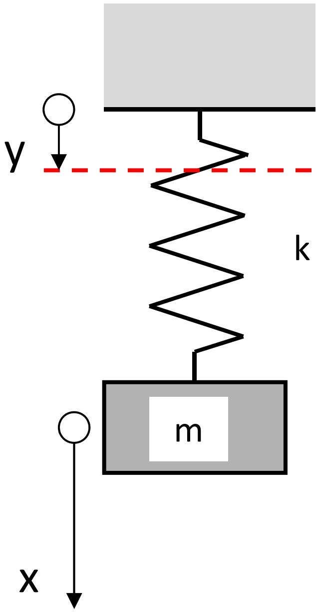
$$m\ddot{x}+kx=ky=kYe^{i\Omega t} \qquad \ddot{x}+\omega_n^2 x=\frac{k}{m}Ye^{i\Omega t}=\omega_n^2 Ye^{i\Omega t}$$

$$x(t)=Xe^{i\Omega t} \qquad \dot{x}(t)=i\Omega Xe^{i\Omega t}$$

$$\ddot{x}(t)=-\Omega^2 Xe^{i\Omega t}$$

$$-\Omega^2 Xe^{i\Omega t}+\omega_n^2 Xe^{i\Omega t}=\omega_n^2 Ye^{i\Omega t}$$

OSCILLAZIONE FORZATA SISTEMA 1 G.D.L. NON SMORZATO
Eccitazione per moto del supporto



$$m\ddot{x}+k\left(x-y\right)=0$$

$$m\ddot{x}+kx=ky=kYe^{i\Omega t} \qquad \ddot{x}+\omega_n^2 x=\frac{k}{m}Ye^{i\Omega t}=\omega_n^2 Ye^{i\Omega t}$$

$$x(t)=Xe^{i\Omega t} \qquad \dot{x}(t)=i\Omega Xe^{i\Omega t}$$

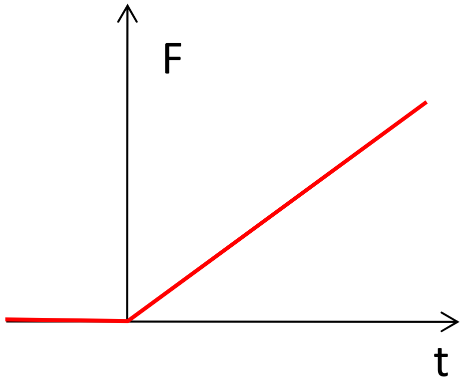
$$\ddot{x}(t)=-\Omega^2 Xe^{i\Omega t}$$

$$-\Omega^2 Xe^{i\Omega t}+\omega_n^2 Xe^{i\Omega t}=\omega_n^2 Ye^{i\Omega t}$$

$$X=\frac{\omega_n^2 Y}{\omega_n^2-\Omega^2}=\frac{Y}{1-\frac{\Omega^2}{\omega_n^2}}$$

OSCILLAZIONE SISTEMA 1 G.D.L. NON SMORZATO

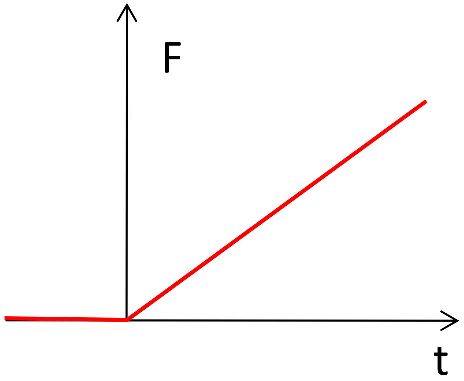
Sollecitazione con forza variabile "a rampa"



$$m\ddot{x} + kx = Bt$$

OSCILLAZIONE SISTEMA 1 G.D.L. NON SMORZATO

Sollecitazione con forza variabile “a rampa”

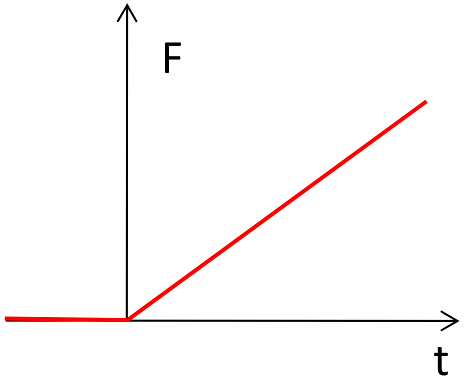


$$m\ddot{x} + kx = Bt$$

$$x(t) = A_1 e^{i\omega_n t} + B_1 e^{-i\omega_n t} + \frac{B}{k} t$$

OSCILLAZIONE SISTEMA 1 G.D.L. NON SMORZATO

Sollecitazione con forza variabile “a rampa”



