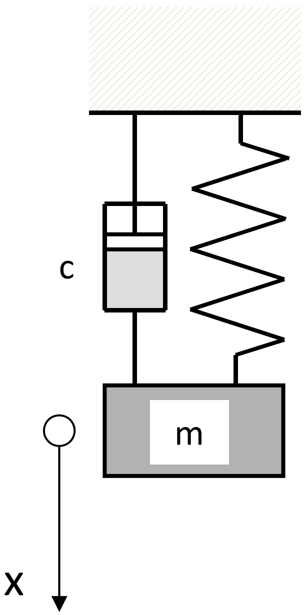
OSCILLAZIONE LIBERA SISTEMA 1 G.D.L. SMORZATO

Sistema ad 1 g.d.l.



$$m\ddot{x} + c\dot{x} + kx = \ddot{x} + \frac{c}{m}\dot{x} + \frac{k}{m}x = 0$$

$$x(t) = A_1 \cdot e^{a_1 t} + A_2 \cdot e^{a_2 t}$$

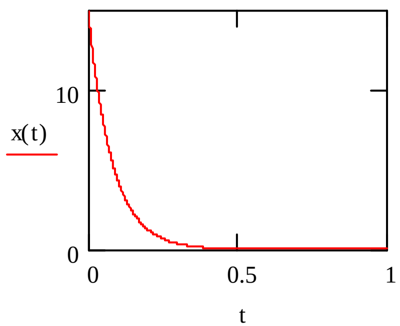
$$a^2 + \frac{c}{m}a + \frac{k}{m} = 0$$

$$\Delta = \frac{c^2}{m^2} - 4\frac{k}{m} = 0 \quad \rightarrow \quad c = c_{cr} = 2\sqrt{km}$$

$$a_{1,2} = -\frac{c}{2m} \pm \frac{1}{2}\sqrt{\frac{c^2}{m^2} - 4\frac{k}{m}}$$

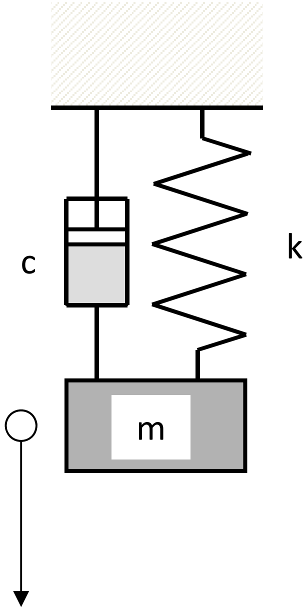
$$c > c_{cr} \quad \rightarrow \quad \Delta > 0$$

$$a_1,\ a_2 \quad reali < 0$$



OSCILLAZIONE LIBERA SISTEMA 1 G.D.L. SMORZATO

Sistema ad 1 g.d.l. $c < c_{cr} \quad \rightarrow \quad \Delta < 0$



$$a_1,\ a_2 \quad complesse \quad coniugate = -\frac{c}{2m} \pm i \cdot \frac{1}{2}\sqrt{4\frac{k}{m} - \frac{c^2}{m^2}}$$

$$\xi = \frac{c}{c_{cr}}$$

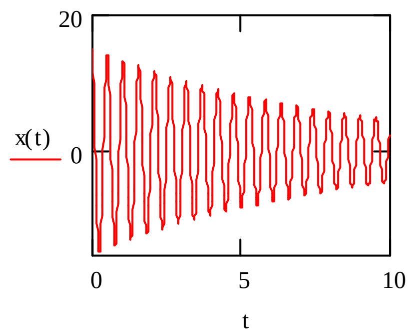
$$\frac{c}{2m} = \frac{c\sqrt{k}}{2m\sqrt{k}} = \frac{c}{2\sqrt{km}}\sqrt{\frac{k}{m}} = \frac{c}{c_{cr}}\omega_n = \xi\omega_n$$

$$\frac{1}{2}\sqrt{4\frac{k}{m} - \frac{c^2}{m^2}} = \sqrt{\frac{k}{m}\left(1 - \frac{c^2}{4mk}\right)} = \sqrt{\frac{k}{m}}\sqrt{1 - \frac{c^2}{c_{cr}^2}} = \omega_n\sqrt{1-\xi^2} = \omega_s$$

$$a_{1,2} = -\xi\omega_n \pm i\omega_s$$

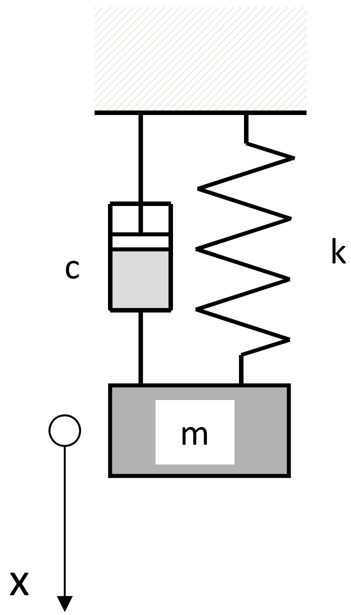
$$x(t) = A_1 e^{(-\xi\omega_n + i\omega_s)t} + B_1 e^{(-\xi\omega_n - i\omega_s)t} = e^{-\xi\omega_n t}\left(A_1 e^{i\omega_s t} + B_1 e^{-i\omega_s t}\right)$$

$$x(t) = e^{-\xi\omega_n t}\left(A\cos(\omega_s t) + B\sin(\omega_s t)\right)$$



OSCILLAZIONE LIBERA SISTEMA 1 G.D.L. SMORZATO

Sistema ad 1 g.d.l.



$$\xi = \frac{c}{c_{cr}}$$

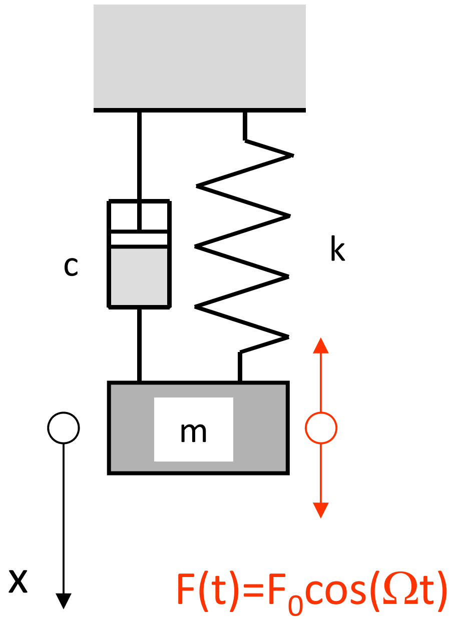
Per la maggior parte dei sistemi meccanici è piuttosto piccolo (< 0.1)

$$\left.\begin{array}{l}\omega_s = \omega_n\sqrt{1-\xi^2} \\ \xi = 0.1\end{array}\right\rangle \omega_s = \omega_n\sqrt{1-0.1^2} = 0.99\omega_n$$

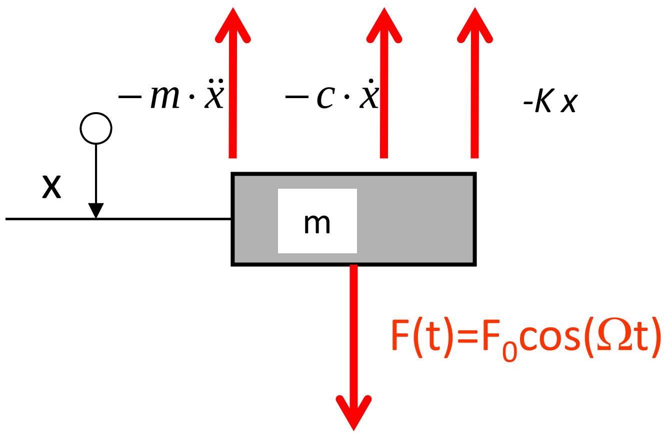
Per questo è solitamente possibile trascurare l’effetto dello smorzamento **sul valore dei modi propri**

OSCILLAZIONE FORZATA SISTEMA 1 G.D.L. SMORZATO

Sistema ad 1 g.d.l.



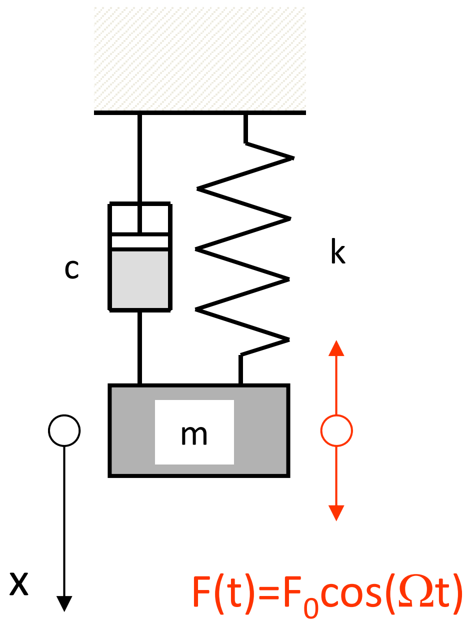
Analisi delle forze agenti



$$m\ddot{x}+c\dot{x}+kx=F_0\cos(\Omega t)$$

OSCILLAZIONE FORZATA SISTEMA 1 G.D.L. SMORZATO

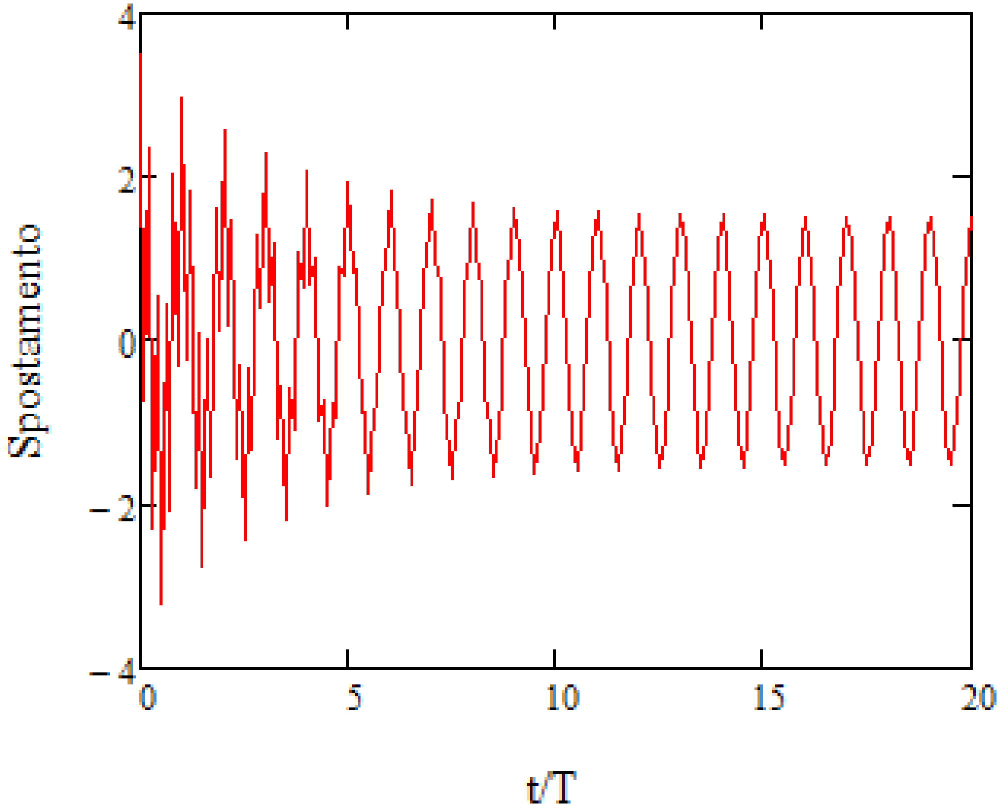
Sistema ad 1 g.d.l.



