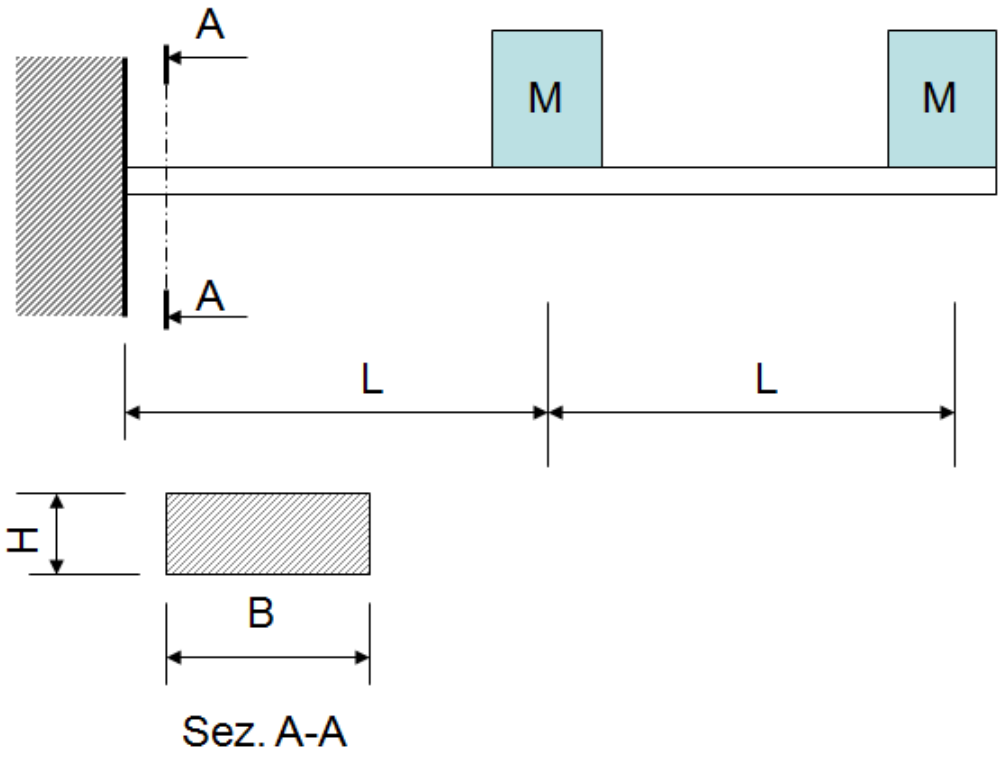
CORSO DI LAUREA MAGISTRALE IN INGEGNERIA MECCANICA
CORSO DI COSTRUZIONE DI MACCHINE

CALCOLO MODI PROPRI DI SISTEMA A 2 GDL

Si calcolino i modi propri del sistema mostrato nella Figura, ipotizzando trascurabile la massa della trave.



Dati

$L_0 := 1 \cdot m$ $M_0 := 5 \cdot kg$ $E := 2.1 \cdot 10^{11} \cdot Pa$ $\upsilon := 0.3$

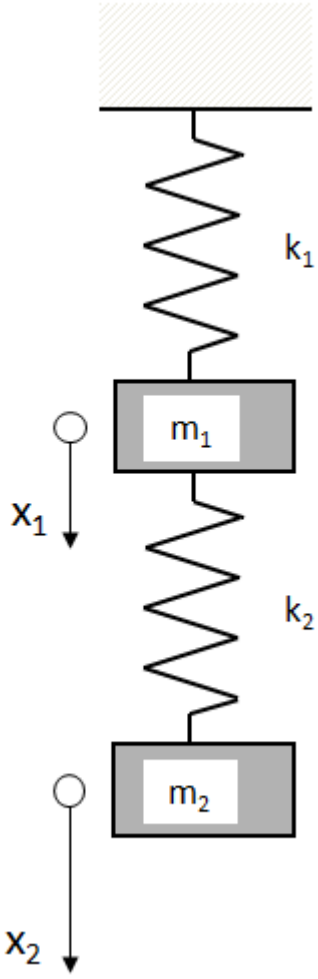
$H := 0.01 \cdot m$ $B := 0.05 \cdot m$

Calcolo parametri geometrici sezione

$$A_0 := H \cdot B = 5 \times 10^{-4}\,m^2 \qquad J_x := \frac{B \cdot H^3}{12} = 4.167 \times 10^{-9}\,m^4$$