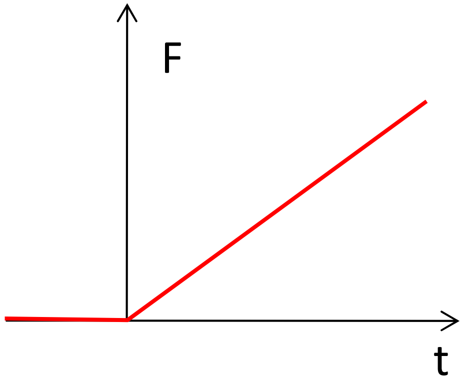
$$m\ddot{x} + kx = Bt$$

$$x(t) = \boxed{A_1 e^{i\omega_n t} + B_1 e^{-i\omega_n t}} + \frac{B}{k}t$$

Integrale generale
omogenea associata

OSCILLAZIONE SISTEMA 1 G.D.L. NON SMORZATO

Sollecitazione con forza variabile “a rampa”



$$m\ddot{x} + kx = Bt$$

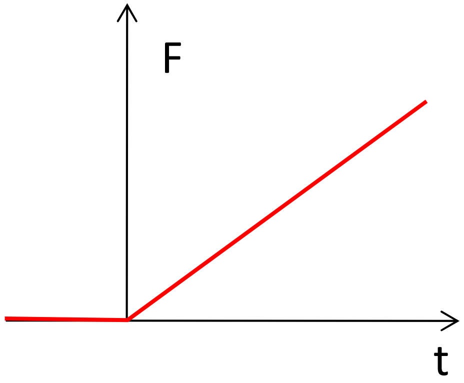
$$x(t) = \boxed{A_1 e^{i\omega_n t} + B_1 e^{-i\omega_n t}} + \boxed{\frac{B}{k} t}$$

Integrale generale omogenea associata

Integrale particolare non omogenea

OSCILLAZIONE SISTEMA 1 G.D.L. NON SMORZATO

Sollecitazione con forza variabile “a rampa”



$$m\ddot{x} + kx = Bt$$

$$x(t) = \boxed{A_1 e^{i\omega_n t} + B_1 e^{-i\omega_n t}} + \boxed{\frac{B}{k}t}$$

Integrale generale omogenea associata

Integrale particolare non omogenea

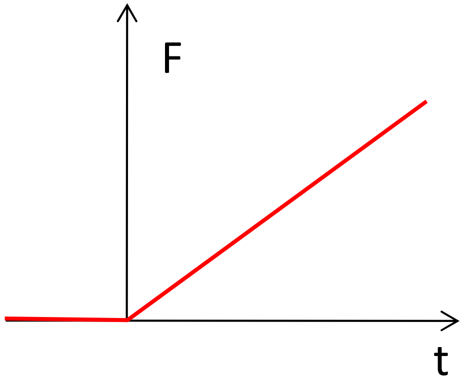
$$x(t) = \dot{X}t$$

$$\dot{x}(t) = \dot{X}$$

$$\ddot{x}(t) = 0$$

OSCILLAZIONE SISTEMA 1 G.D.L. NON SMORZATO

Sollecitazione con forza variabile “a rampa”



$$m\ddot{x}+kx=Bt$$

$$x(t)=\boxed{A_1e^{i\omega_n t}+B_1e^{-i\omega_n t}}+\boxed{\frac{B}{k}t}$$

Integrale generale omogenea associata

Integrale particolare non omogenea

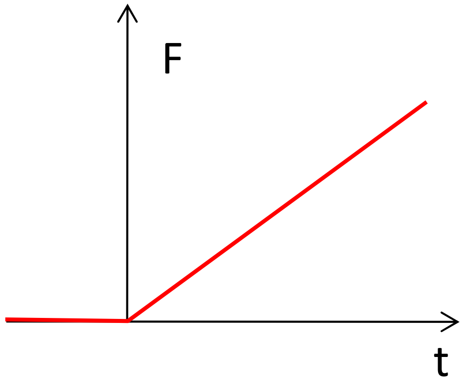
$$x(t)=\dot{X}t \qquad \dot{x}(t)=\dot{X}$$

$$\ddot{x}(t)=0$$

$$k\dot{X}t=Bt \quad \Rightarrow \quad \dot{X}=\frac{B}{k}$$

OSCILLAZIONE SISTEMA 1 G.D.L. NON SMORZATO

Sollecitazione con forza variabile “a rampa”

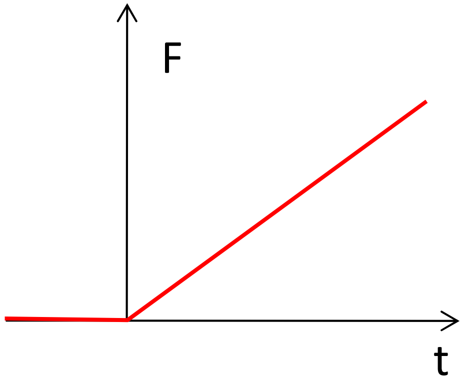


$$m\ddot{x} + kx = Bt$$

$$x(t) = A_1 e^{i\omega_n t} + B_1 e^{-i\omega_n t} + \frac{B}{k} t$$

OSCILLAZIONE SISTEMA 1 G.D.L. NON SMORZATO

Sollecitazione con forza variabile “a rampa”