$$T = \frac{2\pi}{\omega_n}$$

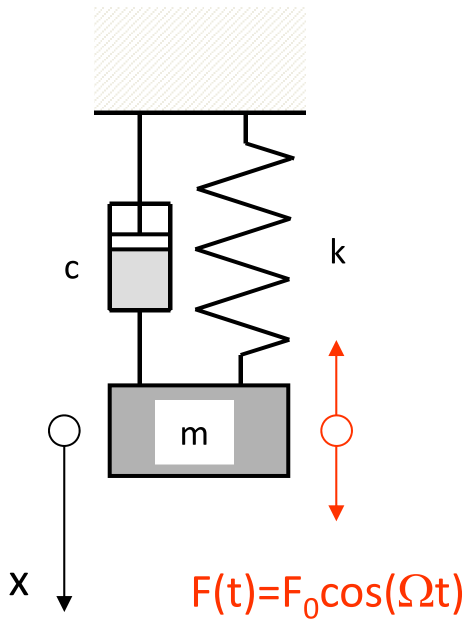
$$m\ddot{x} + c\dot{x} + kx = F_0 \cos(\Omega t)$$

$$x(t) = X \cdot \cos(\Omega t - \varphi) + e^{-\xi\omega_n t} A \sin(\omega_s t + \phi)$$



OSCILLAZIONE FORZATA SISTEMA 1 G.D.L. SMORZATO

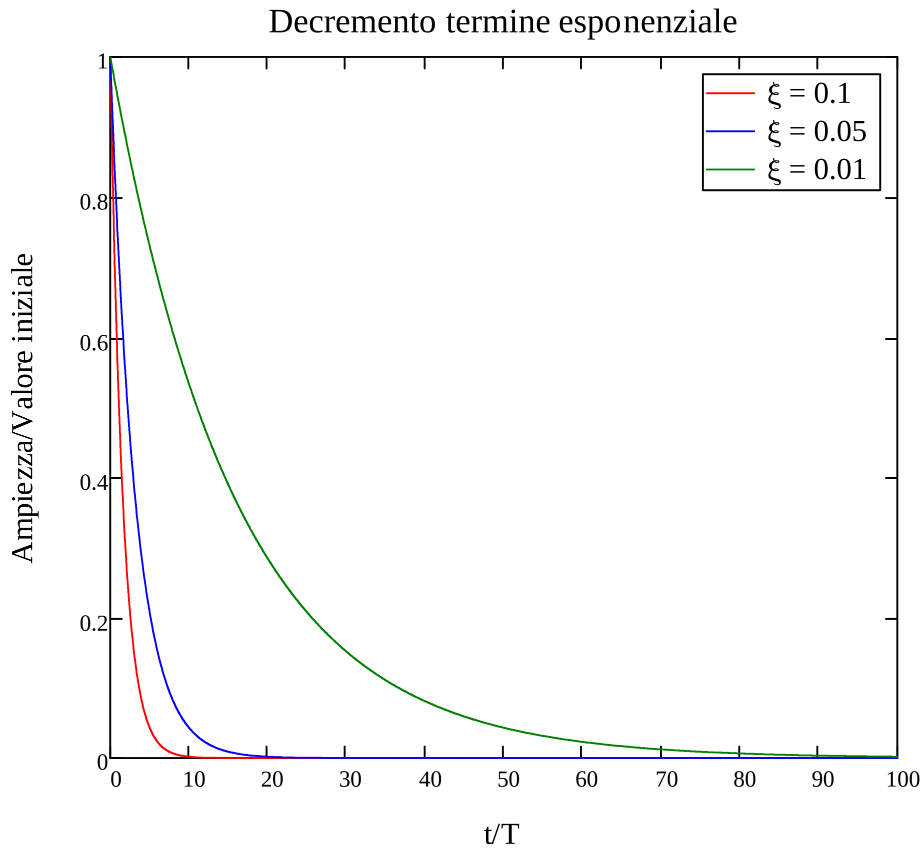
Sistema ad 1 g.d.l.



$$T = \frac{2\pi}{\omega_n}$$

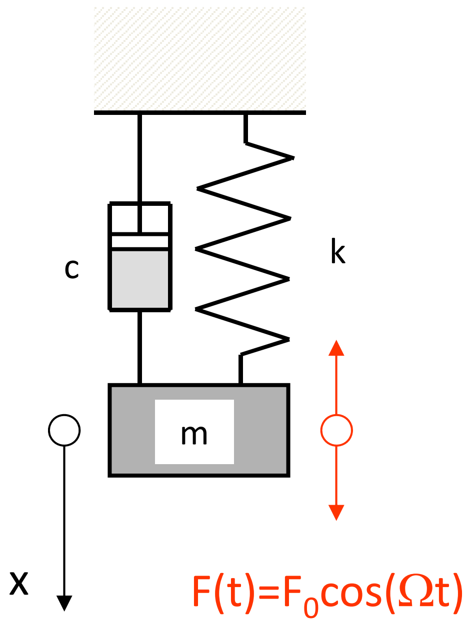
$$m\ddot{x} + c\dot{x} + kx = F_0 \cos(\Omega t)$$

$$x(t) = X \cdot \cos(\Omega t - \varphi) + e^{-\xi\omega_n t} A\sin(\omega_s t + \phi)$$



OSCILLAZIONE FORZATA SISTEMA 1 G.D.L. SMORZATO

Sistema ad 1 g.d.l.



$$m\ddot{x} + c\dot{x} + kx = F_0 \cos(\Omega t)$$

$$\frac{c}{m} = 2\xi\omega_n$$

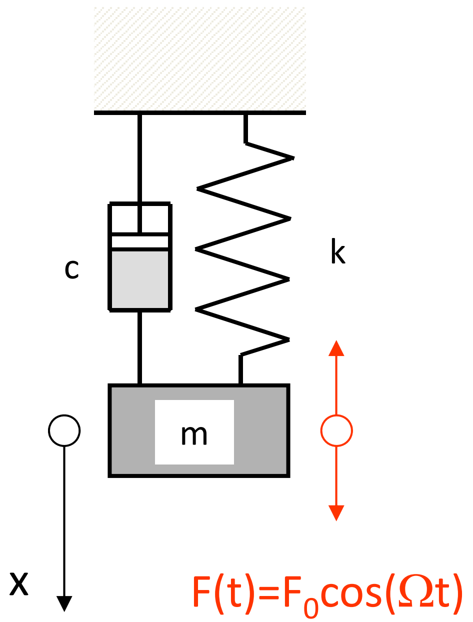
$$\frac{k}{m} = \omega_n^2$$

$$m\ddot{x} + c\dot{x} + kx = \ddot{x} + \frac{c}{m}\dot{x} + \frac{k}{m}x = \ddot{x} + 2\xi\omega_n\dot{x} + \omega_n^2 x$$

$$\ddot{x} + 2\xi\omega_n\dot{x} + \omega_n^2 x = \frac{F_0}{m}\cos(\Omega t)$$

OSCILLAZIONE FORZATA SISTEMA 1 G.D.L. SMORZATO

Sistema ad 1 g.d.l.



$$\ddot{x} + 2\xi\omega_n\dot{x} + \omega_n^2 x = \frac{F_0}{m}\cos(\Omega t)$$

$$x(t) = X \cdot \cos(\Omega t - \varphi) + e^{-\xi\omega_n t} A\sin(\omega_s t + \phi)$$

$$x(t) \cong X \cdot \cos(\Omega t - \varphi) \qquad per\ t > t_{trans}$$

$$\dot{x}(t) \cong -\Omega X \cdot \sin(\Omega t - \varphi)$$

$$\ddot{x}(t) \cong -\Omega^2 X \cdot \cos(\Omega t - \varphi)$$

$$-\Omega^2 X \cdot \cos(\Omega t - \varphi) - 2\xi\omega_n \Omega X \cdot \sin(\Omega t - \varphi) + \omega_n^2 X \cdot \cos(\Omega t - \varphi) = \frac{F_0}{m}\cos(\Omega t)$$

OSCILLAZIONE FORZATA SISTEMA 1 G.D.L. SMORZATO

$$-\Omega^2 X \cdot \cos(\Omega t-\varphi) - 2\xi\omega_n \Omega X \cdot \sin(\Omega t-\varphi) + \omega_n^2 X \cdot \cos(\Omega t-\varphi) = \frac{F_0}{m}\cos(\Omega t)$$

$$\omega_n^2 X \cdot \cos(\Omega t)\cos(\varphi) + \omega_n^2 X \cdot \sin(\Omega t)\sin(\varphi) -$$

$$-\Omega^2 X \cdot \cos(\Omega t)\cos\varphi - \Omega^2 X \cdot \sin(\Omega t)\sin\varphi +$$

$$+2\xi\omega_n \Omega X \cdot \cos(\Omega t)\sin(\varphi) - 2\xi\omega_n \Omega X \cdot \sin(\Omega t)\cos(\varphi) = \frac{F_0}{m}\cos(\Omega t)$$

$$\left[\omega_n^2 \cos(\varphi) - \Omega^2 \cos\varphi + 2\xi\omega_n \Omega \sin(\varphi)\right] \cdot X \cos(\Omega t) +$$

$$+\left[\omega_n^2 \sin(\varphi) - \Omega^2 \sin\varphi - 2\xi\omega_n \Omega \cos(\varphi)\right] \sin(\Omega t) = \frac{F_0}{m}\cos(\Omega t)$$

OSCILLAZIONE FORZATA SISTEMA 1 G.D.L. SMORZATO

$$\begin{cases} \left[\omega_n^2 \cos(\varphi) - \Omega^2 \cos\varphi + 2\xi\omega_n\Omega \sin(\varphi)\right] X = \dfrac{F_0}{m} \\ \omega_n^2 \sin(\varphi) - \Omega^2 \sin\varphi - 2\xi\omega_n\Omega\cos(\varphi) = 0 \end{cases}$$

$$\tan(\varphi) = \frac{2\xi \dfrac{\Omega}{\omega_n}}{1 - \left(\dfrac{\Omega}{\omega_n}\right)^2}$$

$$\begin{cases} \left(\omega_n^2 - \Omega^2\right)^2 \cos^2(\varphi) + 4\xi^2\omega_n^2\Omega^2 \sin^2(\varphi) + 4\xi\omega_n\Omega \sin(\varphi)\cos(\varphi) = \left(\dfrac{F_0}{Xm}\right)^2 \\ \left(\omega_n^2 - \Omega^2\right)^2 \sin^2(\varphi) + 4\xi^2\omega_n^2\Omega^2 \cos^2(\varphi) - 4\xi\omega_n\Omega \sin(\varphi)\cos(\varphi) = 0 \end{cases}$$

OSCILLAZIONE FORZATA SISTEMA 1 G.D.L. SMORZATO

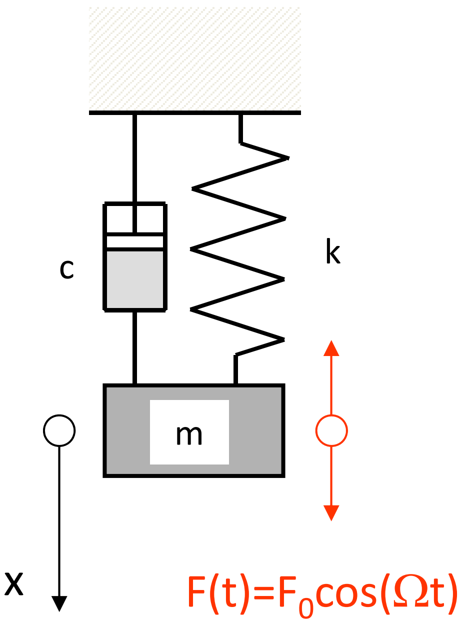
$$\begin{cases} \left(\omega_n^2-\Omega^2\right)^2\cos^2(\varphi)+4\xi^2\omega_n^2\Omega^2\sin^2(\varphi)+4\xi\omega_n\Omega\sin(\varphi)\cos(\varphi)=\left(\dfrac{F_0}{Xm}\right)^2 \\ \left(\omega_n^2-\Omega^2\right)^2\sin^2(\varphi)+4\xi^2\omega_n^2\Omega^2\cos^2(\varphi)-4\xi\omega_n\Omega\sin(\varphi)\cos(\varphi)=0 \end{cases}$$

$$\left(\omega_n^2-\Omega^2\right)^2+4\xi^2\omega_n^2\Omega^2=\left(\frac{F_0}{Xm}\right)^2$$

$$X=\frac{\dfrac{F_0}{m}}{\sqrt{\left(\omega_n^2-\Omega^2\right)^2+4\xi^2\omega_n^2\Omega^2}}=\frac{\dfrac{F_0}{k}}{\sqrt{\left(1-\left(\dfrac{\Omega}{\omega_n}\right)^2\right)^2+\left(2\xi\dfrac{\Omega}{\omega_n}\right)^2}}$$