Modellazione dinamica

Si vuole costruire un modello di sistema dinamico a 2 gdl equivalente, del tipo mostrato nella Figura



$$C_{11} := \frac{L_0^{\,3}}{3 \cdot E \cdot J_x} = 3.81 \times 10^{-4} \cdot \frac{m}{N} \qquad C_{22} := \frac{(2 \cdot L_0)^3}{3 \cdot E \cdot J_x} = 3.048 \times 10^{-3} \cdot \frac{m}{N}$$

$$C_{21} := \int_0^{L_0} \frac{(2 \cdot L_0 - \xi) \cdot (L_0 - \xi)}{E \cdot J_x} \, d\xi = 9.524 \times 10^{-4} \cdot \frac{m}{N}$$

$$C_{12} := C_{21}$$

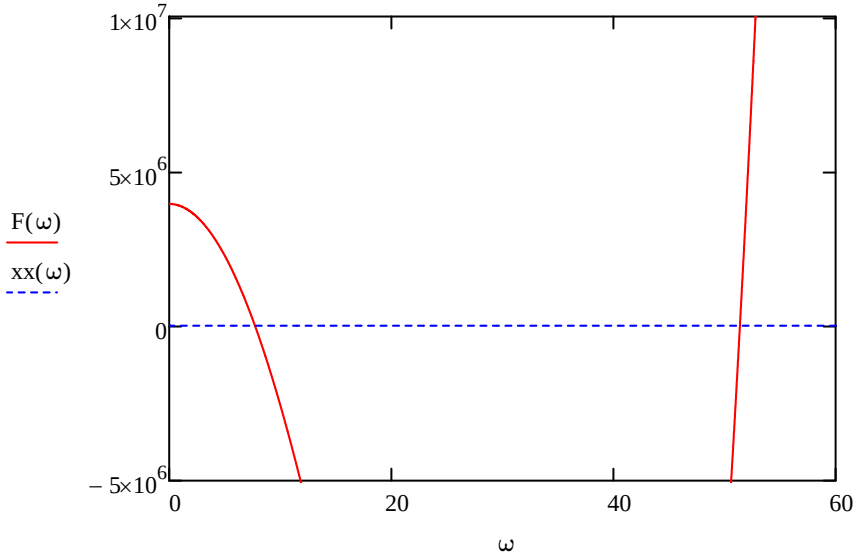
$$C := \begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{pmatrix}$$

$$K := C^{-1} = \begin{pmatrix} 1.2 \times 10^{4} & -3.75 \times 10^{3} \\ -3.75 \times 10^{3} & 1.5 \times 10^{3} \end{pmatrix} \cdot \frac{N}{m}$$

$$M := \begin{pmatrix} M_0 & 0 \\ 0 & M_0 \end{pmatrix} = \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix} kg$$

$$\omega := 0 \cdot \frac{1}{s}, 0.01 \frac{1}{s} .. 100 \cdot \frac{1}{s}$$

$$F(\omega) := \left(K_{1,1} - \omega^2 M_{1,1}\right) \cdot \left(K_{2,2} - \omega^2 \cdot M_{2,2}\right) - K_{1,2} \cdot K_{2,1} \qquad xx(\omega) :=$$



$$\omega := 5 \cdot \frac{1}{s}$$

$\omega_1 := \text{root}(F(\omega), \omega)$ $\qquad \omega_1 = 7.723 \frac{1}{s}$

$$\omega := 51 \cdot \frac{1}{s}$$

$\omega_2 := \text{root}(F(\omega), \omega)$ $\qquad \omega_2 = 51.384 \frac{1}{s}$

$$r_1 := -\left[\frac{\left(K_{1,1} - \omega_1^{\,2} M_{1,1}\right)}{K_{1,2}}\right] = 3.12$$

$$r_2 := -\left[\frac{\left(K_{1,1} - \omega_2^{\,2} M_{1,1}\right)}{K_{1,2}}\right] = -0.32$$