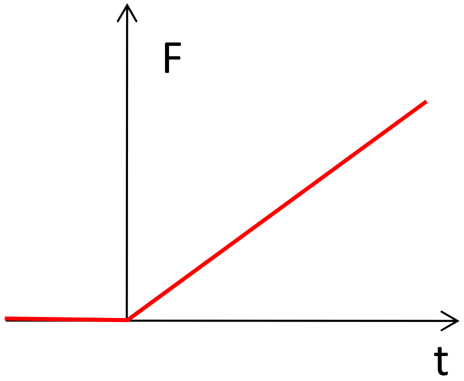
$$m\ddot{x}+kx=Bt$$

$$x(t)=A_1e^{i\omega_n t}+B_1e^{-i\omega_n t}+\frac{B}{k}t$$

Condizioni iniziali $\begin{cases} x(0)=0 \\ \dot{x}(0)=0 \end{cases}$

OSCILLAZIONE SISTEMA 1 G.D.L. NON SMORZATO

Sollecitazione con forza variabile "a rampa"



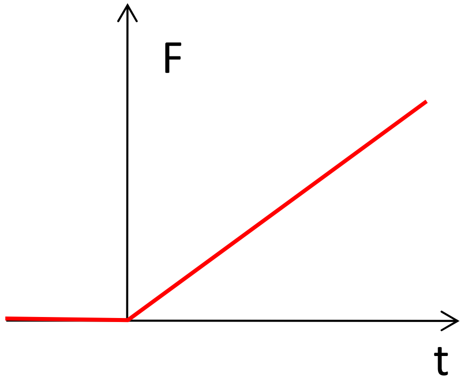
$$m\ddot{x}+kx=Bt$$

$$x(t)=A_1e^{i\omega_n t}+B_1e^{-i\omega_n t}+\frac{B}{k}t$$

Condizioni iniziali
$$\begin{cases} x(0)=0 \\ \dot{x}(0)=0 \end{cases}$$

$$\left.\begin{array}{l} x(0)=A_1+B_1=0 \\ \dot{x}(0)=i\omega_n A_1-i\omega_n B_1+\dfrac{B}{k} \end{array}\right\}\Rightarrow\begin{cases} A_1=i\dfrac{B}{2k\omega_n} \\ B_1=-i\dfrac{B}{2k\omega_n} \end{cases}$$

OSCILLAZIONE SISTEMA 1 G.D.L. NON SMORZATO
Sollecitazione con forza variabile "a rampa"



$$m\ddot{x}+kx=Bt$$

$$x(t)=A_1e^{i\omega_n t}+B_1e^{-i\omega_n t}+\frac{B}{k}t$$

Condizioni iniziali
$$\begin{cases} x(0)=0 \\ \dot{x}(0)=0 \end{cases}$$

$$\left.\begin{aligned} x(0)&=A_1+B_1=0 \\ \dot{x}(0)&=i\omega_n A_1-i\omega_n B_1+\frac{B}{k} \end{aligned}\right\} \Rightarrow \begin{cases} A_1=i\dfrac{B}{2k\omega_n} \\ B_1=-i\dfrac{B}{2k\omega_n} \end{cases}$$

$$x(t)=\frac{B}{k}\left(i\frac{e^{i\omega_n t}}{2\omega_n}-i\frac{e^{-i\omega_n t}}{2\omega_n}+t\right)=\frac{B}{k}\left(\frac{i}{2\omega_n}2iSin(\omega_n t)+t\right)=\frac{B}{k}\left(t-\frac{Sin(\omega_n t)}{\omega_n}\right)$$

OSCILLAZIONE SISTEMA 1 G.D.L. NON SMORZATO

Sollecitazione con forza variabile “a rampa”

$$x(t)=\frac{B}{k}\left(t-\frac{Sin(\omega_n t)}{\omega_n}\right)=\frac{BT}{k}\left(\frac{t}{T}-\frac{Sin(2\pi\frac{t}{T})}{2\pi}\right)$$

OSCILLAZIONE SISTEMA 1 G.D.L. NON SMORZATO

Sollecitazione con forza variabile “a rampa”

$$x(t) = \frac{B}{k}\left(t - \frac{Sin(\omega_n t)}{\omega_n}\right) = \frac{BT}{k}\left(\frac{t}{T} - \frac{Sin(2\pi \frac{t}{T})}{2\pi}\right)$$

Spostamento normalizzato

$$\frac{x(t)}{\frac{BT}{k}}$$

OSCILLAZIONE SISTEMA 1 G.D.L. NON SMORZATO

Sollecitazione con forza variabile “a rampa”

$$x(t) = \frac{B}{k}\left(t - \frac{Sin(\omega_n t)}{\omega_n}\right) = \frac{BT}{k}\left(\frac{t}{T} - \frac{Sin(2\pi \frac{t}{T})}{2\pi}\right)$$

Spostamento normalizzato

$$\frac{x(t)}{\frac{BT}{k}}$$

Spostamento/valore statico

$$\frac{x(t)}{\frac{Bt}{k}} = 1 - \frac{T}{t}\frac{Sin(2\pi \frac{t}{T})}{2\pi}$$

OSCILLAZIONE SISTEMA 1 G.D.L. NON SMORZATO

Sollecitazione con forza variabile "a rampa"