OSCILLAZIONE FORZATA SISTEMA 1 G.D.L. SMORZATO

Sistema ad 1 g.d.l.



$$m\ddot{x} + c\dot{x} + kx = F_0 \cos(\Omega t)$$

$$x(t) = X \cdot \cos(\Omega t - \varphi) + e^{-\xi\omega_n t} A \sin(\omega_s t + \phi)$$

$$x(t) \cong X \cdot \cos(\Omega t - \varphi) \qquad per\ t > t_{trans}$$

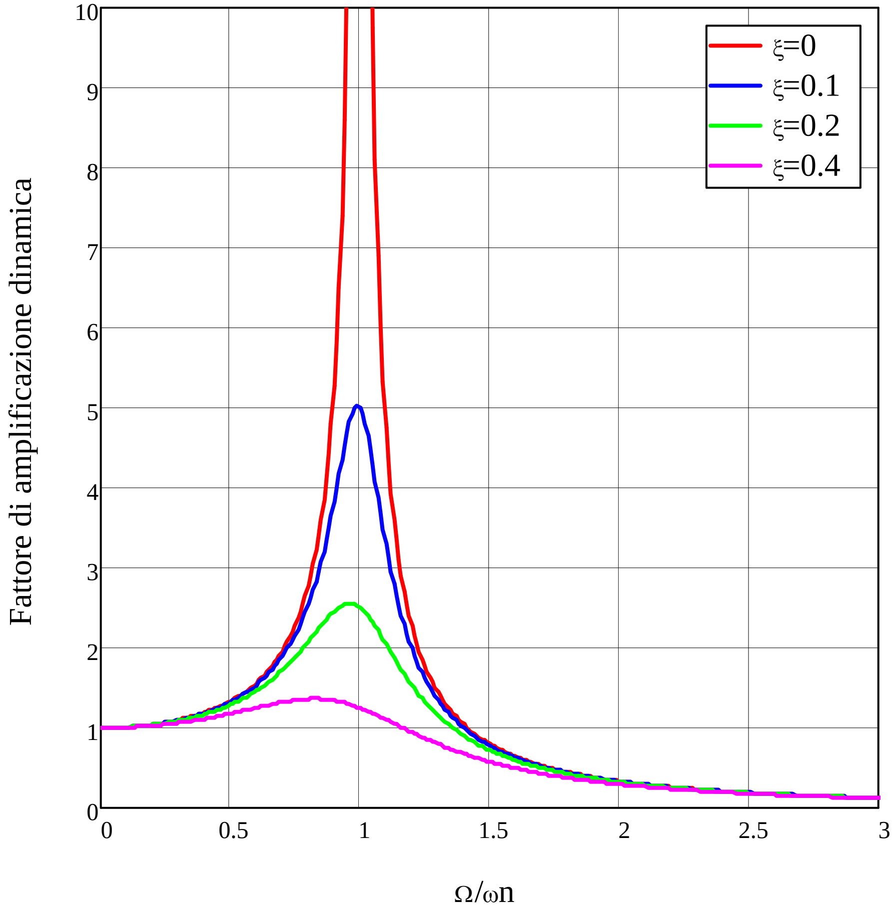
$$X = \frac{F_0}{K} \frac{1}{\sqrt{\left(1 - \frac{\Omega^2}{\omega_n^2}\right)^2 + \left(2\xi \frac{\Omega}{\omega_n}\right)^2}}$$

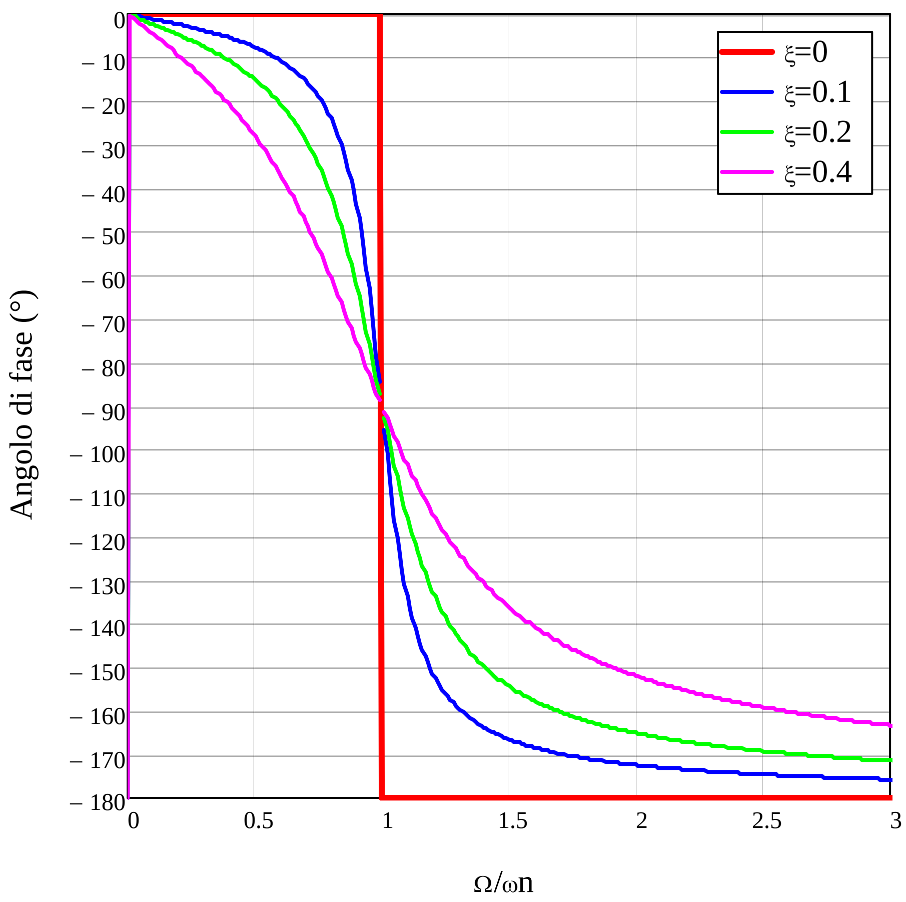
$$\varphi = \arctan\left(\frac{\xi \frac{\Omega}{\omega_n}}{1 - \frac{\Omega^2}{\omega_n^2}}\right)$$

$$\omega_n = \sqrt{\frac{k}{m}} \qquad \omega_s = \omega_n \sqrt{1 - \xi^2}$$

OSCILLAZIONE FORZATA SISTEMA 1 G.D.L. SMORZATO

$$X = \frac{F_0}{K} \frac{1}{\sqrt{\left(1 - \frac{\Omega^2}{\omega_n^2}\right)^2 + \left(2\xi \frac{\Omega}{\omega_n}\right)^2}}$$





$$\varphi = \arctan\left(\frac{-2\xi \frac{\Omega}{\omega_n}}{1 - \frac{\Omega^2}{\omega_n^2}}\right)$$

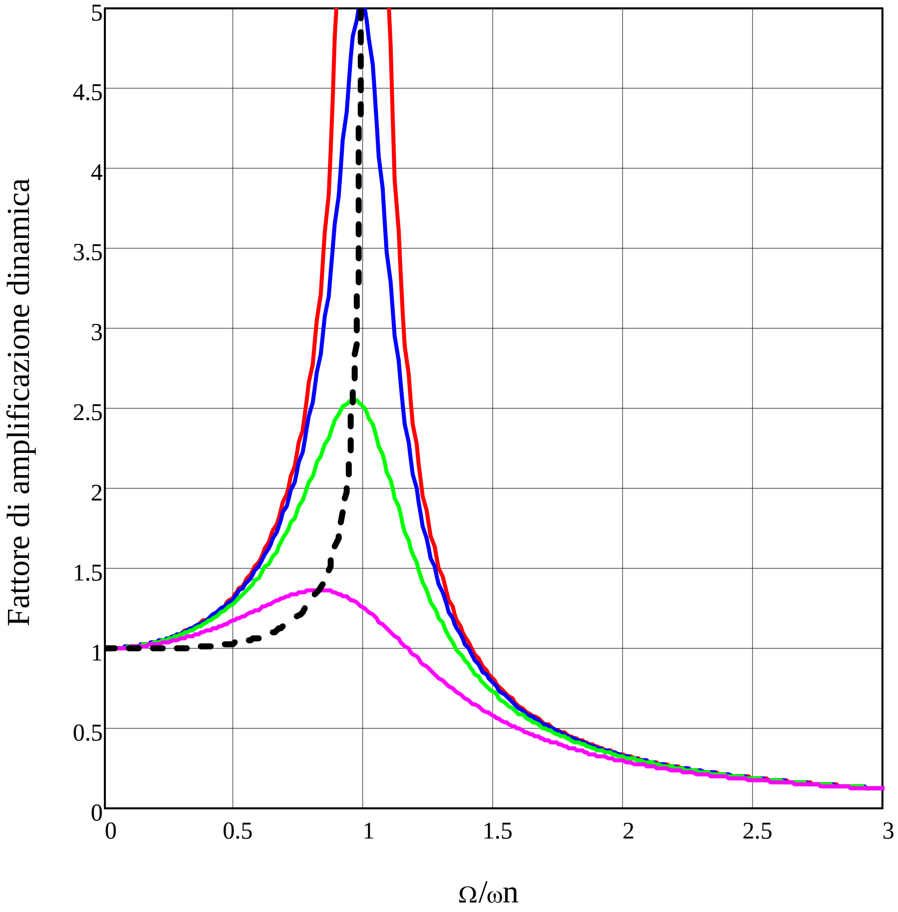
OSCILLAZIONE FORZATA SISTEMA 1 G.D.L. SMORZATO

Rapporto di frequenza per il quale si ha il massimo valore del fattore di amplificazione dinamica:

$$\left(\frac{\Omega}{\omega_n}\right)_{max} = \sqrt{1-2\xi^2}$$

Massimo valore del fattore di amplificazione dinamica:

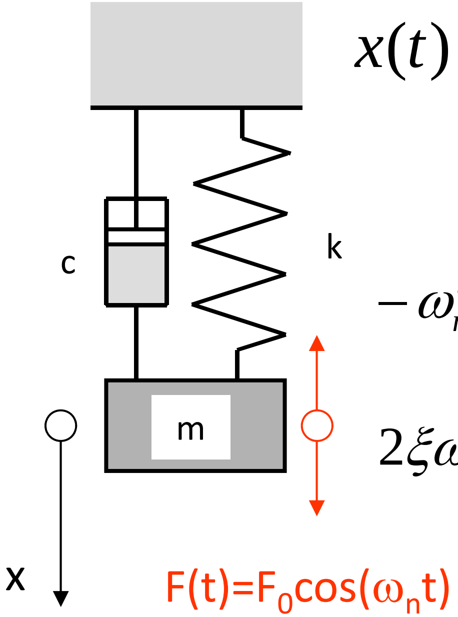
$$D_{max} = \frac{1}{2\xi\sqrt{1-\xi^2}}$$