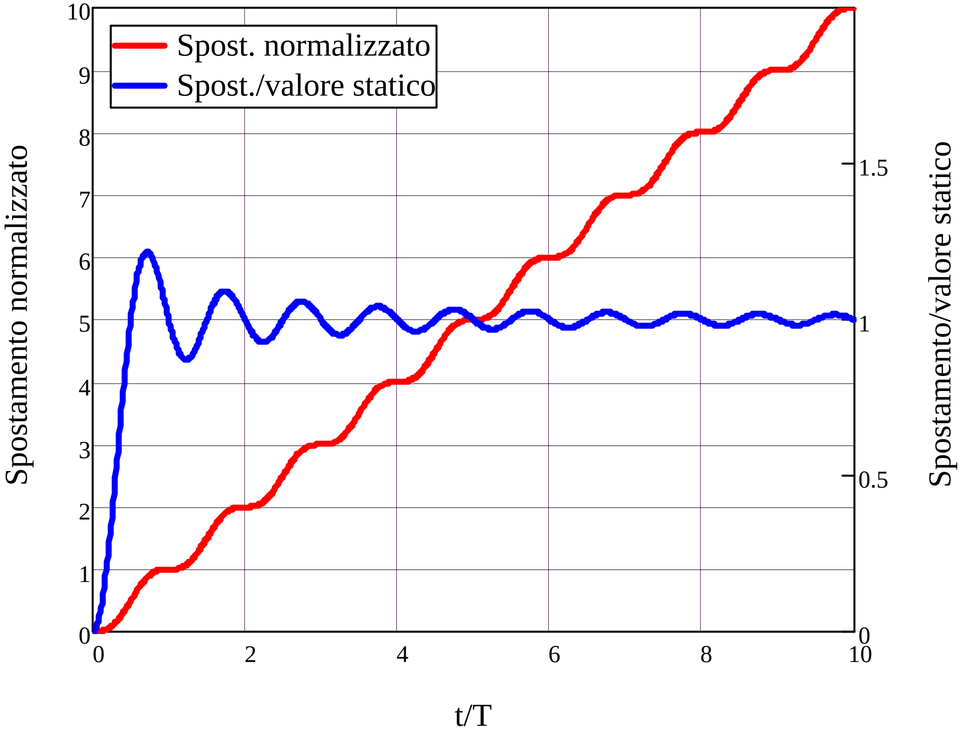
$$x(t) = \frac{B}{k}\left(t - \frac{Sin(\omega_n t)}{\omega_n}\right) = \frac{BT}{k}\left(\frac{t}{T} - \frac{Sin(2\pi\frac{t}{T})}{2\pi}\right)$$

Spostamento normalizzato

$$\frac{x(t)}{\frac{BT}{k}}$$

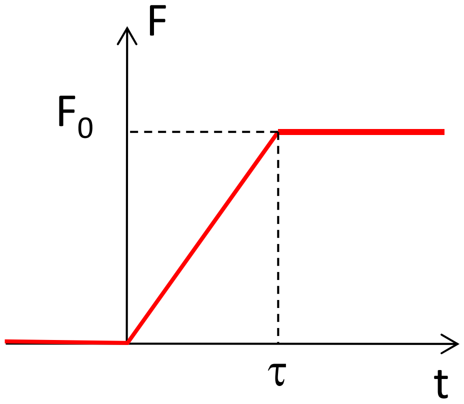
Spostamento/valore statico

$$\frac{x(t)}{\frac{Bt}{k}} = 1 - \frac{T}{t}\frac{Sin(2\pi\frac{t}{T})}{2\pi}$$



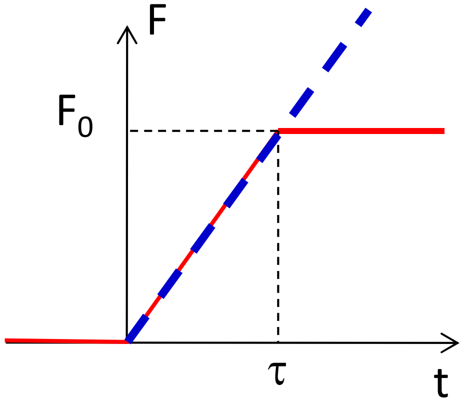
OSCILLAZIONE SISTEMA 1 G.D.L. NON SMORZATO

Sollecitazione con forza variabile a gradino con rampa iniziale



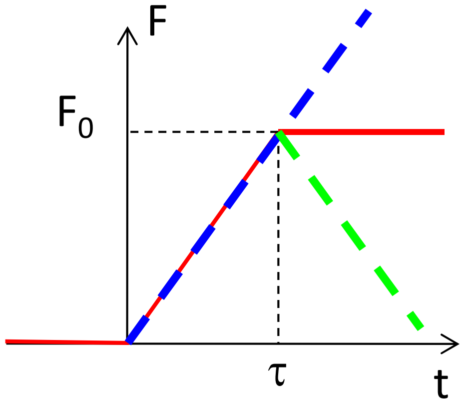
OSCILLAZIONE SISTEMA 1 G.D.L. NON SMORZATO

Sollecitazione con forza variabile a gradino con rampa iniziale



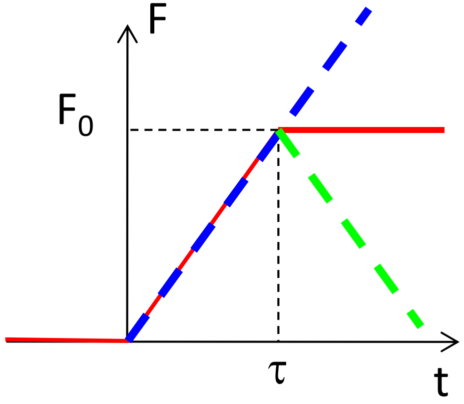
OSCILLAZIONE SISTEMA 1 G.D.L. NON SMORZATO

Sollecitazione con forza variabile a gradino con rampa iniziale



OSCILLAZIONE SISTEMA 1 G.D.L. NON SMORZATO

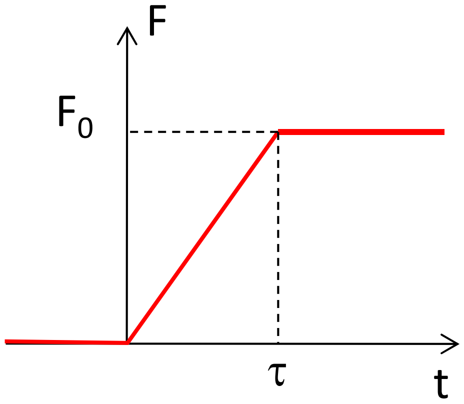
Sollecitazione con forza variabile a gradino con rampa iniziale



$$\begin{cases} F = \dfrac{F_0}{\tau} t = Bt & 0 \leq t \leq \tau \\ F = Bt - B(t-\tau) = F_0 & \tau < t \end{cases}$$

OSCILLAZIONE SISTEMA 1 G.D.L. NON SMORZATO

Sollecitazione con forza variabile a gradino con rampa iniziale



$$\begin{cases} F = \dfrac{F_0}{\tau} t = Bt & 0 \le t \le \tau \\ F = Bt - B(t-\tau) = F_0 & \tau < t \end{cases}$$

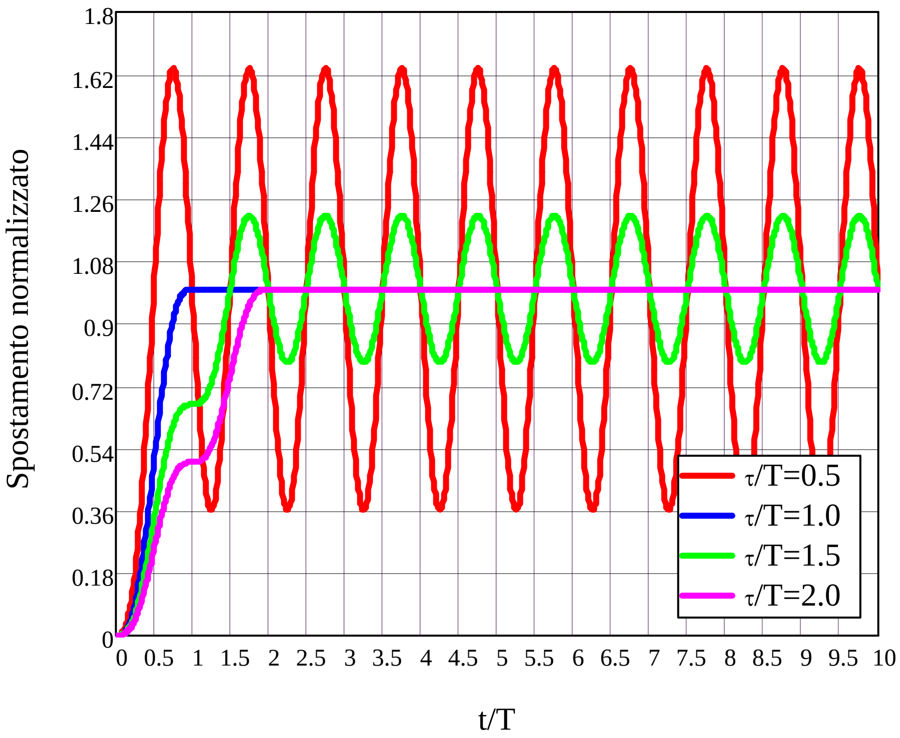
$$\begin{cases} 0 \le t \le \tau & x(t) = \dfrac{B}{k}\left(t - \dfrac{Sin(\omega_n t)}{\omega_n} \right) \\ \tau < t & x(t) = \dfrac{B}{k}\left(t - \dfrac{Sin(\omega_n t)}{\omega_n} \right) - \dfrac{B}{k}\left((t-\tau) - \dfrac{Sin(\omega_n (t-\tau))}{\omega_n} \right) = \\ & = \dfrac{B}{k}\left(\tau - \dfrac{Sin(\omega_n t)}{\omega_n} + \dfrac{Sin(\omega_n (t-\tau))}{\omega_n} \right) \end{cases}$$

OSCILLAZIONE SISTEMA 1 G.D.L. NON SMORZATO
Sollecitazione con forza variabile a gradino con rampa iniziale