OSCILLAZIONE FORZATA IN RISONANZA SISTEMA 1 G.D.L. SMORZATO

Sistema ad 1 g.d.l.

$$\ddot{x} + \xi\omega_n\dot{x} + \omega_n^2 x = \frac{F_0}{m}\cos(\omega_n t)$$



$$x(t) = X\sin(\omega_n t)$$

Integrale particolare non omogenea

$$\dot{x}(t) = \omega_n X\cos(\omega_n t)$$

$$\ddot{x}(t) = -\omega_n^2 X\sin(\omega_n t)$$

$$-\omega_n^2 X\sin(\omega_n t) + 2\xi\omega_n^2 X\cos(\omega_n t) + \omega_n^2 X\sin(\omega_n t) = \frac{F_0}{m}\cos(\omega_n t)$$

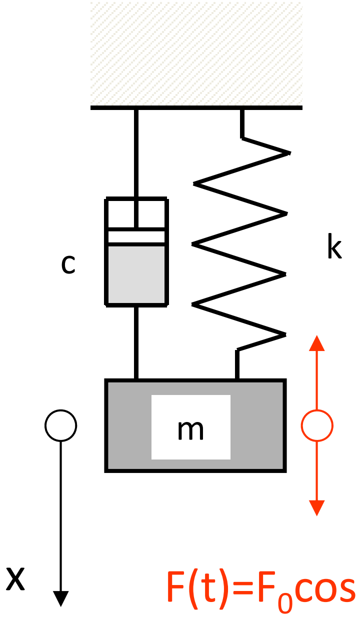
$$2\xi\omega_n^2 X\cos(\omega_n t) = \frac{F_0}{m}\cos(\omega_n t)$$

$$X = \frac{F_0}{m}\frac{1}{2\xi\omega_n^2} = \frac{F_0}{km}\frac{k}{2\xi\omega_n^2} = \frac{F_0}{k}\frac{1}{2\xi}$$

OSCILLAZIONE FORZATA IN RISONANZA SISTEMA 1 G.D.L. SMORZATO

Sistema ad 1 g.d.l.

$$x(t) = e^{-\xi\omega_n t}\left(A_1 \cos(\omega_s t) + B_1 \sin(\omega_s t)\right) + \frac{F_0}{k}\frac{1}{2\xi}\sin(\omega_n t)$$



$F(t)=F_0\cos(\omega_n t)$

Condizioni iniziali

$$\begin{cases} x(0) = 0 \\ \dot{x}(0) = 0 \end{cases}$$

$$x(0) = A_1 = 0$$

$$\dot{x}(0) = -\omega_n A_1 + \omega_s B_1 + \frac{F_0}{k}\frac{1}{2\xi}\omega_n = 0$$

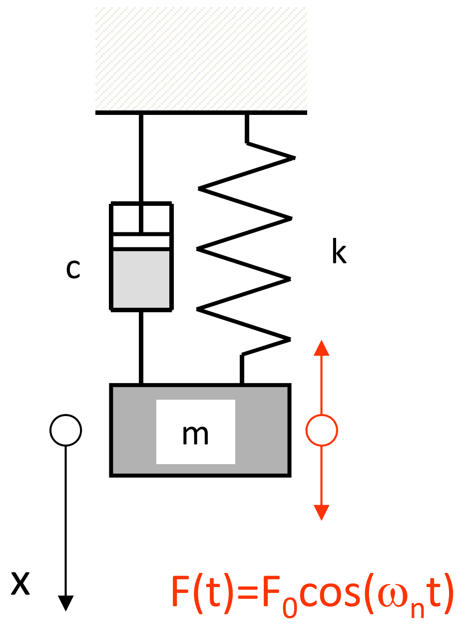
$$B_1 = \frac{F_0}{2k\xi\sqrt{1-\xi^2}}$$

$$A_1 = 0$$

$$x(t) = -e^{-\xi\omega_n t}\frac{F_0}{2k\xi\sqrt{1-\xi^2}}\sin(\omega_s t) + \frac{F_0}{k}\frac{1}{2\xi}\sin(\omega_n t)$$

OSCILLAZIONE FORZATA IN RISONANZA SISTEMA 1 G.D.L. SMORZATO

Sistema ad 1 g.d.l.



$$x(t) = -e^{-\xi\omega_n t}\frac{F_0}{2k\xi\sqrt{1-\xi^2}}\sin(\omega_s t) + \frac{F_0}{k}\frac{1}{2\xi}\sin(\omega_n t)$$

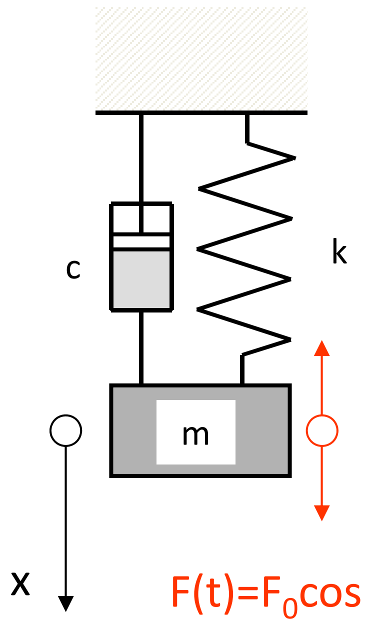
$$x(t) = \frac{F_0}{2k\xi}\left(\sin(\omega_n t) - e^{-\xi\omega_n t}\frac{1}{\sqrt{1-\xi^2}}\sin(\omega_s t)\right)$$

$$x(t) \approx \frac{F_0}{2k\xi}\sin(\omega_n t)\left(1 - e^{-\xi\omega_n t}\frac{1}{\sqrt{1-\xi^2}}\right)$$

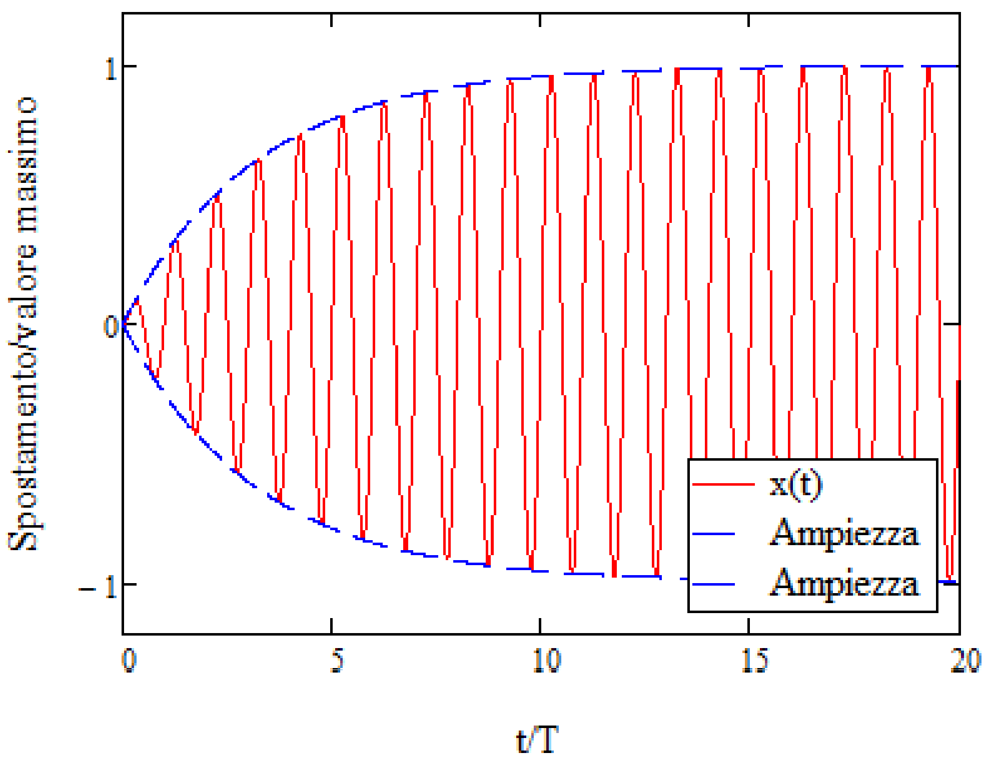
OSCILLAZIONE FORZATA IN RISONANZA SISTEMA 1 G.D.L. SMORZATO

Sistema ad 1 g.d.l.

$$\frac{x(t)}{\frac{F_0}{2k\xi}} \approx \sin(\omega_n t)\left(1 - e^{-\xi\omega_n t}\frac{1}{\sqrt{1-\xi^2}}\right)$$

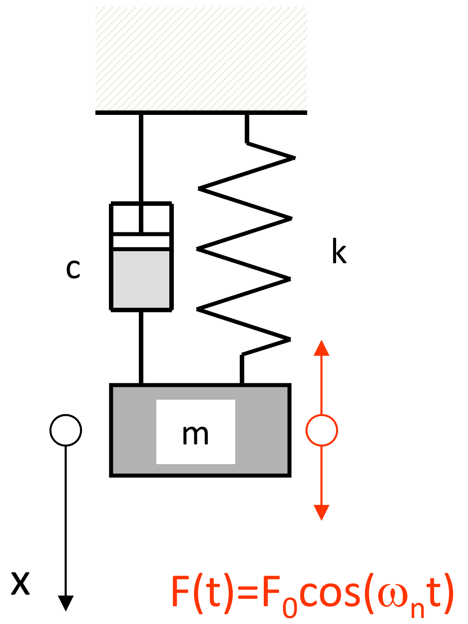


$F(t)=F_0\cos(\omega_n t)$

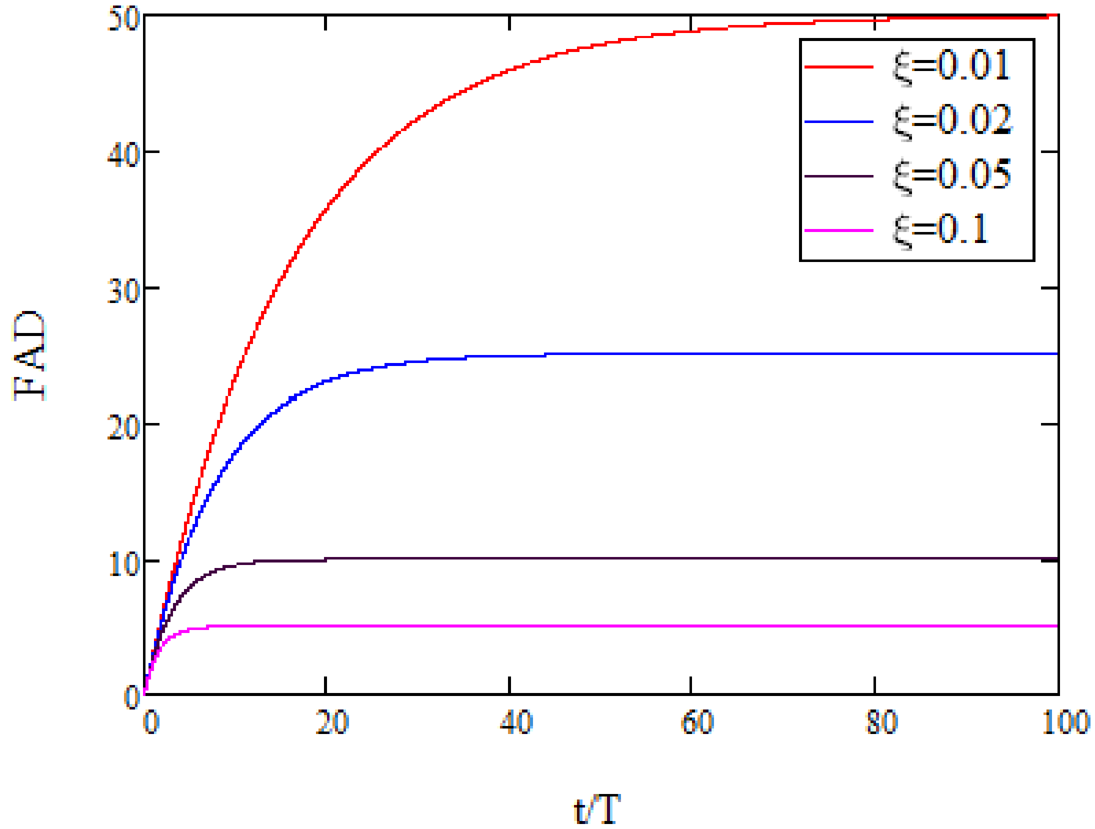


OSCILLAZIONE FORZATA IN RISONANZA SISTEMA 1 G.D.L. SMORZATO

Sistema ad 1 g.d.l.