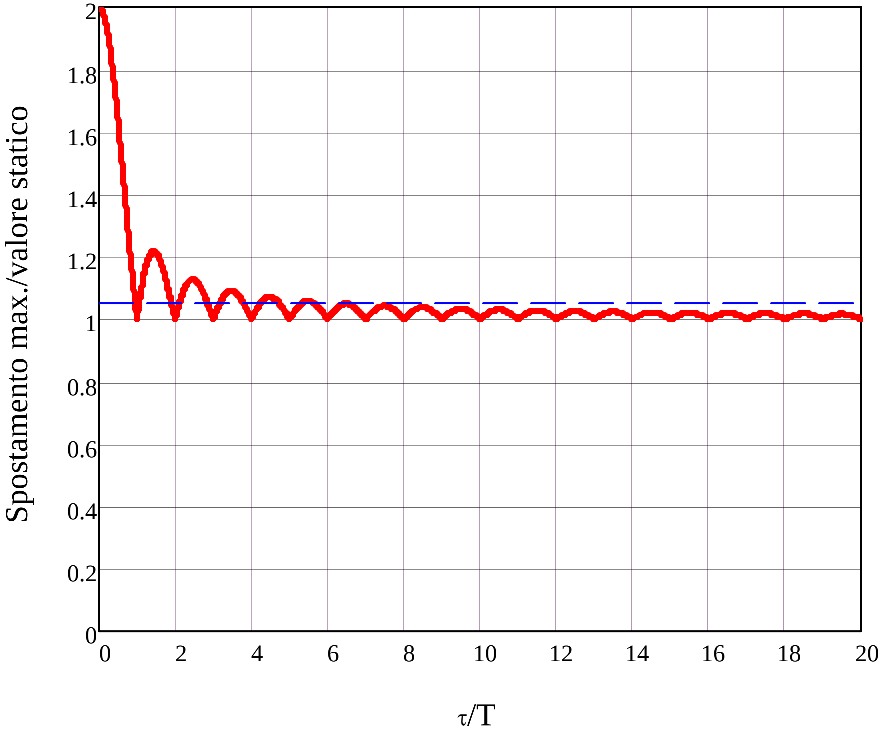
$$\begin{cases} 0 \leq t \leq \tau \qquad \dfrac{x(t)k}{F_0} = \dfrac{T}{\tau} . \left(\dfrac{t}{T} - . \dfrac{Sin(2\pi \frac{t}{T})}{2\pi} \right) \\ \tau < t \qquad \dfrac{x(t)k}{F_0} = 1 - \dfrac{Sin(\omega_n t)}{\tau \omega_n} + \dfrac{Sin(\omega_n (t - \tau))}{\tau \omega_n} = \\ \qquad = 1 + \dfrac{T}{\tau} \left(\dfrac{Sin(\omega_n (t - \tau))}{2\pi} - \dfrac{Sin(\omega_n t)}{2\pi} \right) \end{cases}$$

OSCILLAZIONE SISTEMA 1 G.D.L. NON SMORZATO

Sollecitazione con forza variabile a gradino con rampa iniziale

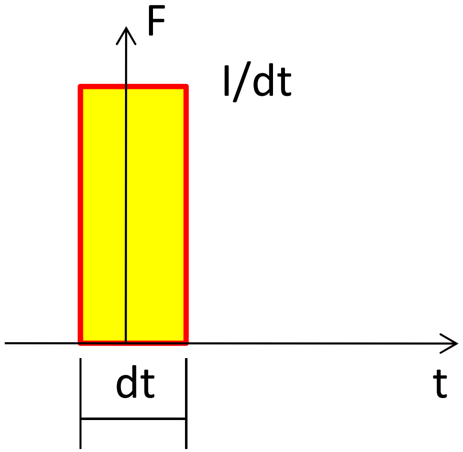


OSCILLAZIONE SISTEMA 1 G.D.L. NON SMORZATO
Sollecitazione con forza variabile a gradino con rampa iniziale



OSCILLAZIONE SISTEMA 1 G.D.L. NON SMORZATO

Sollecitazione con impulso I al tempo t=0

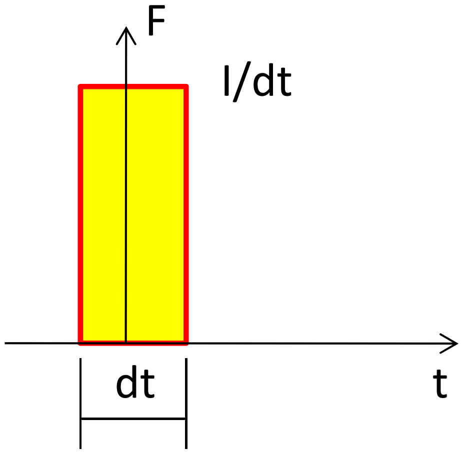


$$F = \lim_{dt \to 0} \frac{I}{dt}$$

$$I = mv \quad \Rightarrow \quad v = \frac{I}{m}$$

OSCILLAZIONE SISTEMA 1 G.D.L. NON SMORZATO

Sollecitazione con impulso I al tempo t=0



$$F = \lim_{dt \to 0} \frac{I}{dt}$$

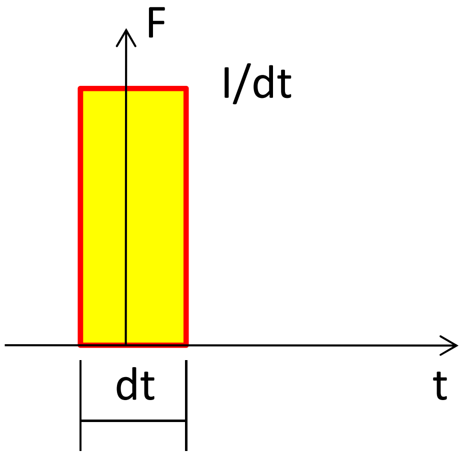
$$I = mv \quad \Rightarrow \quad v = \frac{I}{m}$$

Condizioni iniziali

$$\begin{cases} x(0) = 0 \\ \dot{x}(0) = v \end{cases}$$

OSCILLAZIONE SISTEMA 1 G.D.L. NON SMORZATO

Sollecitazione con impulso I al tempo t=0



$$F = \lim_{dt \to 0} \frac{I}{dt}$$

$$I = mv \quad \Rightarrow \quad v = \frac{I}{m}$$

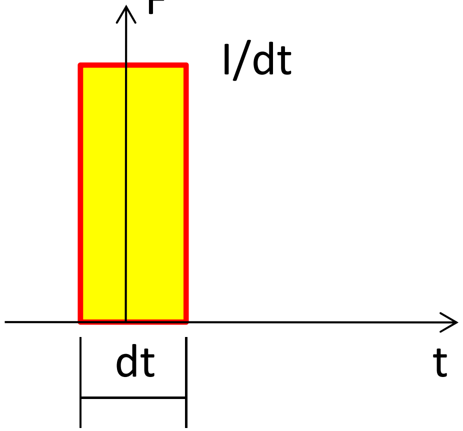
$$x(t) = A_1 e^{i\omega_n t} + B_1 e^{-i\omega_n t}$$

Condizioni iniziali

$$\begin{cases} x(0) = 0 \\ \dot{x}(0) = v \end{cases}$$

OSCILLAZIONE SISTEMA 1 G.D.L. NON SMORZATO

Sollecitazione con impulso I al tempo t=0



$$F = \lim_{dt \to 0} \frac{I}{dt}$$

$$I = mv \quad \Rightarrow \quad v = \frac{I}{m}$$

Condizioni iniziali

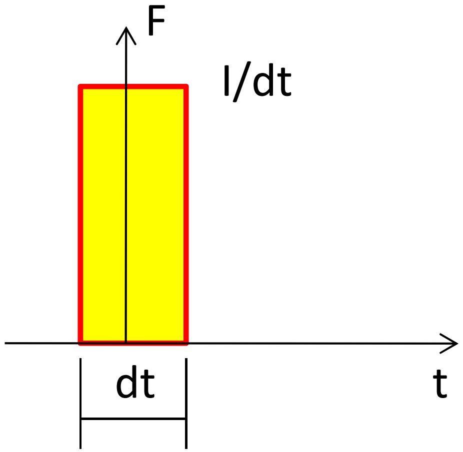
$$\begin{cases} x(0) = 0 \\ \dot{x}(0) = v \end{cases}$$

$$x(t) = A_1 e^{i\omega_n t} + B_1 e^{-i\omega_n t}$$

$$\left. \begin{aligned} x(0) &= A_1 + B_1 = 0 \\ \dot{x}(0) &= i\omega_n A_1 - i\omega_n B_1 = \frac{I}{m} \end{aligned} \right\} \Rightarrow \begin{cases} A_1 = -i \dfrac{I}{2m\omega_n} \\ B_1 = i \dfrac{I}{2m\omega_n} \end{cases}$$

OSCILLAZIONE SISTEMA 1 G.D.L. NON SMORZATO

Sollecitazione con impulso I al tempo t=0



$$F = \lim_{dt \to 0} \frac{I}{dt}$$

$$I = mv \quad \Rightarrow \quad v = \frac{I}{m}$$

Condizioni iniziali

$$\begin{cases} x(0) = 0 \\ \dot{x}(0) = v \end{cases}$$

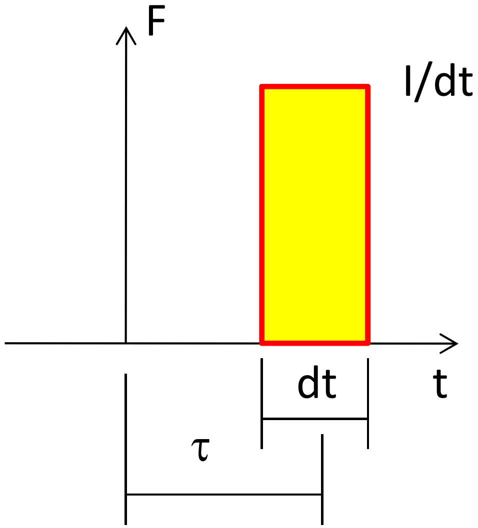
$$x(t) = A_1 e^{i\omega_n t} + B_1 e^{-i\omega_n t}$$

$$\left. \begin{aligned} x(0) &= A_1 + B_1 = 0 \\ \dot{x}(0) &= i\omega_n A_1 - i\omega_n B_1 = \frac{I}{m} \end{aligned} \right\} \Rightarrow \begin{cases} A_1 = -i\dfrac{I}{2m\omega_n} \\ B_1 = i\dfrac{I}{2m\omega_n} \end{cases}$$

$$x(t) = \frac{I}{2\omega_n m}\left(ie^{-i\omega_n t} - ie^{i\omega_n t}\right) = \frac{I}{2\omega_n m}\left(-i2iSin(\omega_n t)\right) = \frac{ISin(\omega_n t)}{\omega_n m}$$