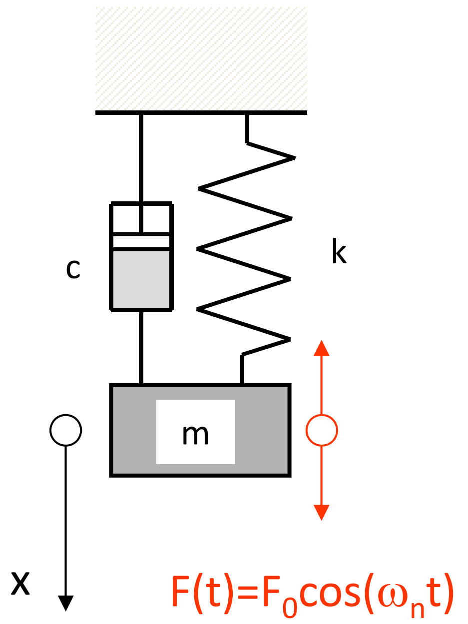
$$FAD = \frac{x(t)}{\dfrac{F_0}{k}} = \frac{1}{2\xi}\left(1 - e^{-\xi\omega_n t}\frac{1}{\sqrt{1-\xi^2}}\right)$$

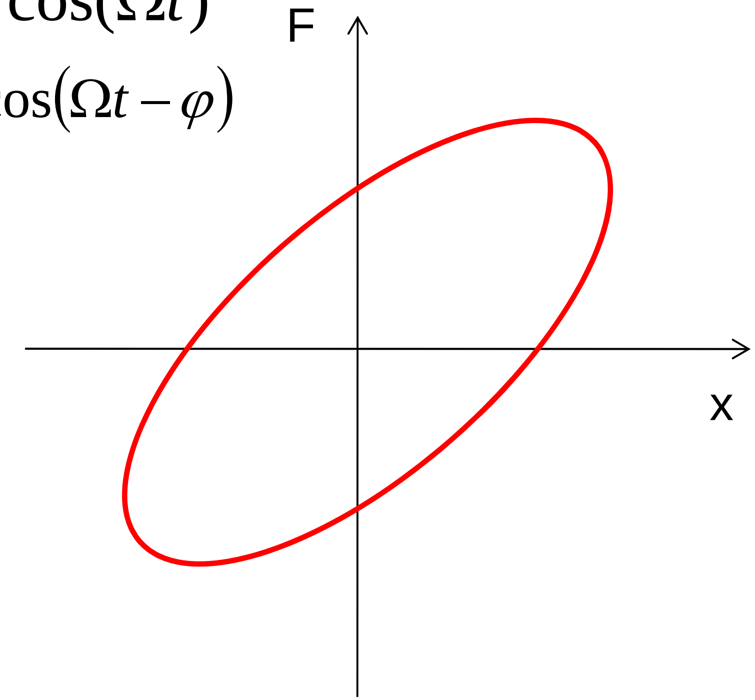


LAVORO DI UNA FORZA ARMONICA IN UN CICLO

$$F(t) = F_0 \cos(\Omega t)$$

$$x(t) = X \cdot \cos(\Omega t - \varphi)$$

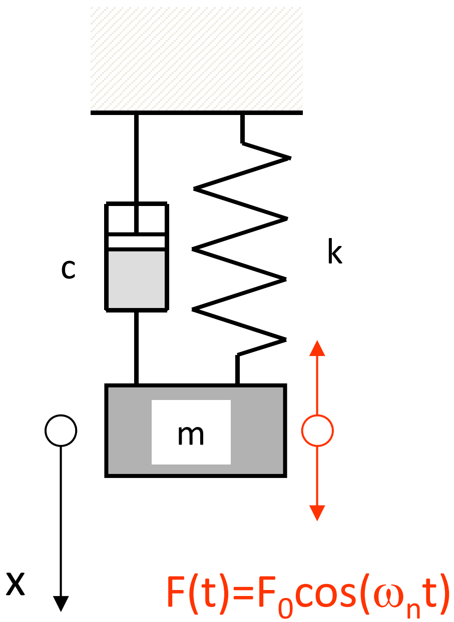


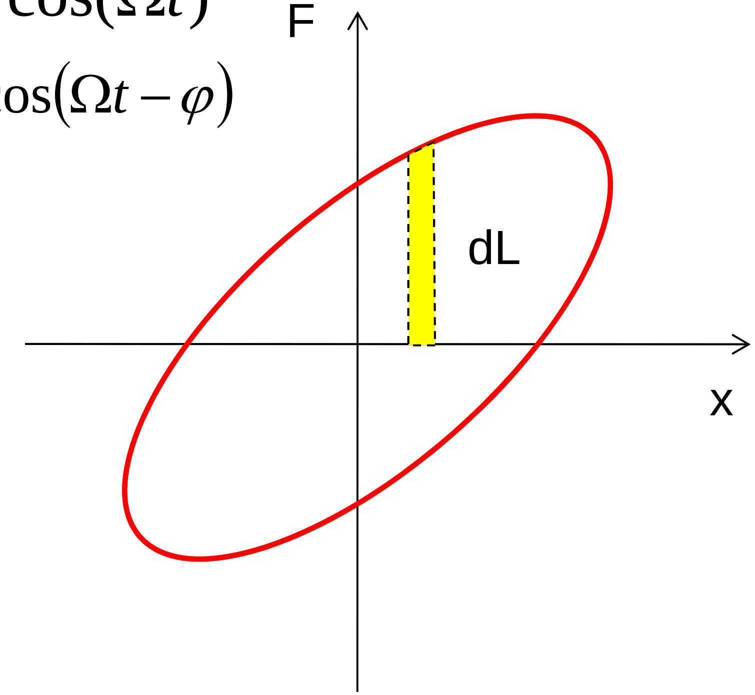


LAVORO DI UNA FORZA ARMONICA IN UN CICLO

$$F(t) = F_0 \cos(\Omega t)$$

$$x(t) = X \cdot \cos(\Omega t - \varphi)$$



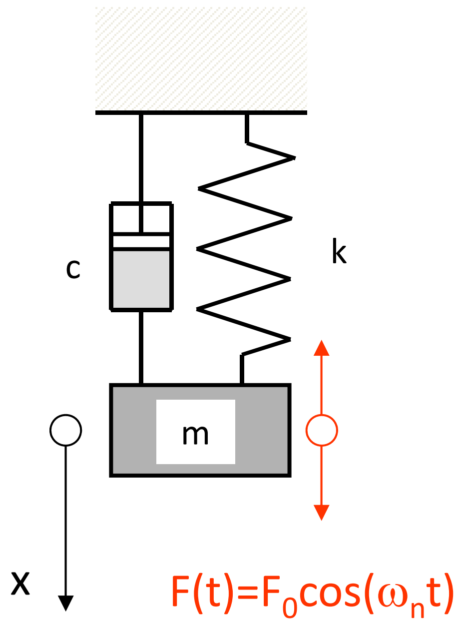


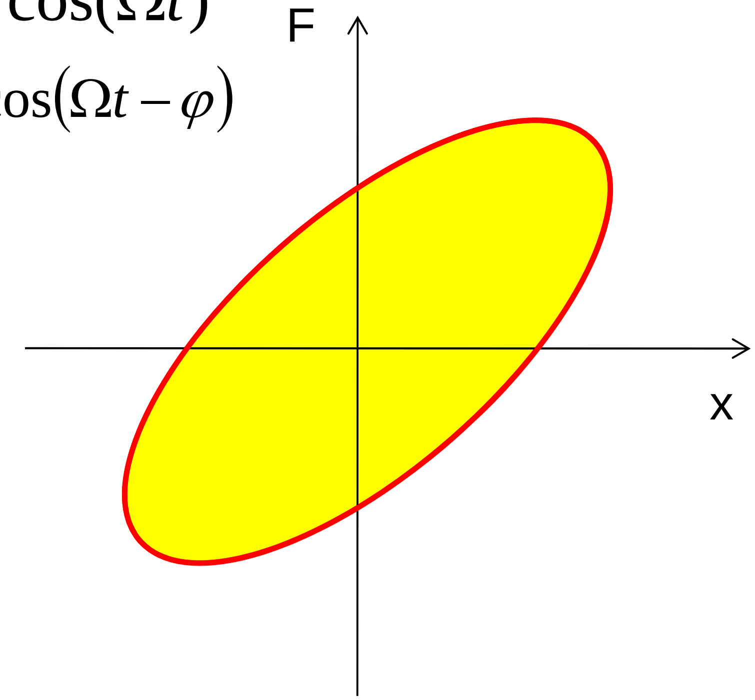
$$dL = F(t)dx = F(t)\dot{x} \cdot dt$$

LAVORO DI UNA FORZA ARMONICA IN UN CICLO

$$F(t) = F_0 \cos(\Omega t)$$

$$x(t) = X \cdot \cos(\Omega t - \varphi)$$

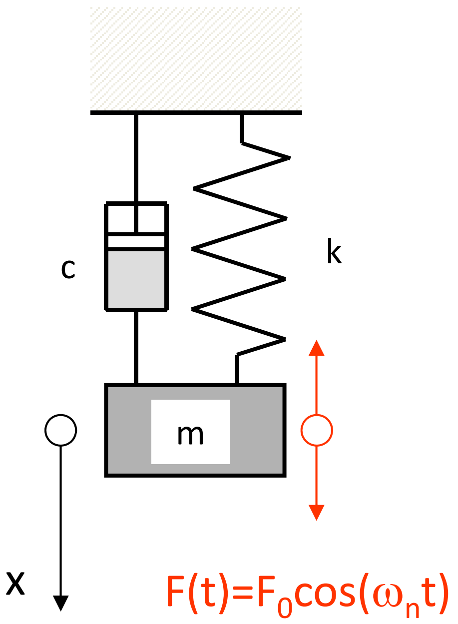




$$dL = F(t)dx = F(t)\dot{x} \cdot dt$$

$$L = \int_0^T F(t)\dot{x} \cdot dt$$

LAVORO DI UNA FORZA ARMONICA IN UN CICLO



$$L = \int_0^T F(t)\dot{x}\cdot dt = \int_0^T F_0 \cos(\Omega t)\Omega X \sin(\Omega t - \varphi)\cdot dt$$

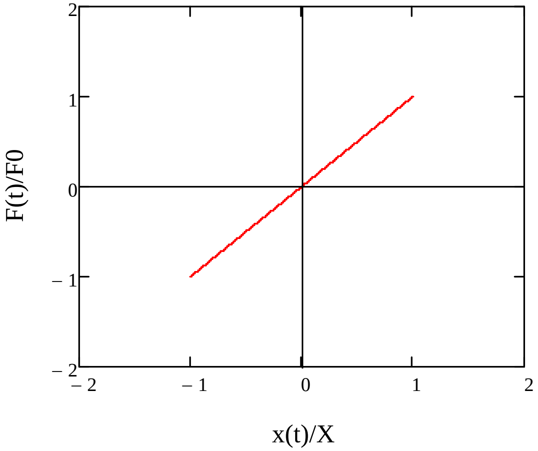
$$= F_0 \Omega X \int_0^T \cos(\Omega t)\sin(\Omega t - \varphi)\cdot dt$$

$$= F_0 \Omega X \int_0^T \frac{1}{2}\left[\sin(2\Omega t - \varphi) + \sin(-\varphi)\right]\cdot dt$$

$$= \frac{F_0 \Omega X}{2}\sin(\varphi)\int_0^T dt \; = \frac{F_0 \Omega X}{2}\sin(\varphi)\frac{2\pi}{\Omega} =$$