OSCILLAZIONE SISTEMA 1 G.D.L. NON SMORZATO

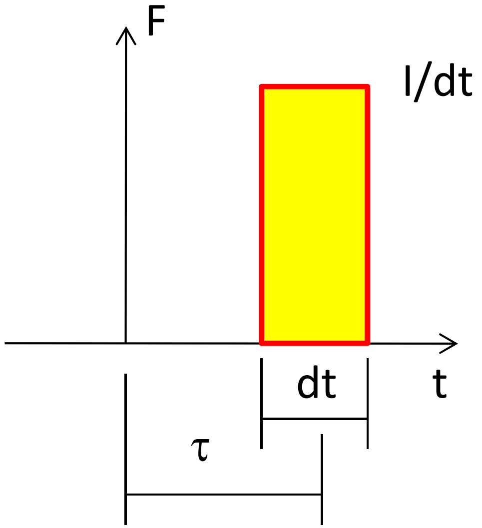
Sollecitazione con impulso I al tempo t=τ



$$\cdot\begin{cases} x(t) = \dfrac{ISin(\omega_n(t-\tau))}{\omega_n m} & t \geq \tau \\ x(t) = 0 & t < \tau \end{cases}$$

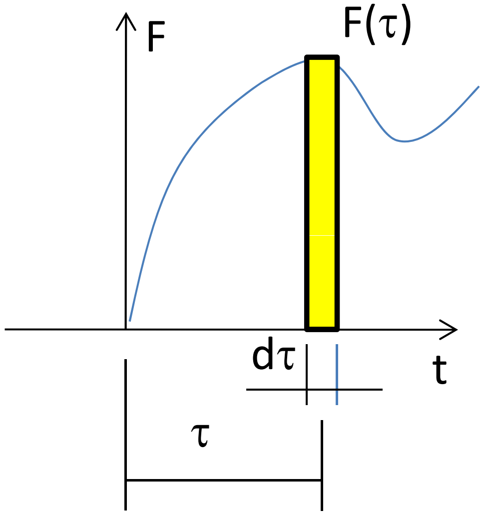
OSCILLAZIONE SISTEMA 1 G.D.L. NON SMORZATO

Sollecitazione con impulso I al tempo t=τ

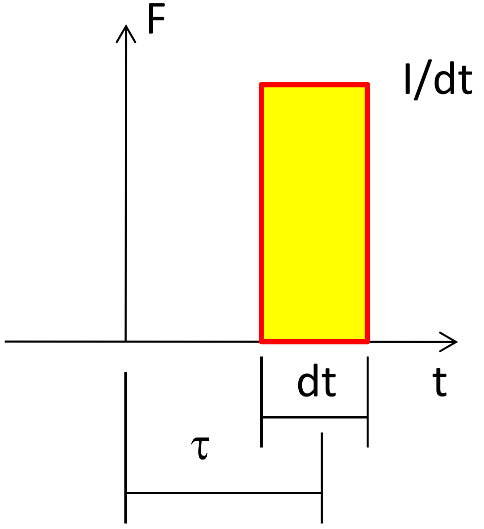


$$\cdot\begin{cases} x(t) = \dfrac{ISin(\omega_n(t-\tau))}{2\omega_n m} & t \geq \tau \\ x(t) = 0 & t < \tau \end{cases}$$

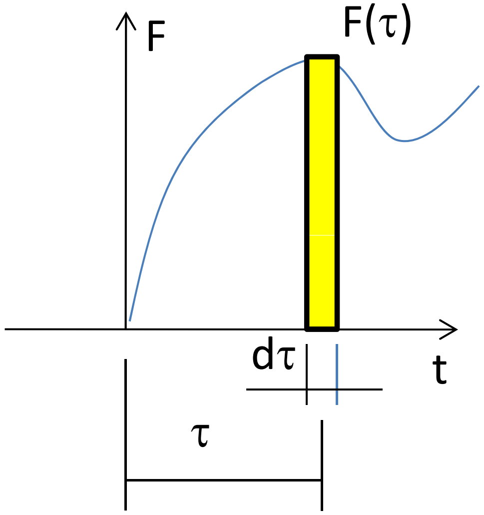
La forza di andamento generico può essere vista come una successione di impulsi di valore F(τ) dτ.



OSCILLAZIONE SISTEMA 1 G.D.L. NON SMORZATO
Sollecitazione con impulso I al tempo t=τ



$$\begin{cases} x(t) = \dfrac{ISin(\omega_n(t-\tau))}{2\omega_n m} & t \geq \tau \\ x(t) = 0 & t < \tau \end{cases}$$



La forza di andamento generico può essere vista come una successione di impulsi di valore F(τ) dτ.

$$x(t) = \frac{1}{\omega_n m} \int_0^t F(\tau) Sin(\omega_n(t-\tau)) \cdot d\tau$$

Integrale di convoluzione o di Duhamel