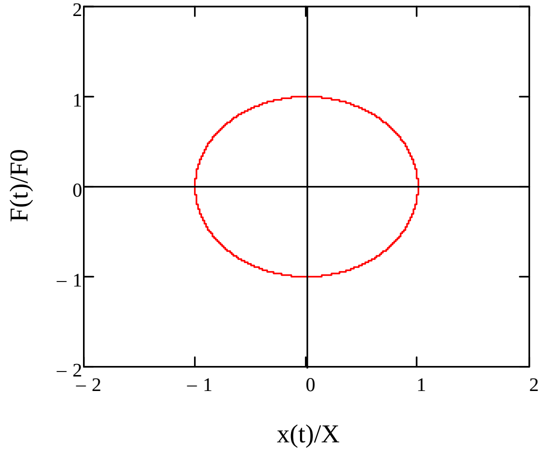
$$= \pi F_0 X \sin(\varphi)$$

LAVORO DI UNA FORZA ARMONICA IN UN CICLO



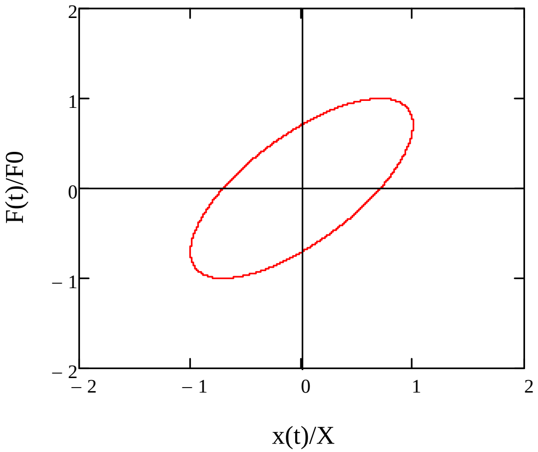
$$\varphi = 0° \; o \; 180°$$

$$L = 0$$



$$\varphi = 90°$$

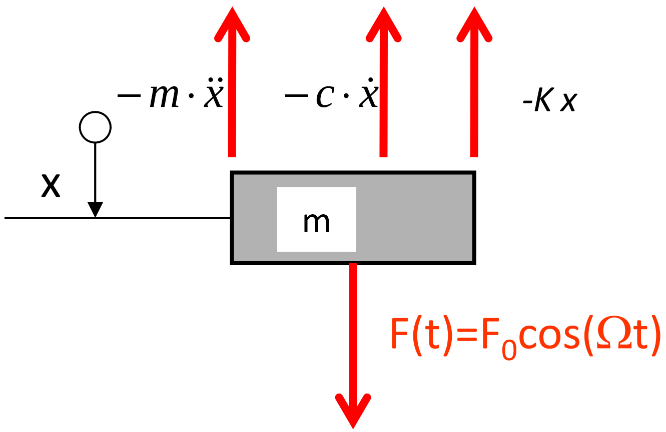
$$L = \pi F_0 X$$



$$\varphi = 45°$$

$$L = \pi F_0 X \frac{\sqrt{2}}{2}$$

LAVORO DI UNA FORZA ARMONICA IN UN CICLO



$$x(t) = X\cdot\cos(\Omega t-\varphi)$$

$$\dot{x}(t) = -\Omega X\cdot\sin(\Omega t-\varphi)$$

$$\ddot{x}(t) = -\Omega^2 X\cdot\cos(\Omega t-\varphi)$$

$$Forza\,elastica\;\;molla = -kx \rightarrow \;fase\;con\;x(t) = 180° \rightarrow L_k = 0$$

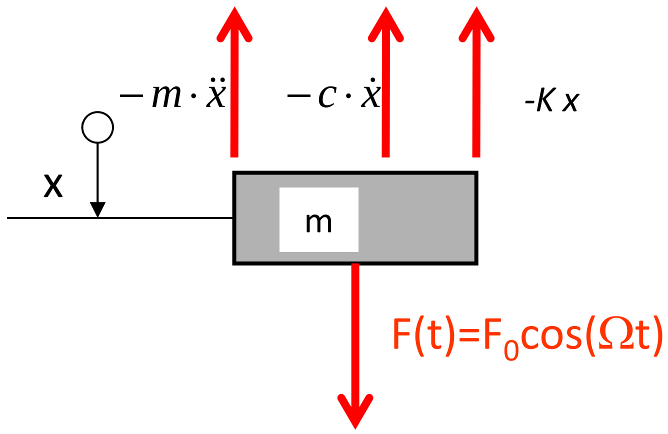
$$Forza\,smorzatore = -c\dot{x} \rightarrow \;fase\;con\;x(t) = 270° \rightarrow L_c = \pi c\Omega X^2$$

$$Forza\,inerzia = -m\ddot{x} \rightarrow \;fase\;con\;x(t) = 0° \rightarrow L_i = 0$$

⇓

$$Lavoro\;\;F\;\;esterna = \pi F_0 X\sin(\varphi) = L_c$$

LAVORO DI UNA FORZA ARMONICA IN UN CICLO



$$\pi F_0 X\sin(\varphi) = \pi c\Omega X^2$$

$$X = \frac{F_0}{c\Omega}\sin(\varphi)$$

$$Se\ \Omega = \omega_n \rightarrow \varphi = 90°$$

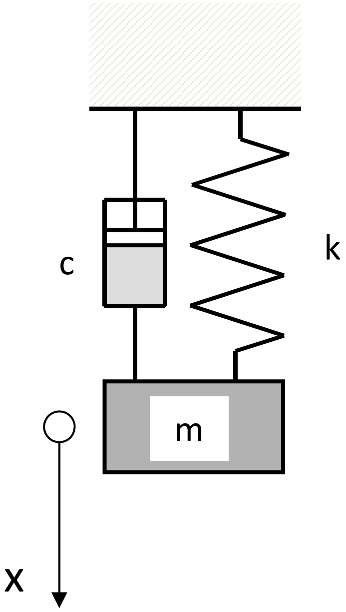
$$X = \frac{F_0}{c\omega_n} = \frac{F_0}{c\sqrt{\frac{k}{m}}} = \frac{F_0}{\frac{c\cdot 2\cdot k}{2\sqrt{km}}} = \frac{F_0}{k}\frac{1}{2\xi}$$

Da soluzione generale

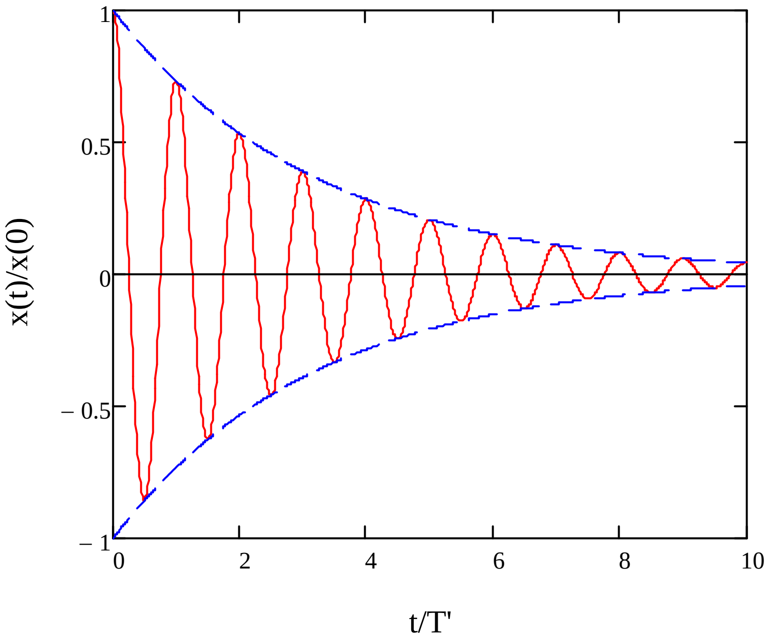
$$X = \frac{F_0}{k}\frac{1}{\sqrt{\left(1-\frac{\Omega^2}{\omega_n^2}\right)^2+\left(2\xi\frac{\Omega}{\omega_n}\right)^2}} =$$

$$= \frac{F_0}{k}\frac{1}{2\xi}$$

DETERMINAZIONE SPERIMENTALE DELLO SMORZAMENTO RELATIVO
METODO DEL DECREMENTO LOGARITMICO



Si basa sull'andamento delle ampiezze di oscillazione rilevate sulla struttura, in seguito ad una perturbazione iniziale.



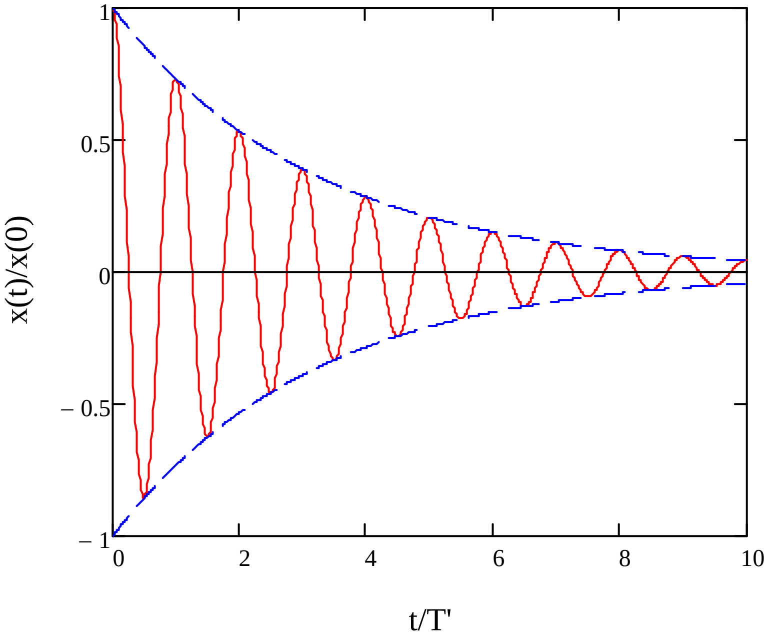
$$x(t) = e^{-\xi\omega_n t}\left(A\cos(\omega_s t) + B\sin(\omega_s t)\right)$$

DETERMINAZIONE SPERIMENTALE DELLO SMORZAMENTO RELATIVO
METODO DEL DECREMENTO LOGARITMICO

Rapporto di ampiezza tra due picchi successivi

$$T' = \frac{2\pi}{\omega_s}$$

$$R = \frac{e^{-\xi\omega_n t}\left(A\cos(\omega_s t) + B\sin(\omega_s t)\right)}{e^{-\xi\omega_n (t+T')}\left(A\cos(\omega_s (t+T')) + B\sin(\omega_s (t+T'))\right)} = \frac{e^{-\xi\omega_n t}}{e^{-\xi\omega_n (t+T')}}$$



Decremento Logaritmico

$$\delta = \ln\left(\frac{e^{-\xi\omega_n t}}{e^{-\xi\omega_n (t+T')}}\right) = \xi\omega_n T' = \xi\omega_n \frac{2\pi}{\omega_s}$$

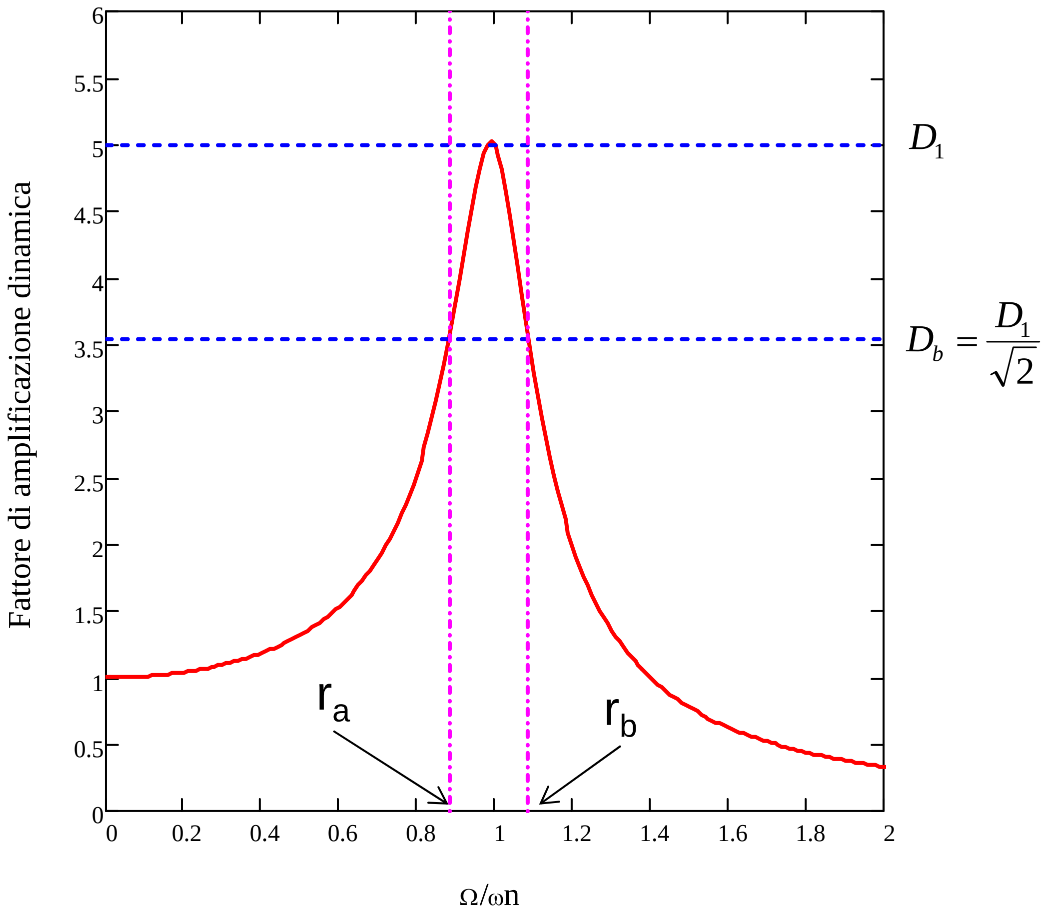
$$= \xi\omega_n \frac{2\pi}{\omega_n\sqrt{1-\xi^2}} = \frac{2\pi\xi}{\sqrt{1-\xi^2}}$$

$$\Downarrow$$

$$\xi = \frac{\delta}{\sqrt{4\pi + \delta^2}}$$

DETERMINAZIONE SPERIMENTALE DELLO SMORZAMENTO RELATIVO
METODO DELLA LARGHEZZA DI BANDA

Si basa sull'andamento del coefficiente di amplificazione dinamica del sistema al variare della frequenza della forzante.



DETERMINAZIONE SPERIMENTALE DELLO SMORZAMENTO RELATIVO
METODO DELLA LARGHEZZA DI BANDA

Calcolo di r_a ed r_b

$$D_1 = \frac{1}{2\xi}$$

$$D_b = \frac{D_1}{\sqrt{2}} = \frac{1}{2\sqrt{2}\xi}$$

$$r = \frac{\Omega}{\omega_n}$$

$$D = \frac{1}{\sqrt{\left(1-r^2\right)^2 + \left(2\xi r\right)^2}}$$

$$\frac{1}{\sqrt{\left(1-r^2\right)^2 + 4\xi^2 r^2}} = \frac{1}{2\sqrt{2}\xi}$$

Elevando al quadrato

$$\frac{1}{\left(1-r^2\right)^2 + 4\xi^2 r^2} = \frac{1}{8\xi^2}$$

$$r^4 + 2r^2\left(2\xi^2 - 1\right) + \left(1 - 8\xi^2\right) = 0$$

$$r_{a,b}^2 = 1 - 2\xi^2 \pm \sqrt{\left(2\xi^2 - 1\right)^2 - \left(1 - 8\xi^2\right)} = 1 - 2\xi^2 \pm \sqrt{\left(4\xi^4 - 4\xi^2 + 1\right) - \left(1 - 8\xi^2\right)} = 1 - 2\xi^2 \pm 2\xi\sqrt{1 + \xi^2}$$

DETERMINAZIONE SPERIMENTALE DELLO SMORZAMENTO RELATIVO
METODO DELLA LARGHEZZA DI BANDA

$$r_{a,b}^2 = 1 - 2\xi^2 \pm 2\xi\sqrt{1+\xi^2}$$

Per ξ<<1

$$r_{a,b}^2 \approx 1 - 2\xi^2 \pm 2\xi$$

$$r_{a,b} = \sqrt{1 - 2\xi^2 \pm 2\xi}$$

Per x<<1 si può porre

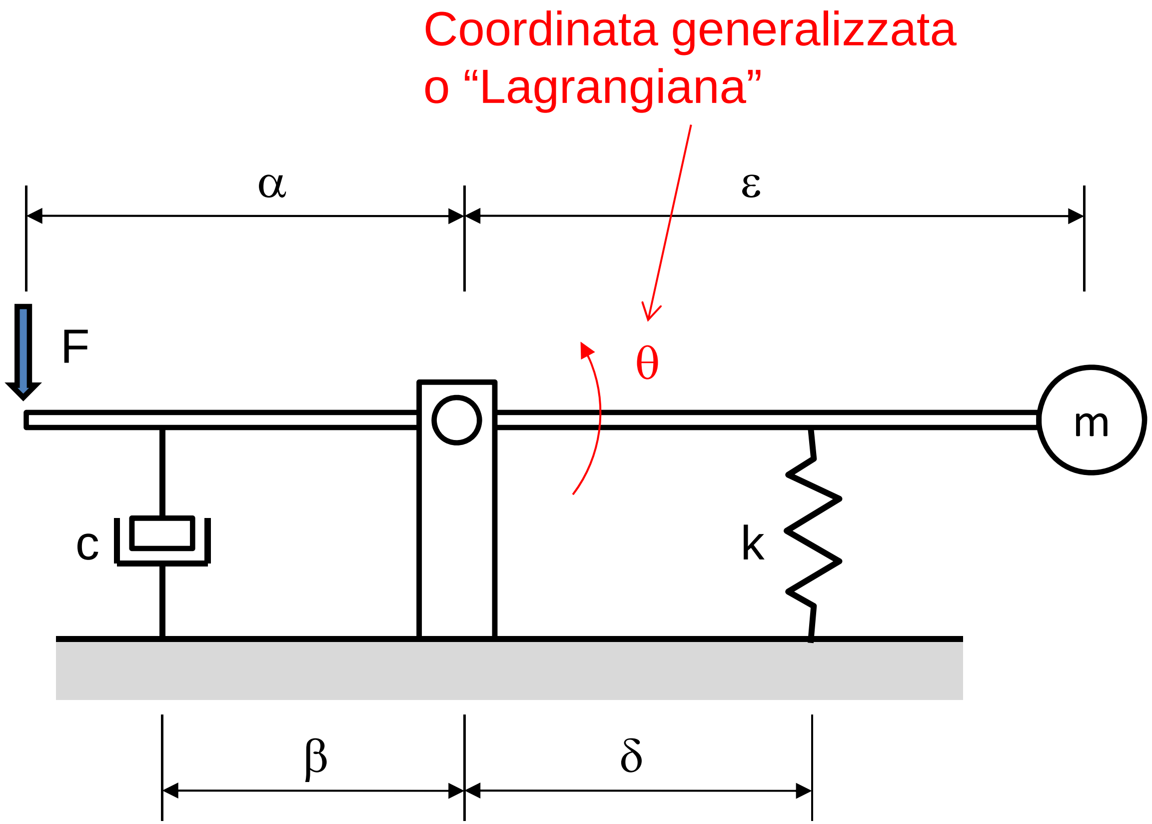
$$\sqrt{1+x} \approx 1 + \frac{x}{2} + \dots \quad \Rightarrow \quad r_{a,b} \approx 1 - \xi^2 \pm \xi$$

$$r_a = 1 - \xi^2 - \xi \qquad r_b = 1 - \xi^2 + \xi$$

$$\xi \approx \frac{r_b - r_a}{2}$$

RIDUZIONE DI SISTEMI COMPLESSI AD UN SISTEMA MASSA-MOLLA-SMORZATORE EQUIVALENTE - SISTEMI DI CORPI RIGIDI AD 1 GDL

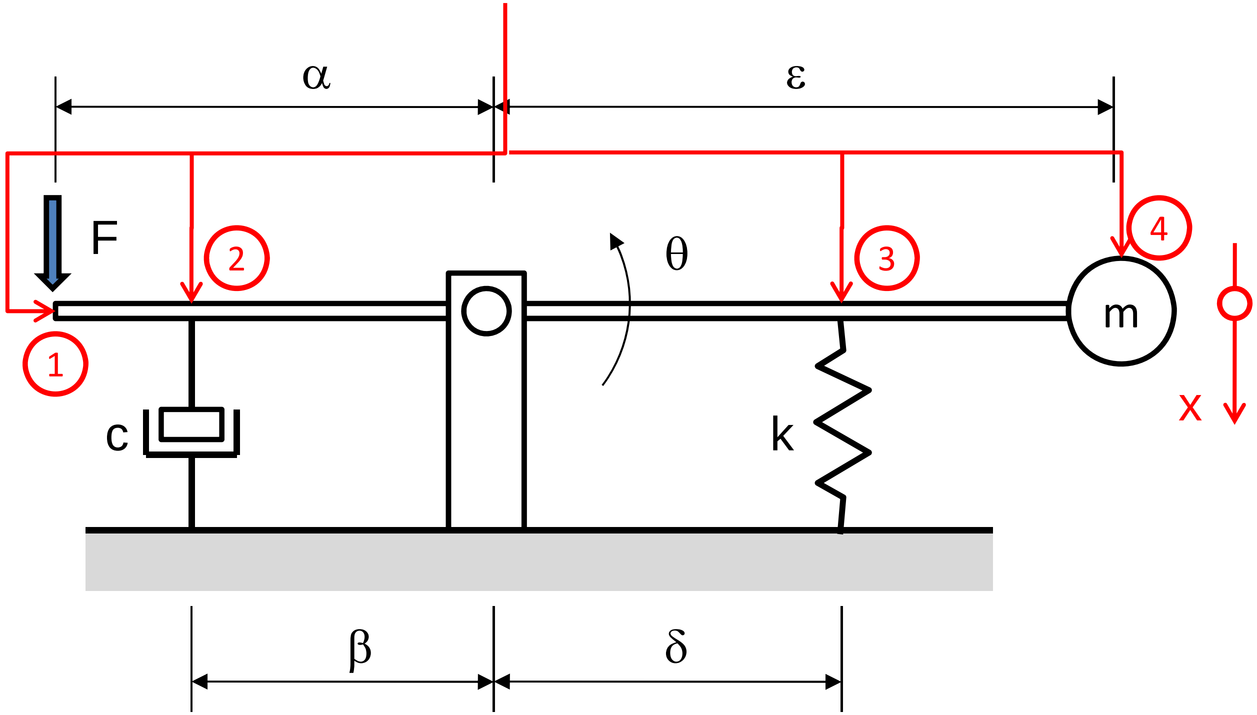
Dato un sistema meccanico formato da corpi rigidi, uniti a masse concentrate, molle e smorzatori, il cui moto sia rappresentabile con il valore di una sola grandezza.



RIDUZIONE DI SISTEMI COMPLESSI AD UN SISTEMA MASSA-MOLLA-SMORZATORE EQUIVALENTE - SISTEMI DI CORPI RIGIDI AD 1 GDL

Rappresentazione degli spostamenti

Punti "significativi" del sistema



$$\{x\} = \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{Bmatrix}$$

$$\{x\} \Leftrightarrow \vartheta \ ?$$

$$\begin{cases} x_1 = \alpha \cdot \vartheta \\ x_2 = \beta \cdot \vartheta \\ x_3 = -\delta \cdot \vartheta \\ x_4 = -\varepsilon \cdot \vartheta \end{cases}$$

$$\{x\} = [d]\vartheta$$

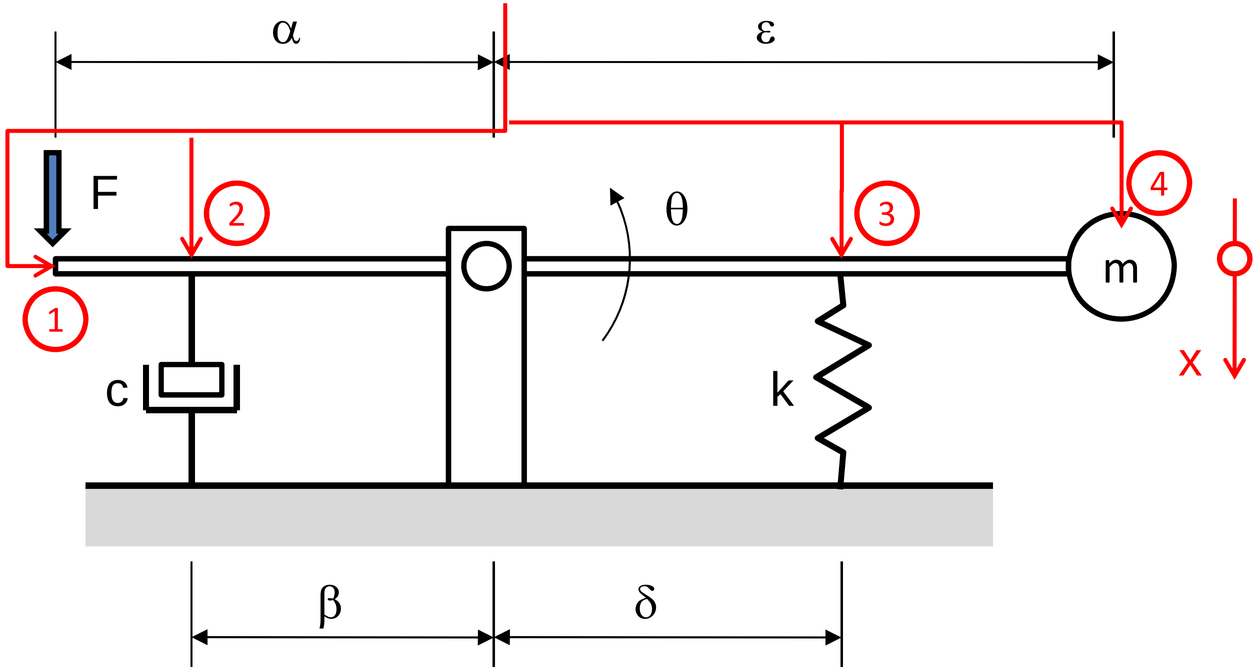
$$[d] = \begin{bmatrix} \alpha \\ \beta \\ -\delta \\ -\varepsilon \end{bmatrix}$$

RIDUZIONE DI SISTEMI COMPLESSI AD UN SISTEMA MASSA-MOLLA-SMORZATORE EQUIVALENTE - SISTEMI DI CORPI RIGIDI AD 1 GDL

Punti "significativi" del sistema

"Riduzione" delle forze

$$\{f\} = \begin{Bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{Bmatrix} = \begin{Bmatrix} F \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$



Lavoro forze effettive x spost. effettivi = Lavoro forza ridotta x coord. lagrangiana

$$Q \cdot \vartheta = \{x\}^T \{f\} \qquad \{x\} = [d]\vartheta$$

$$Q \cdot \vartheta = \vartheta [d]^T \{f\}$$

$$Q = [d]^T \{f\}$$

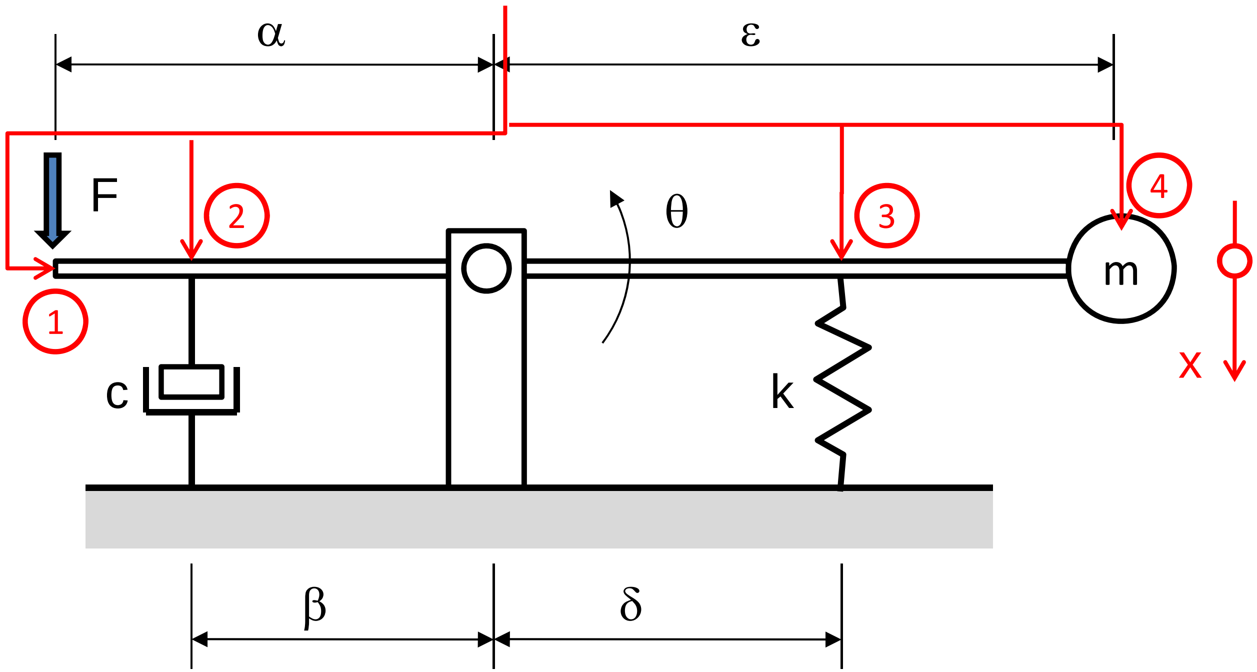
RIDUZIONE DI SISTEMI COMPLESSI AD UN SISTEMA MASSA-MOLLA-SMORZATORE EQUIVALENTE - SISTEMI DI CORPI RIGIDI AD 1 GDL

Punti "significativi" del sistema

"Riduzione" delle rigidezze

$$[K]=\begin{bmatrix}k_{11} & k_{12} & k_{13} & k_{14}\\ k_{21} & k_{22} & k_{23} & k_{24}\\ k_{31} & k_{32} & k_{33} & k_{34}\\ k_{41} & k_{42} & k_{43} & k_{44}\end{bmatrix}=$$

$$=\begin{bmatrix}0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & k & 0\\ 0 & 0 & 0 & 0\end{bmatrix}$$



Lavoro forze effettive x spost. effettivi = Lavoro forza ridotta x coord. lagrangiana

$$k^* \cdot \vartheta \cdot \vartheta = \{x\}^T [k] \{x\}$$

$$\{x\} = \{d\} \vartheta$$

$$k^* \cdot \vartheta \cdot \vartheta = \vartheta \{d\}^T [K] \{d\} \vartheta$$

$$k^* = \{d\}^T [K] \{d\}$$

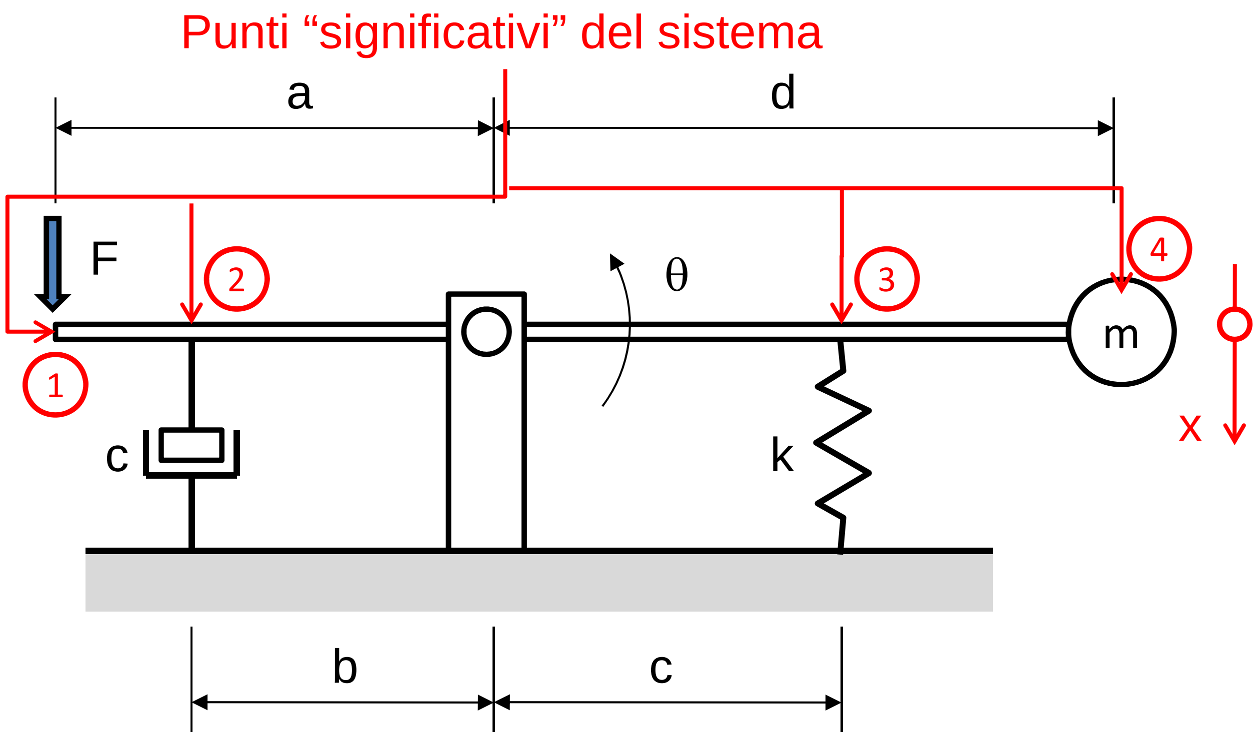
$$k^* = \{\alpha \quad \beta \quad -\delta \quad -\varepsilon\}^T \begin{bmatrix}0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & k & 0\\ 0 & 0 & 0 & 0\end{bmatrix} \begin{Bmatrix}\alpha\\ \beta\\ -\delta\\ -\varepsilon\end{Bmatrix} = \delta^2 k$$

RIDUZIONE DI SISTEMI COMPLESSI AD UN SISTEMA MASSA-MOLLA-SMORZATORE EQUIVALENTE - SISTEMI DI CORPI RIGIDI AD 1 GDL

“Riduzione” delle masse e degli smorzamenti

$$m^* = \{d\}^T [M] \{d\}$$

$$c^* = \{d\}^T [C] \{d\}$$



Equazione di equilibrio dinamico del sistema ridotto

$$m^* \ddot{\vartheta} + c^* \dot{\vartheta} + k^* \vartheta = Q$$